Level set methods to find separators

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Abstract

Our object is to find a vertex separator. Given a graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ are vertices and $\mathcal{E}$ are edges, we want to partition the vertices $\mathcal{V} = \mathcal{B} \sqcup \mathcal{S} \sqcup \mathcal{W}$ where $\mathcal{S}$ separates $\mathcal{B}$ from $\mathcal{W}$. The boundaries of $\mathcal{B}$ and $\mathcal{W}$, nodes adjacent to but not included in $\mathcal{B}$ and $\mathcal{W}$, are subsets of $\mathcal{S}$. Our goal is to find a partition where the weight of $\mathcal{S}$ is small and the weights of $\mathcal{B}$ and $\mathcal{W}$ are close.

Level sets can be used effectively to find separators. Given a source node s, we find the distance between s and every other node, $\mathsf{dist}(u, s)$. One or two source nodes can be used to find separators. Here is a comparison.

Level Sets	**Half-Level Sets**
• one source node s	• two source nodes s and t
• $\phi(u) = \mathsf{dist}(u, s)$	• $\psi(u) = \mathsf{dist}(u, s) - \mathsf{dist}(u, t)$
• $\phi : \mathcal{V} \mapsto \{0, 1, \ldots, r\}$	• $\psi : \mathcal{V} \mapsto \{-\mathsf{dist}(s, t), \ldots, \mathsf{dist}(s, t)\}$.
• $0 \le \lvert\phi(u) - \phi(v)\rvert \le 1$ for $(u, v) \in \mathcal{E}$	• $0 \le \lvert\psi(u) - \psi(v)\rvert \le 2$ for $(u, v) \in \mathcal{E}$
• reduced matrix is tridiagonal	• reduced matrix is pentadiagonal
• $S = \phi^{-1}(k)$ for some $0 < k < r$	• $S = \psi^{-1}(k) \sqcup \psi^{-1}(k+1)$ for some $0 < k < k+1 < r$
• S is minimal	• S may not be minimal

GENAND from George and Liu (1981) uses "Level Sets" on the left. Our new contribution uses "Half-Level Sets" on the right.

Half-level sets have advantages. Their separators are more "straight" or "planar" than the curved separators from single source level sets. Their non-minimal property does require a second step to smooth the separator into a minimal separator. This can be done with a simple greedy "trimming" method, or by solving a max flow problem to find minimum weight separators.

We improve a partition by exploring a nearby space in search of better separators. Level sets and half-level sets can be used to determine these nearby "wide" separators, and form the basis for trimming and max flow algorithms to find better separators.

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