Building O(N) linear solvers using nested dissection

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In the pioneering work of George and others, it was shown how nested dissection can be used to optimally order the unknowns during the Gaussian elimination. Despite this, in 2D for example the computational cost for a simple structured grid with N nodes remains N^(3/2). In 3D this goes up to N^2. Is it possible to reduce the computational cost further to O(N)? One approach is to consider the factorization of the dense blocks that result from the elimination. For example, in 2D, the final dense block (Schur complement) has size N^(1/2). We will present linear solvers for dense matrices that scale like O(N) by using the low-rank hierarchical structure of the matrix (exploiting the low-rank property of certain off-diagonal blocks). These types of methods apply for example when solving elliptic partial differential equations. If these methods are used to factor the dense blocks in nested dissection (for example the last block of size N^(1/2)) we can achieve a solver with an overall computational cost of O(N).