Errata

Automatic Sequences: Theory, Applications, Generalizations

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Page xiii: Replace "introduced in Chapter 5" with "introduced in Chapter 4". (Joel Noche, December 6 2011)

Page xv: The correct address for the errata page is
https://cs.uwaterloo.ca/~shallit/asas.html .
(P. Stockmeyer, January 27 2005; JOS, March 31 2020)

Page 5: In line 12, replace the first k with $-k$. (D. Pal, September 21 2006)

Page 6: The correct definition for D should be

$$D = \{D_{j,a} : j \geq 0, a \in \Sigma\}.$$

(P. Stockmeyer, January 27 2005)

Page 5: Insert $\cdots$ before a_{k-2} in the rhs of line 13. (Joel Noche, December 6 2011)

Page 6: In line -8, the domain of d should be $A \times A$, not A. (D. Pal, September 21 2006)

Page 22: In line 11, remove the P. (Kalle Saari, September 5 2006)

Page 24: In Line 1, the definition of critical exponent is correct, but poorly worded. It should read that "e is defined to be the sup, over all $\alpha > 1$, such that $\mathbf{w}$ contains an α-power". (V. Linek, May 19 2004)

Page 24: In Exercise 2, add the requirement that the set S be closed. (Kalle Saari, November 5 2004)

Page 28: In exercise 48 (b), replace φ with μ. (Joel Noche, December 6 2011)

Page 29: Exercise 57 is phrased poorly. It means that the maximum is to be taken over all words of length n. (Kalle Saari, September 5 2006)

Page 31: In Open Problem 12 (c), replace "infinitely many indices" with "infinitely many even indices". (Kalle Saari, September 5 2006)

Page 33: In line -14, replace the second big-O with Ω. (Kalle Saari, September 5 2006)

Page 33: In line 13, change "weakly cubefree" to "weakly squarefree". (Narad Rampersad, April 12 2004)

Page 42: In line -6, replace $a_1 p$ with $a_1 p q^{n-1}$ and a_0 with $a_0 q^n$. (Kalle Saari, September 5 2006)

Page 44: In Section 2.4, we should have proved that continued fraction expansions of real numbers are essentially unique (so that the statement on page 61, line -12 would be true). (Kalle Saari, September 5 2006)

Page 46: The term "partial quotient" is used for the first time after Theorem 2.4.3, but it was not defined yet. (Dalia Krieger, January 24 2006)

Page 49: In line 4, append "in $[0, 1)$" to the end of the sentence. (Kalle Saari, September 5 2006)

Page 54: In line 13, remove the extra occurrence of "of length". (Kalle Saari, September 5 2006)

Page 55: In line 3, replace "II" with "I". In line 8, replace "I" with "II". (Kalle Saari, September 5 2006)

Page 58: In line 3, the leftmost "2" in the last row of the matrix should be "3". (Kalle Saari, September 5 2006)

Page 62: In the third line from the bottom, replace "where $0 \le a_i < p$ for $i \ge 0$" with "where $0 \le a_i < p$ for $i \ge -m$". (Joseph Meleshko, July 18 2020)

Page 63: In line -10, replace "reminder" with "remind". (Eric Rowland, September 4 2008)

Page 63: In Example 2.11.1, last line, the second $0 \cdot 3^5$ should be $0 \cdot 3^6$. (Joel Noche, December 6 2011)

Page 69: In line 3, replace "2.11" with "2.13.5". (Kalle Saari, September 5 2006)

Page 84: In line -8, the notation for suffix "$\triangleright$" was introduced but not defined. (Kalle Saari, September 5 2006)

Page 86: If line -10, replace $k^{-j}[x_n]$ with $k^{-n}[x_n]$. (Kalle Saari, September 5 2006)

Page 99: In line -2, in the statement of Theorem 3.5.6, replace $(-1)^e_{2;11}(n)$ with $(-1)^{e_{2;11}(n)}$. (Kalle Saari, September 5 2006)

Page 114: In exercise 23, the summation should be $\sum_{0 \le i \le n} r_n$. (Andrej Vukovic, March 31 2020)

Page 115: In exercise 37, the inequality on a_i should read $0 \le a_i \le i$. (Stepan Holub, October 23 2012)

