See No Evil, Hear No Evil: Banks, Universities, and Risky Investments

Peter Forsyth*, Irene Huang†, Ken Vetzal‡
University of Waterloo

*Cheriton School of Computer Science
†Electrical and Computer Engineering
‡School of Accounting and Finance

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Some Personal History

This all began about eight years ago.

• In Computer Science at Waterloo, we had several endowment funds used for graduate student scholarships

• The University invested these funds with a money manager, and we were told that

"The income from the endowment is 5% per year."

•Based on this, we would work out how much income we would get from the endowment, and then hand out a number of scholarships each year.

History II

In 2001 (after the dot-com crash), half way through the year, after we had handed out the scholarships, we received the following email from Financial Services

"The income from the endowment this year is zero. Please give us an account number we can use to pay for your committed scholarships."

- Suddenly, we were on the hook for a not-inconsiderable sum.
- The Director of the School of CS phoned me up

"Peter, don't you know something about finance? Can you tell me what is going on here?"

What Happens When an Endowment is Set up?

Example: donor agrees to fund an endowed chair

- An academic unit is informed that the real cash flow from the Endowment is $\simeq X\%$ per year (typically $X = 4 - 5\%$).
- Unit hires prominent professor, agrees to cover salary and research costs until retirement *(promised cash flows)*
- Salary and research costs are expected to increase at a known academic inflation rate.
- Endowment is invested in risky assets $\rightarrow$ reserve account set up to a cushion against poor returns
- University assures donor that **real value of endowment will not decrease**.

University Endowment Policies

Details of Endowment Management vary from institution to institution

- Policy is encapsulated in a set of *Spending Rules*
- I will describe a *typical* set of spending rules
- These are not the exact rules used by any one institution, but are prototypical of the rules used (e.g. at Waterloo, Toronto, etc.)

Important: it is not possible to find a riskless investment which produces a real return of $4 - 5\%$ per year. Currently, inflation protected bonds yield about 2% real per year.

Spending Rules

The Endowment fund consists of a capital account and a reserve account

• Each year, the real gain in the capital account is determined

Case 1: A good year: real investment gain is positive, this gives a possible *Disbursement amount*

• If the disbursement amount is larger than the promised cash flows, then the excess is first added to the reserve account (to some maximum level)

• If the reserve account reaches its maximum, then the excess is added to the capital account

Spending Rules

Case 2: A bad year: real investment gain is negative

• First, cash is withdrawn from the reserve account to reduce the real loss of the capital account to zero.

• If there is any cash left over, from the reserve, this is then applied to the promised cash flows

Important:

• Cash can never be withdrawn from the capital account.

• There is no guarantee that the promised cash flows will be met.

Problem

From (Dybvig, *Financial Analysts Journal* (1999)) who discusses endowments invested in risky assets

> *"[There is] a significant probability of a shortfall. To assert otherwise is to state that the fund is certain that stocks will go up and that going long stocks and short in the riskless asset is, in effect, a riskless arbitrage. Such cheerful optimism may be an appealing personality trait, but it is not a healthy attitude for an investment manager."*

$\hookrightarrow$ So, the University is taking on some risk to attract endowments.

Objective

I will determine the *no-arbitrage* value of this risk.

What is the no-arbitrage value?

• The cost of hedging this risk

• Or, what it would cost the University to have someone (i.e. a bank) take this risk off their books

• Or, if the University did a rigorous *mark-to-market* accounting each year, what is the unfunded liability that the University has due to this endowment

Spending Rules: Simplified Example

The spending rule specifies a set of valuation dates $\{t_i\}$ (almost always annually) with $t_{i+1} - t_i = \Delta t = const.$.

$$
\begin{aligned}
S_i = S(t_i) &= \text{ Value of endowed capital at } t_i \\
R_i &= \text{ value of reserve fund at } t_i \\
I_i &= \text{ inflation factor in period } [t_i, t_{i+1}] \\
C_r &= \text{Cap on reserve fund}, (\max = C_r S_i) \\
F^{sp} &= \text{Spending target factor in period } [t_i, t_{i+1}] \\
\text{Maximum spending } &= F^{sp} S_i
\end{aligned}
$$

