Parallel multifrontal methods and their elimination trees

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We discuss our experience with multifrontal methods, and how their behaviour can be impacted by the shapes of the trees associated with the factorization of sparse matrices. Multifrontal methods are a particular class of direct methods aiming at solving sparse systems of linear equations using an elimination (or assembly) tree. They are such that, at each node of the tree, a partial factorization of a dense matrix is computed, producing a Schur complement which must then be stored in memory before being later used at the parent level. We consider three aspects:
(i) the characteristics of the trees resulting from various ordering techniques (including nested dissection, but also local heuristics), and their impact on the memory usage and parallelism of multifrontal methods;
(ii) how appropriate tree traversals may reduce the memory usage in serial and parallel multifrontal methods;
(iii) the impact of tree shapes on the amount of memory and flops compression when using low rank representations to compress the dense matrices arising in multifrontal solvers.
The talk will be illustrated by experiments using the sparse direct solver MUMPS with internal and external ordering packages.