Page 121: In line 10, we attribute to D. McIlroy the connection between $S(n)$ and merging algorithms. However, this connection was noticed earlier by D. E. Knuth, Mathematical Analysis of Algorithms, *Proceedings of IFIP Congress 71*, North-Holland, 1971, pp. 19–27;

see pp. 23–24. This paper is also reprinted in *Selected Papers on Analysis of Algorithms*, Center for the Study of Language and Information, 2000, pp. 1–18. (Donald Knuth, August 19 2006)

Page 124: In line 18, replace "odd than even" with "even than odd". (Kalle Saari, September 5 2006)

Page 135: In the last centered display, the $B_{qq'}$ needs a complement bar over it. (Joel Seiferas, February 18 2014)

Page 140: In line 5, replace "is is" with "is". (Eric Rowland, September 19 2008)

Page 147: In line −15, replace "all inputs" with "for all inputs". (Eric Rowland, September 4 2008)

Page 155: In the table in the center of the page, the entry for 14 has an unneeded +. (Kalle Saari, September 5 2006)

Page 161: In the proof of Theorem 5.2.7, the implication arrows that introduce both parts of the proof are in the wrong direction. (Kalle Saari, September 5 2006)

Page 162: Delete the word "the" before "each of the fibers" in the proof of Theorem 5.3.2. (Olga Miltchman, December 1 2007)

Pages 164–165: The transducers presented in Figures 5.14 and 5.15 are completely wrong. In general you need $k + \ell + 1$ states. (Dominique Perrin, March 23 2023)

Page 165: In the proof of Theorem 5.4.2, we should specify that $q_0 = 0$ and in the last displayed line we should replace q with q_0. (Afshin Amini, January 24 2014)

Page 165: In the proof of Theorem 5.4.3, we should specify that ρ is a map from Γ to Δ, and in the definition of the DFAO M', replace Γ with Δ. (Afshin Amini, January 13 2014)

Page 166: In line 5 of Example 5.5.1, replace "$y = 2^{2m+1}$" with "$y = 2^{2m+1} - 1$". (Kalle Saari, September 5 2006)

Page 166: In line 11, the word "nonautomatic" is misspelled. (Eric Rowland, September 4 2008)

Page 167: In the proof of Corollary 5.5.3, the expression $\overbrace{1 \cdots 1}^{n}$ should be $\overbrace{k-1 \cdots k-1}^{n}$. (Greg Dresden, November 11 2013)

Page 168: In the statement of Theorem 5.6.3, part (d), it should read "$r \in R$", not "$r \in S$". (Narad Rampersad via an anonymous referee, July 7 2005)

Page 169: In the second displayed equation, replace the q with a. Further down, replace "guesses s" with "guessed s". (Eric Rowland, December 20 2011)

Page 171: Ali Aberkane points out that Open problem 1 of section 5.9 is solved, and the critical exponent is 4. This follows from the paper Allouche and Bousquet-Mélou [1994b]

cited on page 483. They prove that the only 4th powers are 0000 and 1111, and there are no 5th powers. It follows that there are no 4^+ powers. (James Currie, February 24 2004)

Page 175: In line -3, change Σ^* to Σ_k^*. (Zaid Shakwet, November 17 2009)

Page 181: In line 8, remove the extra occurrence of the word "that". (Kalle Saari, September 5 2006)

Page 183: In the 2nd line of the proof of Theorem 6.5.4, replace "automatic" with "2-automatic". (Narad Rampersad, July 10 2006)

Page 185: In the last set of displayed equations on this page, each item should be enclosed in $\tau(\cdots)$ instead. (JOS, January 10 2017)

Page 192: In line 9, replace "$\tau'(a)$" with "$\tau(a, q)$". (Kalle Saari, September 5 2006)

Page 195: In line 1, replace "automatic, sequence" with "automatic sequence". (Eric Rowland, December 20 2011)

Page 202: In Exercise 9, correct the first few terms of a_n to be $11264224288\cdots$. (Eric Rowland, April 17 2010)

Page 204: In Exercise 19, the formula in (c) and the two formulas in part (d) are off by a factor of two. Replace the right side of (c) by $2/\pi$; the first in (d) by $2\sqrt{2}/\pi$, and the second by $\sqrt{2}/\pi$. (March 19 2011; August 5 2012)

Page 209: Our discussion of the period-doubling sequence should have mentioned (a) M. Morse and G. A. Hedlund, Symbolic dynamics, *Amer. J. Mathematics* **60** (1938), 815–866, where it appears on p. 844 after the words "Upon reducing..." and (b) M. Garcia and G. A. Hedlund, The structure of minimal sets, *Bull. Amer. Math. Soc.* **54** (1948), 954–964, where it appears under the name β on p. 962.