Spending Rules: Simplified Example

At valuation date t_{i+1} the real gain of the endowment over $[t_i, t_{i+1}]$, denoted by RG_{i+1}, is given by

$$RG_{i+1} = S_{i+1} - S_i I_i$$

S_i is the value of the endowment at t_i

I_i is the inflation factor over$[t_i, t_{i+1}]$

If $RG_{i+1} < 0$

- Reserve fund R_{i+1} is drawn down to preserve real capital
- If the reserve fund is exhausted, and $RG_{i+1} < 0$, then
  $\longrightarrow$ No disbursements

Spending Rules: Simplified Example

If $RG_{i+1} > 0$ or $R_{i+1} > 0$ after transfers to capital account

- Attempt to disburse $F_{sp}S_i$ (spending target)
- First use up RG_{i+1} (real gain)
- Then use up reserve fund R_{i+1}
  - $\rightarrow$ Reserve fund not allowed to go negative
  - $\rightarrow$ Define net disbursed amount $= D_{i+1}$.

If excess after disbursements

- First increase reserve to maximum $C_r S_i$.
- Anything left is added to capital account

Guarantee

The problem:

$$D_{i+1} = \text{Disbursement at } t_{i+1} \text{ from spending rules}$$
$$E_{i+1} = \text{ promised cash flow at } t_{i+1}$$
$$\text{e.g. chairholder salary, inflation adjusted}$$

If $D_{i+1} < E_{i+1}$, cashflow must be made up by academic unit

$$G_{i+1} = \text{ cash flow from unit to make up shortfall}$$
$$= \max(0, E_{i+1} - D_{i+1})$$

Spending Rules: Simplified Example

To precisely define the spending rules, we need the following information:

$$
\begin{aligned}
S(t) &= \text{ Capital Account at time } t \\
R(t) &= R(t_{i-1}) \ t \in [t_{i-1}, t_i] \\
&= \text{ value of reserve at previous valuation date} \\
P(t) &= S(t_{i-1}) \ t \in [t_{i-1}, t_i] \\
&= \text{ value of capital account at previous valuation date}
\end{aligned}
$$

Spending Rule Functions

All information about spending rules is encapsulated in a set of functions

$$
\begin{aligned}
S^{sp}(t_i) &= S^{sp}(S, R, P, t_i) \\
&= \text{ Capital Account after spending rules applied at } t_i \\
R^{sp}(t_i) &= R^{sp}(S, R, P, t_i) \\
&= \text{ Reserve account after spending rules applied at } t_i \\
G^{sp}(t_i) &= G^{sp}(S, R, P, t_i) \\
&= \text{ cashflow from unit to make up shortfall at } t_i
\end{aligned}
$$

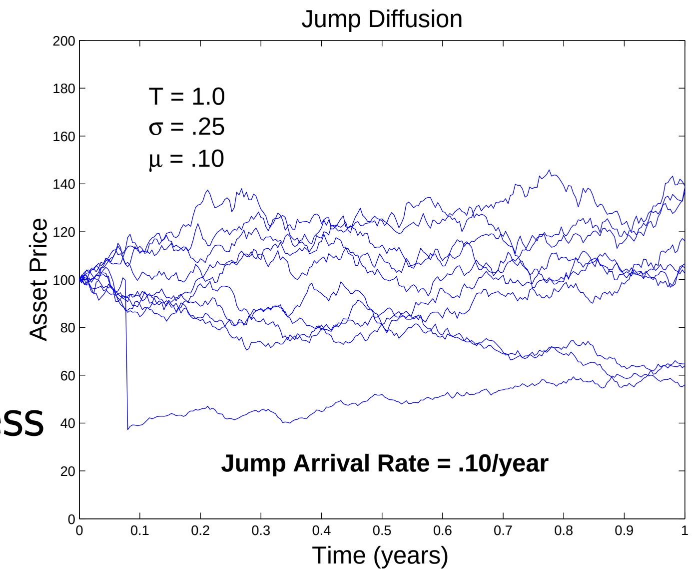
Are You Confused Yet?

