Imre Simon at Waterloo: 1969–72

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Master of Mathematics: 1969-1970

- Imre had some experience with software design
- but was eager to work on mathematical problems in CS
- I had many difficult problems at that time

Problems I was presenting in graduate courses:

- star height
- generalized star height
- limited languages
- star-free languages
- non-counting languages
- dot-depth hierarchy
- locally testable languages

Star-free languages

- regular languages: smallest class containing finite languages and closed under union, concatenation, and star
- regular languages are also closed under complement, so we could also define:
- regular languages: smallest class containing finite languages and closed under boolean operations, concatenation, and star
- star-free languages smallest class containing finite languages and closed under boolean operations and concatenation
  - feedback-free circuits
  - permutation-free automata
  - group-free semigroups
  - many other nice characterizations

Non-counting languages

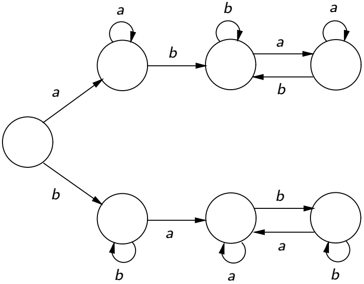
- L is non-counting of order n if

$$xy^n z \in L \Leftrightarrow xy^{n+1} z \in L$$

- L is non-counting if there exists an n such that L is non-counting of order n
- every star-free language is non-counting
- non-counting languages are not necessarily regular

Non-counting languages of order 1 over $\Sigma = \{a, b\}$

Here we have $x = x^2$, that is we have idempotent monoids
For two letters there are only 7 classes



So the case $x^2 = x^3$ can't be much more complicated!

Free Burnside semigroups

- satisfy $x^n = x^{n+m}$
- Thue: infinite
  - if $n \geq 2$, $m \geq 1$ and at least three generators
  - if $n \geq 3$, $m \geq 1$ and two generators
- the case $x^2 = x^3$ is the hardest one - Imre's Master's essay!
- Conjecture: each congruence class of this relation is regular

- I. Simon. Notes on non-counting languages of order 2, 1970.
- briefly in: J. Brzozowski, Open problems about regular languages, In: Formal Language Theory - Perspectives and Open Problems R. V. Book, ed., Academic Press, New York, NY, pp. 23–47, 1980.
- A. Pereira do Lago, I. Simon: Free Burnside semigroups. Theor. Inform. Appl 35, 575–595, 2001.

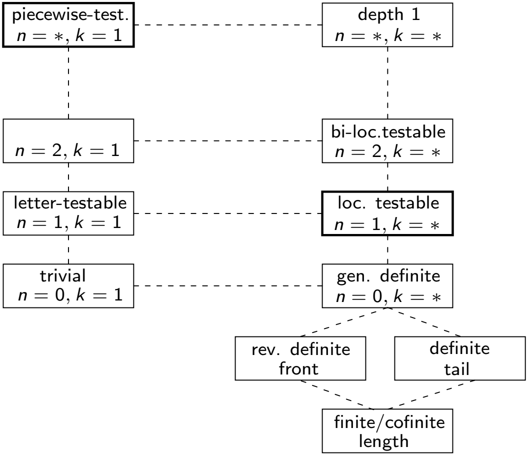
PhD degree: 1970–1972

- Finite/cofinite languages are a boolean algebra $\mathbb{B}_0$
- close $\mathbb{B}_0$ under concatenation; let $\mathbb{M}_1 = \mathbb{B}_0 M$
- close $\mathbb{M}_1$ under boolean operations; let $\mathbb{B}_1 = \mathbb{B}_0 MB$
- let $\mathbb{B}_2 = \mathbb{B}_1 MB$, etc.
- In the limit we get the star-free languages
- a language in $\mathbb{B}_i$ is of dot-depth i
- Imre's PhD thesis was on $\mathbb{B}_1$, languages of dot-depth one

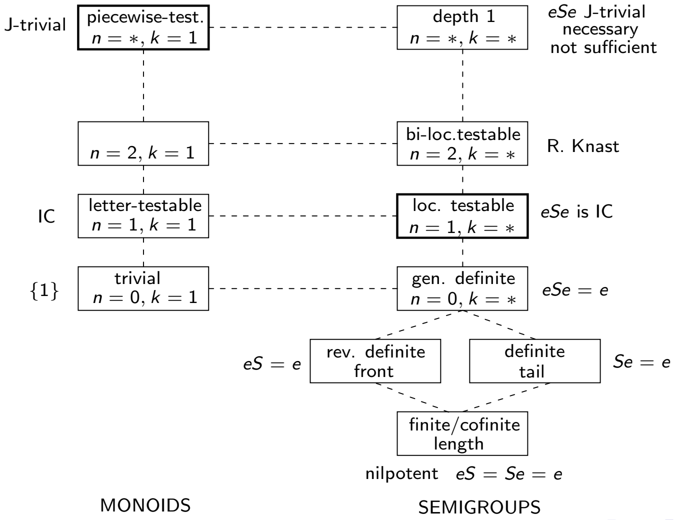
The congruence $\sim_{n,k}$

- $W = (w_1, \ldots w_n)$ $n-$ tuple of words of length k
- W occurs in x, if there are $u_1, \ldots, u_n$, $v_1, \ldots, v_n$ such that $|u_1| < |u_2| < \cdots < |u_n|$ and $x = u_i w_i v_i$
- Example: $x = ababb$. The pairs of length 2 occurring in x are $(ab, ba), (ab, ab), (ab, bb), (ba, ab), (ba, bb)$
- n-tuples of length k: $T_{n,k}(x)$
- front of length k: $f_k(x)$ prefix of length k of x, or x if $|x| < k$
- tail of length k: $t_k(x)$ suffix of length k of x, or x if $|x| < k$
- $x \sim_{n,k} y$ iff $f_{k-1}(x) = f_{k-1}(y)$, $t_{k-1}(x) = t_{k-1}(y)$, $T_{n,k}(x) = T_{n,k}(y)$
- L is of depth 1 iff it is a $\sim_{n,k}$ language

Simon's depth-1 hierarchies



Semigroups and monoids



Locally testable and piecewise-testable results

- J.A. Brzozowski and I. Simon, "Characterizations of Locally Testable Events," pp. 166–176 in IEEE Computer Society Conference Record of *1971 Twelfth Annual Symposium on Switching and Automata Theory*.
- J.A. Brzozowski and I. Simon, "Characterizations of Locally Testable Events," *Discrete Mathematics*, Vol. 4, No. 3, pp. 243–271, March 1973.
- I. Simon, Piecewise Testable Events. In: 2nd GI Conference, 1975, Kaiserslautern. Automata Theory and Formal languages. Berlin : Springer-Verlag, 1975. p. 214-222.

Imre's Livre-dôcencia (habilitation): 1978

- limited languages: there exists n such that $(\{\varepsilon\} \cup L)^n = L^*$
- Imre solved it using tropical semirings
- Imre made important contributions to this theory
- I. Simon . Limited subsets of a free monoid. In: Symposium on Foundations of Computer Science, 1978, Ann Arbor. Proceedings of the 19th Annual Symposium on Foundations of Computer Science, 1978. v. 19. p. 143-150.

Imre as a person

- my sabbatical in São Paolo in 1983
- A visit to Imre's ranch
- Imre liked to eat and drink well
- Imre as a coffee drinker
- Imre as my teacher of Portuguese