Page 218: In line -12, replace "$h^t(x)$" with "$h^t(y)$". (Kalle Saari, September 5 2006)

Page 220: Delete line 11 (the one that begins "The following theorem is the first...". (Kalle Saari, September 5 2006)

Page 230: In line 6, replace "$w_1w_2\cdots w_r$" with "$b_1b_2\cdots b_r$". (Kalle Saari, September 5 2006)

Page 244: In exercise 23, replace "nonerasing morphism" with "nontrivial nonerasing morphism". (Tim Smith, November 27 2012)

Page 248: In line 7, replace the period after $0 \to 1$ with comma. (Kalle Saari, September 5 2006)

Page 254: The left side of the equation in line 2 is missing the term $|\lambda^{k-1}|$. (Kalle Saari, September 5 2006)

Page 262: The second part of the statement of Theorem 8.3.12 (b) is missing an hypothesis. In order to conclude that $\lim_{n\to\infty} M^n/r^n$ has rational entries, add the extra hypothesis that r is rational. (D. Krieger, March 10 2006)

Page 262: In line -7, replace $\lim_{n\to\infty}$ with $\lim_{k\to\infty}$. (Kalle Saari, September 5 2006)

Page 265: In line -7, add 1 to the exponents of the two rightmost entries on the second row of the matrix M_k^n. (Kalle Saari, September 5 2006)

Page 267: In line 13, replace 2.3^ℓ in the subscript of the summation with $2 \cdot 3^\ell$. In line -2, replace "The first sum" with "As $n \to \infty$, the first sum". (Kalle Saari, September 5 2006)

Page 268: In Proposition 8.4.4 (b), the displayed equation should be $\lim_{s\to 1^+}(s-1)\sum_{n\geq 1; x_n=a}\frac{1}{n^s}$. (Johannes Morgenbesser, August 2 2010)

Page 268: The proof of Theorem 8.4.5 should have been divided more clearly into parts (a) and (b) (to follow the statement). (Kalle Saari, September 5 2006)

Page 286: In line -11, replace f_α with $\mathbf{f}_\alpha$. (Moshe Schwartz, June 12 2016)

Page 300: The definition for T_x should have an intersection, not a union, and the comment about the union being non-disjoint should be removed. (Michel Rigo, March 9 2005)

Page 303: At the end of line 5, delete $\leq k$. (JOS, February 25 2017)

Page 303: Lemma 10.2.7 is known in the literature as Fekete's lemma; we should have cited M. Fekete, Über die Verteilung der Wurzeln bei gewissen algebraischen Gleichungen mit ganzzahligen Koeffizienten, *Math. Zeitschrift* **17** (1923), 228–249.

Page 306: In line -18, the equation $o_n + o_{n+1} = 4n + 4$ holds for $n \geq 2$, not $n \geq 1$. (Kalle Saari, September 5 2006)

Page 309: In line 6, replace "$u_1 \geq 2$" with "$|u_1| \geq 2$". (Kalle Saari, September 5 2006)

Page 309: In line -12, replace "Exercise 35" by "Exercise 34" twice. (Narad Rampersad, February 22 2007)

Page 311: The proof of Theorem 10.4.12 as given is incorrect. (The problem is that j is not uniquely specified for all words of length n.) It should be replaced with the following simpler proof.
Revised proof of Theorem 10.4.12:

Proof. Let i be the least index such that

$$\min_{a\in\Sigma} |h^{i-1}(a)| \leq n \leq \min_{a\in\Sigma} |h^i(a)|.$$

Since $\mathbf{u} = h(\mathbf{u})$, it follows that $\mathbf{u} = h^i(\mathbf{u})$. Write $\mathbf{u} = u_0u_1u_2\cdots$; then $\mathbf{u} = h^i(u_0)h^i(u_1)\cdots$. Let w be a word of length n. Since each block $h^i(u_j)$ is of length at least n, it follows that either w is contained entirely in such a block, or it straddles exactly two such blocks. Thus every word of length n is contained in a word of the form $h^i(bc)$ where $b, c \in \Sigma$. Hence any such word w of length n is uniquely specified by giving b, c and the position within $h^i(bc)$ where w starts. Using Theorem 1.4.3, it follows that there exists an e, depending only on h, such that $|h^i(bc)| \leq 2W^{2e}\min_{a\in\Sigma}|h^{i-1}(a)| \leq 2W^{2e}n$, where $W = \mathrm{Width}(h)$. Hence $p_{\mathbf{u}}(n) \leq 2k^2W^{2e}n$.

Page 319: In line -6, replace ρ with α. Same typo in line -2. (Kalle Saari, September 5 2006)

Page 327: In Example 10.8.10, the orbit closure should also include the word 0^ω. (JOS, November 16 2011)

Page 329: In the proof of Theorem 10.9.2, part (a), the very last inequality should read $R_{\mathbf{x}}(n+1) \geq t+1$. (Michel Rigo, March 9 2005)

Also, we should have said that we would write R in place of $R_{\mathbf{t}}$ for brevity. (Kalle Saari, September 5 2006)

Page 331: In line 7, the statement beginning "It follows that" is not true. A fix will be forthcoming. (Kalle Saari, September 5 2006)

Page 333: In line 8 replace "such for" with "such that for". (Kalle Saari, September 5 2006)

Page 334: In line -18, replace "for ≥ 2" with "for $n \geq 2$". (Kalle Saari, September 5 2006)

Page 335: In the 4th line of Exercise 10, replace $2^a + 2^{a-1} - b$ with $2^a + 2^{a-1} + b$. (Narad Rampersad, July 10 2006)

Page 339: In Exercise 42, replace "$i \geq 1$" with "$i \geq 0$". (Joel Noche, December 6 2011)

Page 346: Lemma 11.1.2 is incorrect. It needs to be replaced by a different result. For a corrected proof, see M. Rigo and L. Waxweiler, A note on syndeticity, recognizable sets, and Cobham's theorem, *Bull. of the EATCS*, No. 88 (February 2006), 169–173. This is also available at http://www.discmath.ulg.ac.be/papers/coblw.pdf. Also see http://www.tucs.fi/publications/attachment.php?fname=TR713.pdf. (Michel Rigo, April 11 2005; JPA, July 15 2006)

Page 347: In the last line on this page, replace $\mathbb{Z}$ with $\mathbb{N}$. (JOS, February 26 2017)

Page 348: Line 19, replace $0 \leq y < l^j$ with $0 \leq t < l^j$. (JOS, February 26 2017)

Page 350: In Theorem 11.2.2, change the mention of $\mathbf{u}$ in the last line to $\mathbf{s}$. (Brent Bostick, June 29 2011)

Pages 354–359, Section 12.2. As presented, the proof of Christol's theorem is very slightly incorrect. The problem is that in the claim "$G = A(X)/B_0(X)$" on page 358, the resulting G is only guaranteed to be a formal Laurent series and not a formal power series. This was pointed out to us by Joost Winter in April 2015.

One way to fix this is as follows:

In Definition 12.2.1, define Λ_r from $GF(q)((X))$ to $GF(q)((X))$.

In Lemma 12.2.2, replace $i \geq 0$ by i (i.e., $i \in \mathbb{Z}$) in the statement, as well as three times in the part of the proof on the same page. In the second part of the proof next page, replace every $j \geq 0$ by j and $k \geq 0$ by k; also every $i \geq 0$ by i.

On page 358, note that G belongs (a priori) to $GF(q)((X))$. In the definition of $\mathscr{H}$, replace $GF(q)[[X]]$ by $GF(q)((X)))$.

Now at the very end of the proof we still invoke Lemma 12.2.4, but only after having noted that A belongs to $\mathscr{H}$ and is an element of $GF(q)[[X]]$, so are all the elements of its orbit under all the Λ's and their compositions (the semigroup generated by the Λ's), so that everything we are now interested in belongs both to $\mathscr{H}$ and $GF(q)[[X]]$.