My first attempt at writing a *Spending Rule* function (for the most general case)

```
spending_rule( input: time, S, R, P
               output: S_sp, R_sp, P_sp, G_sp
             )
  ................
```

$\longrightarrow$ Required nineteen IF statements!
After some work, I got it down to ten IF statements

Risk Neutral Stochastic Process for Asset S



$$\frac{dS}{S} = (r - \lambda\kappa)dt + \sigma dZ + (J-1)dq$$

$r =$ risk free interest rate,

$\sigma =$ volatility,

$dZ =$ increment of a Wiener process

$$dq = \begin{cases} 0 & \text{with probability } 1 - \lambda dt \\ 1 & \text{with probability } \lambda dt, \end{cases}$$

$\lambda =$ mean arrival rate of Poisson jumps; $S \to JS$

$\kappa = E[J-1], E[\cdot]$ expected value

Path Dependent Contingent Claim

The no-arbitrage value of the guarantee is a *path dependent contingent claim*.

• We can solve a 3-d Partial Integro Differential equation (PIDE) to value the claim

• We can also use Monte Carlo (MC) simulation

We will use both methods here.

$\hookrightarrow$ A single solve of the PIDE gives the value of the guarantee for all values of the initial capital.

$\hookrightarrow$ A single MC solution gives us the value of the guarantee for only a single value of the initial capital.

Partial Integro Differential Equation (PIDE)

In the PIDE case, let the no-arbitrage value be given by $V(S, P, R, \tau)$, where

$$\begin{aligned} \tau &= T - t \\ &= \text{ time running backwards} \\ T &= \text{ expiry time of claim} \end{aligned} \tag{1}$$

We take $T = 20$ years, i.e. the time frame for an endowed chair. At $t = 20$ $(\tau = 0)$ we have

$$V(S, P, R, \tau = 0) = 0 \quad \text{(No further obligations)}$$

- At $t = T$, we can always put off hiring a new chairholder

PIDE Solution

In between valuation dates, we solve

$$V_\tau = \frac{\sigma S^2}{2} V_{SS} + (r - \lambda\kappa) S V_S - rV$$
$$+ \lambda\left(\int_0^\infty V(S\eta) g(\eta) d\eta - V\right)$$
$$g(\eta) = \text{ risk neutral jump size density}$$

E.g. (d'Halluin, Forsyth, Labahn, *Numerische Math.* (2004))

PIDE Solution

At valuation dates τ_i (in backwards time), we apply the following *jump conditions*, which can be deduced from *no-arbitrage*

$$V(S,P,R,\tau_i^+) = V(S^{sp},P^{sp},R^{sp},\tau_i^-) - G^{sp}$$

$$\tau_i^+ = \tau_i + \varepsilon \ ; \ \tau_i^- = \tau_i - \varepsilon$$

$$S^{sp},P^{sp},R^{sp} = \text{values from spending rules}$$

$$G^{sp} = \text{ cash flow to make up shortfall}$$

$$V(...,t_i^+) = V(...,t_i^-) + \text{ cash flow}$$

Numerical Example

We will show the value of the guarantee to the university.

