APPROXIMATION OF BEST SEPARATORS BY EIGENFUNCTIONS OF $P-$LAPLACIANS

K. GÄRTNER*

Abstract. Partitioning a graph with n vertices into two parts with minimal deviation from equal size and a minimal number of cut edges can be understood as looking at the geometric problem of dividing some domain Ω into two regions of equal measure by a surface Γ with possibly small surface measure in some non Euclidian metric. In [1] the geometric problem is related to minimizing the functional

$$F_1(u) = \frac{\int_\Omega |Du| \, dV}{||u||_1} \to \min, \; u \in BV, \; \int_\Omega \operatorname{sign} u \, dx = 0.$$

F_1 is still unpleasant for algorithmic purposes, hence the problem is replaced by

$$F_p(u) = \frac{||\nabla u||_p^p}{||u||_p^p} \to \min, \; u \in H^{1,p}, \; \int_\Omega |u|^{p-2} u \, dV = 0, \; u \neq 0, \; p \in (1,2].$$

Minimizers $u \in H^{1,p}(\Omega)$ satisfy necessarily the Euler Lagrange equation

$$-\nabla \cdot (|\nabla u|^{p-2} \nabla u) = F_p(u)|u|^{p-2} u \text{ in } \Omega, \; \nu \cdot \nabla u = 0 \text{ on } \partial\Omega,$$

ν outer normal (see [1] for convergence results). This can be seen as a nonlinear eigenvalue problem, which can be solved by a steepest descent or Newton's method for fixed $p > 1$ and embedding starting form $p = 2$. The case $p = 2$ is the usual spectral bisection, minimizing the L^2-norm of the gradient but not fulfilling $\int_\Omega \operatorname{sign} u \, dx = 0$. The figure shows a grid and a separator approximation. The main

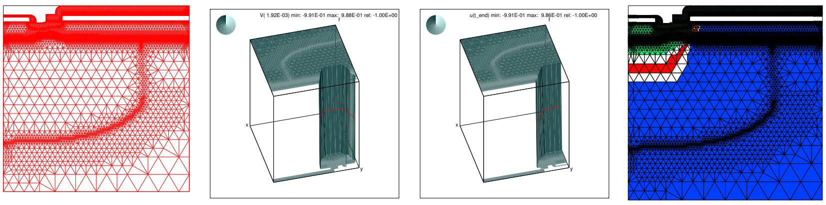

FIG. 0.1. *The grid (left), initial guess and solution for $p = 1.001$, grid with separator (right, red: triangles with sign change). Number of cut edges metis 88, p=1.03: 86, p=1.01: 76, p=1.003: 71, p=1.001: 70.*

practical interest in these computational expensive separators is seen in comparing heuristic methods in cases without obvious symmetry to best approximations. Another question is how much weight is given to minimal operation count and to load balance in case of non convex 'domains'. Remark: Phase separation models minimizing 'surface energy' in the limit (see [2]) are well suited to approximate the total minimal surface for devinding a domain (or graph) into n partitions with specified measures m_i, $i = 1, \ldots, n$.

[1] GAJEWSKI, H., GÄRTNER, K. *Domain separation by means of sign changing eigenfunctions of p-Laplacians*, Appl. Anal. 79, 483-501 (2001).
[2] GAJEWSKI, H., GRIEPENTROG, J. A. *A descent method for the free energy of multicomponent systems*, Discrete and Continuous Dynamical Systems 15, 505-528 (2006).

*Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstr. 39, 10117 Berlin, Germany (gaertner@wias-berlin.de).