Another way to do this is to use the alternative solution of Joost Winter, available at christol.pdf.

Page 355: In Lemma 12.2.3, the list of polynomials on line 3 should include $B_0(X)$. (JOS, January 24 2017)

Page 361: the last line on the page should read "it suffices to prove that the sequence $(c(m) \bmod p)_{m\geq 1}$ is not q-automatic" (JOS, January 23 2017).

Page 371: Insert "(recall that $Q(0,0) \neq 0$)" on the first line of page 371 just before "and so is zero". (Jean Berstel, January 31 2008)

Page 381: In line 6, replace $e_t = b-1$ with $e_t \geq b-1$. In line 14, replace $e_n \neq b-1$ with $e_n < b-1$. In line -4, replace $\tau(q) \neq b-1$ with $\tau(q) < b-1$. (Dan Roche, November 2006)

Page 382: In line 2, replace $e_n = b-1$ with $e_n \geq b-1$. (Dan Roche, November 2006)

Page 395: In line -1, replace $1/q_{|U_kV_k|^2}$ with $1/q^2_{|U_kV_k|}$. (Amy Glen, February 19 2004)

Page 396: In the statement of Proposition 13.7.4, part (b) the limit should read $\lim_{k\to\infty}$. (M. Mendès France, November 21 2003)

Page 398: In lines 4 and 7, replace the limit $n \to \infty$ with $k \to \infty$. (Joel Noche, June 9 2009)

Page 399: Replace 3.2^k with $3 \cdot 2^k$ twice in line -4 and one time in line -8. (Hanna Uscka-Wehlou, November 22 2008)

Page 404: In line -17, replace k in the subscript of the product with n. (Kalle Saari, September 5 2006)

Page 409: In lines 4,6, and 8 replace $u_{n,m}$ with $u_{m,n}$. (Eric Rowland, December 20 2011)

Page 415: In line 7, remove the extra "can" in the sentence. (Kalle Saari, September 5 2006)

Page 425: In line 4, replace "an squarefree" with "a squarefree". (Joel Noche, June 8 2009)

Page 425: Open Problem 14.8.2 was already solved by A. Carpi, Multidimensional unrepetitive configurations, *Theoret. Comput. Sci.* **56** (1988), 233–241. (Also, the problem should impose the restriction $\gcd(c, e) = 1$.) (N. Rampersad, May 29 2006)

Page 432: In line 11, replace "a DFA" with "an NFA". (Kalle Saari, September 5 2006)

Page 441: Replace "theorem" with "section" in the first line of section 16.2. (JOS, March 16 2006)

Page 442: In the statement of Theorem 16.2.3 (b), change Σ^k to Σ_k. (Eric Rowland, August 29 2015)

Page 443: In the second line of Theorem 16.2.5, change Σ^k to Σ_k. (Eric Rowland, August 29 2015)

Page 444: In line 11, replace $e = \max_{0 \le i < t}$ with $e = \max_{0 \le i < k}$. (Eric Rowland, April 17 2010)

Pages 452–453: Exercise 35 should specify that you are counting partitions into *distinct* Fibonacci number parts. (Eric Rowland, May 23 2012)

Page 453: Exercise 36 duplicates Exercise 27(a) on Page 451. (Eric Rowland, December 20 2011)

Pages 476–477: The exercise numbers given for the solutions in A.10 Ch. 10 are shifted: 6, 7, 10, 12 are ok, 22 should be 21, and from 28 to 52 on, n should be replaced by $n-1$. (JPA, April 26 2008)

Page 478: In the hint for Exercise 32 of Chapter 16, we should have cited the solution to Problem 10906 in *Amer. Math. Monthly* **110** (2003), 642–643. (K. Stolarsky, July 15 2005)

Page 495: In the title of Büchi [1960], replace "secord" with "second". (Eric Rowland, May 16 2014).

Page 503: In line 2, remove the word "checked". (Joel Noche, June 8 2009)

Page 528: The reference [de Luca 1990] should read A. de Luca, On the Burnside problem for semigroups, in *Mots: Mélanges Offerts à M.-P. Schützenberger*, Hermès, 1990, ed. M. Lothaire, pp. 185-200. (M.-w. Wang, March 18 2004)