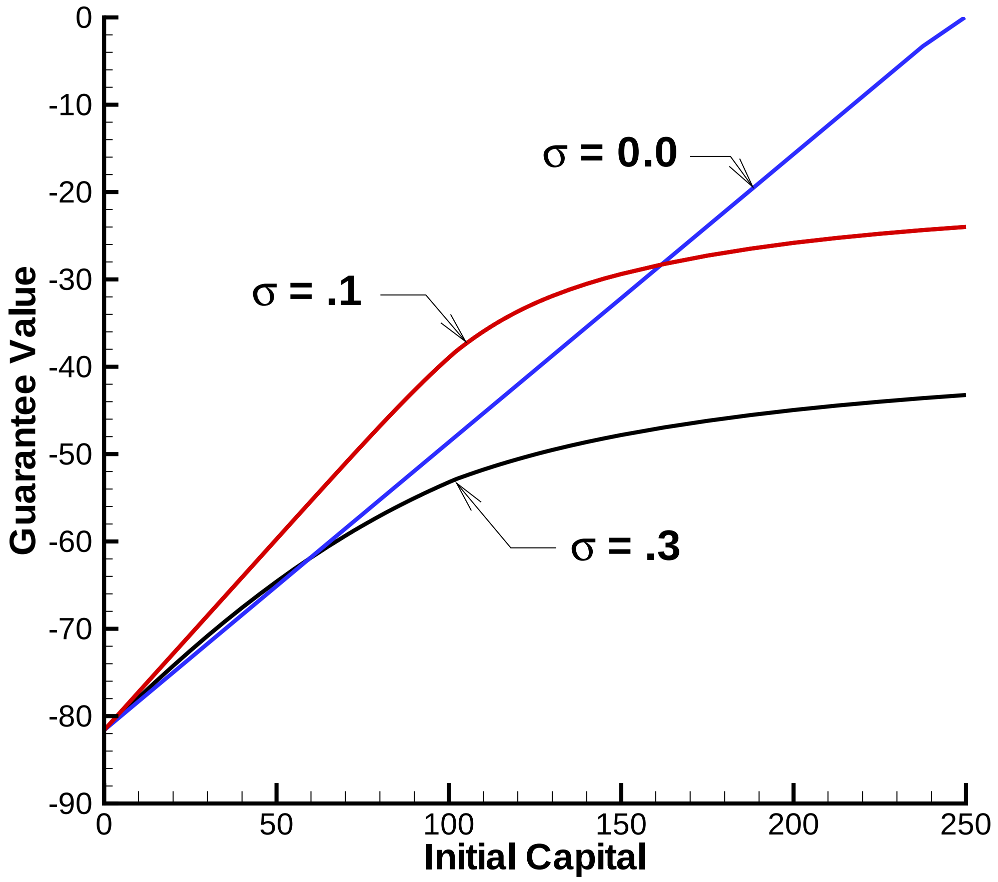
It will be a negative quantity (indicating a liability), i.e. the larger in absolute value, the worse it is.

All results will be scaled by the initial value of the capital
• We will solve problem for initial capital in $[0, 250]$, and initial promised cash flow of 5
• E.g., if initial capital is 100, promised cash flow is 5 per year, then this corresponds to a 5% spending target.

Base Case

Parameter	Value
Initial Promised cash flow/year	5
Initial reserve R_0	0
Reserve cap C_r	.15 (larger than usual)
Time horizon (T)	20 years
Valuation frequency	yearly
General inflation rate I_i^{rate}	.02
Academic inflation rate A_f^{rate}	.02 (optimistic)
σ	.10 (low risk)
Risk free interest rate r	.04
λ	0 (no jumps)

No-arbitrage value of guarantee: Base Case



Convergence Test

PIDE		
Nodes $(n_x \times n_y \times n_z)$	Timesteps	Guarantee value at Initial Capital $= 100$
$59 \times 59 \times 18$	240	-39.15
$117 \times 117 \times 35$	480	-38.96
$233 \times 233 \times 69$	960	-38.90

Monte Carlo	
Number of paths	Guarantee Value at Initial Capital $= 100$
1000	-39.11
5000	-38.96
10000	-38.88
25000	-38.90
100000	-38.89

Example:

Suppose university receives an endowment of $4 million for an endowed chair

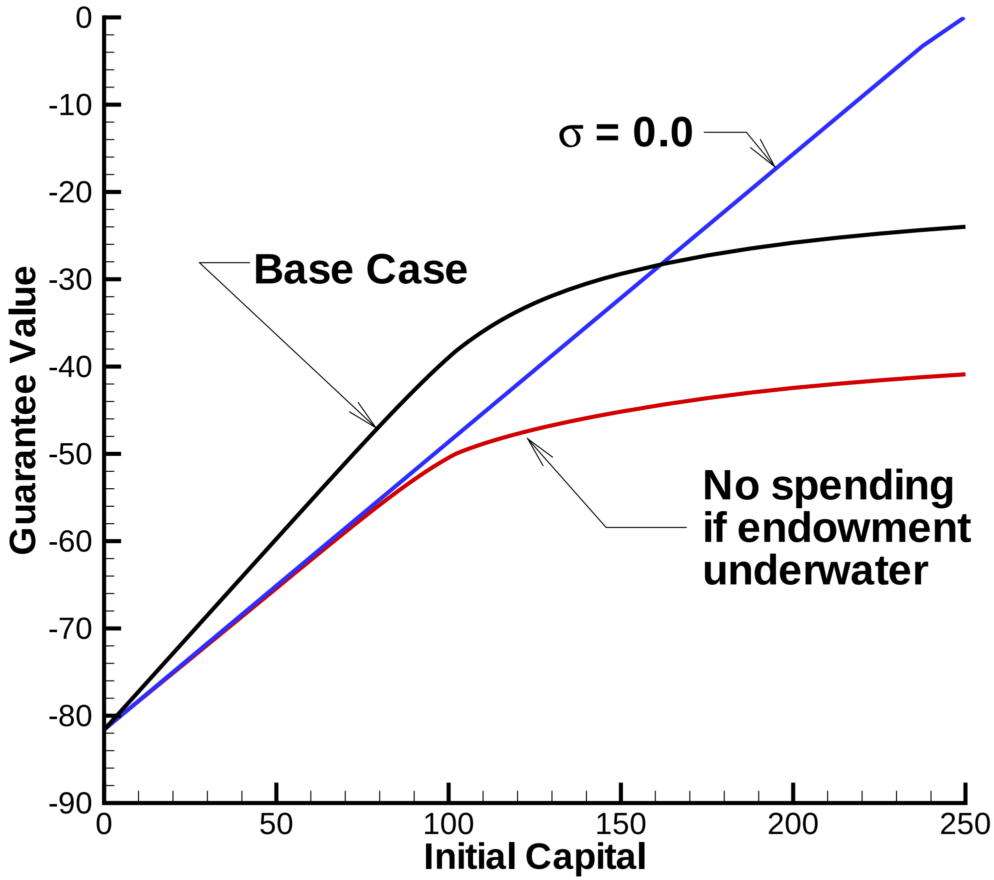
- At 4% real return, this should generate $160,000 per year (in constant dollars) for chairholder's salary and benefits
- Endowment is invested in low-risk assets ($\sigma = .10$), and follows prototype spending rules
- The no-arbitrage value of the unfunded guarantee is about
$$(33/125) \times 4 \text{ Million} \simeq 1 \text{ Million}$$
- The unfunded liability is about 25% of the value of the endowments

No Spending if Endowment Underwater

Note that the spending rules we have used do not guarantee that the real value of the endowment is preserved (only best efforts)
$\longrightarrow$ If the reserve is depleted, and the real gain is negative, then the capital account is *underwater*
$\longrightarrow$ Our base case spending rule does not require that this be made up in future years.

Some Universities require that no disbursements can take place in future years until the capital account is above water.

No Spending if Endowment Underwater



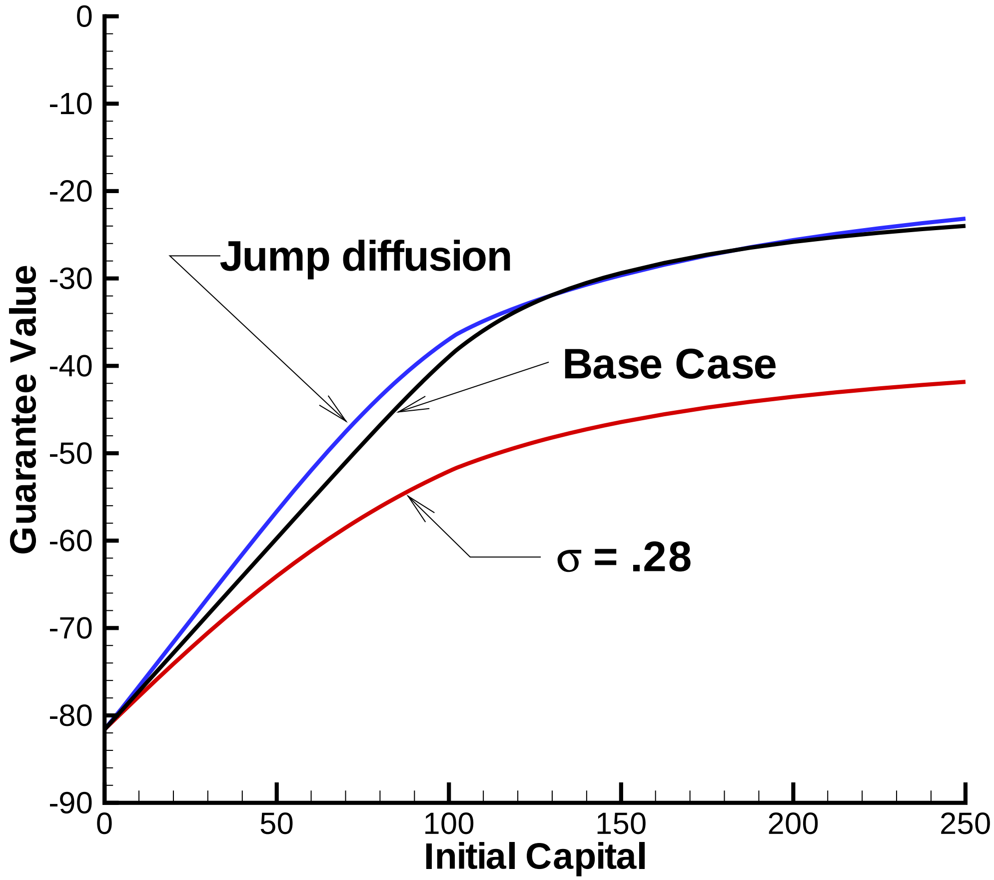
Adding Jumps

Base Case has no jumps: consider two new cases

- Add non-zero jumps to base case, jump parameters from Andersen et al (2000), fit to S&P 500 data
- No jumps, but use implied volatility which matches the price of a 20 year European call option at the money priced under a jump diffusion model with the above parameters ($\sigma_{imp} = .2811.$)
- Conventional wisdom: over long horizons, jumps look like increased volatility

Jumps vs No-jumps

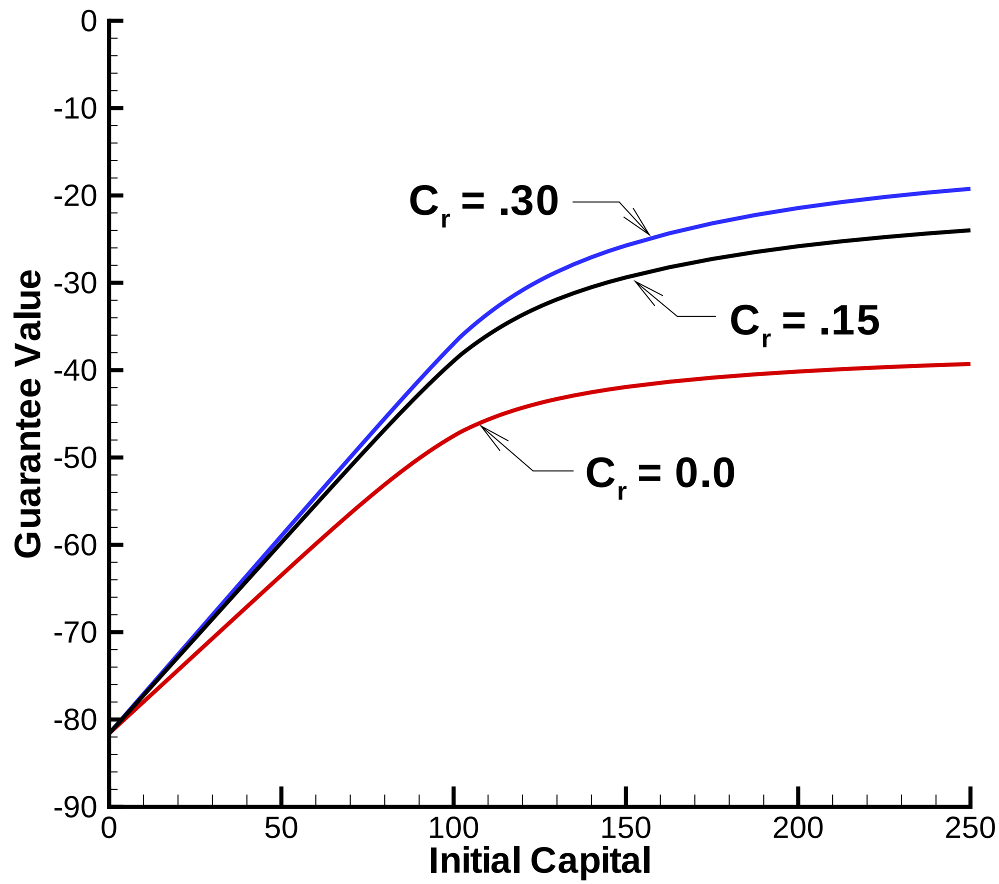
Conventional wisdom
not correct for
path dependent claims



Increasing Reserve Cap

Maximum reserve

$= C_r \times$ capital

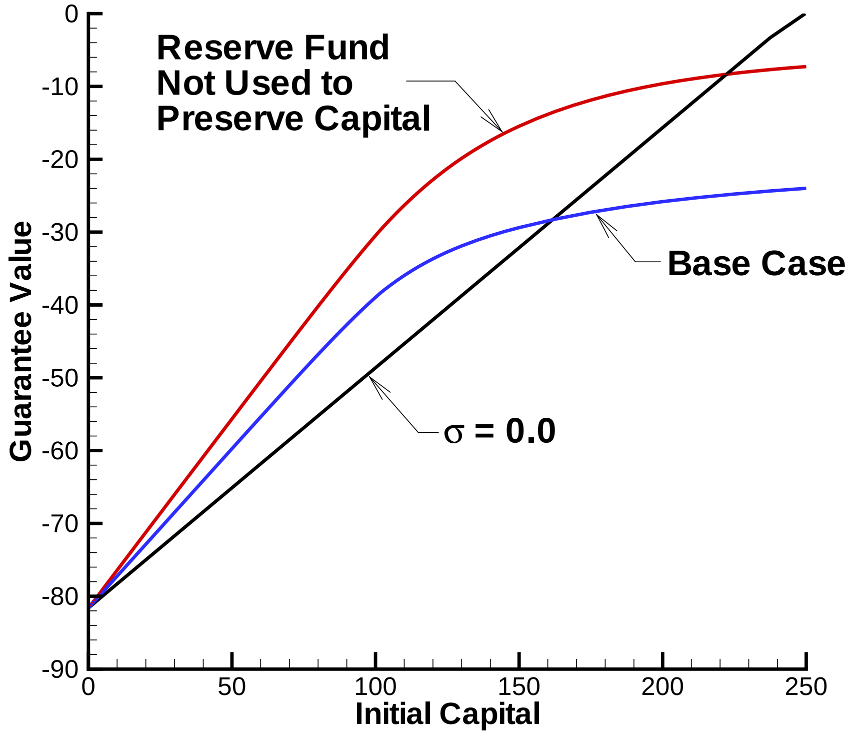


Why is the Base Case So Bad?

Although at times the reserve fund may be several times promised cash flows (examine MC simulations)
$\longrightarrow$ A small relative loss of a large endowment capital quickly eliminates the reserve

What about modifying the spending rule? Change rule so that:
$\hookrightarrow$ First priority is to paying the promised cash flow
$\hookrightarrow$ After this, attempt made to preserve real capital
$\hookrightarrow$ Higher probability of not preserving real capital

Pay promised cash flow before preserving real capital



Why Are Universities Taking on These Risks?

Charles Prince, ex CEO of Citigroup[1], explaining why Citigroup was aggressively involved in credit derivatives (2007)

> *"As long as the music is playing, you've got to get up and dance."*

Why are Universities making unrealistic promises to donors (in effect taking a naked (unhedged) position in stock market puts)?

> *"It's a competitive market out there for endowments. If we don't offer this kind of deal, the University down the street is going to get the donation."*

[1] Prince received a $105 Million exit payment. Citi shares have lost 95% of their value

Why do Universities, Banks Follow the Same Risky Policies?

- Inadequate mark-to-market accounting allows inflated balance sheets
- Unrealistic promises to donors/shareholders.
- Performance indicators based on short-term results, no accounting for long term risks.
- Decision makers have no *"long term skin in the game."*
- E.g. increased endowments $\rightarrow$ increase institutional prestige, development officers *"hit their numbers,"* when problems arise, someone else has to clean up the mess.

Other Applications of These Results

Conventional Wisdom: *"Over the long term, investing in the stock market produces higher returns than bonds, with low risk."*

• This is highly risky if we need to generate revenue each year
• For long term investors, with no need to generate income each year, the **order** of random returns is irrelevant.
• On the other hand, investors who need to generate income each year
$\rightarrow$ Are exposed to risk due to the order of the random returns
$\rightarrow$ Bad returns at the start of the investment period are much worse than bad returns at the end of the investment period

This has implications for retirement planning: risky to invest in the stock market once you have retired and must withdraw cash each year.

Conclusions

- University Endowment Spending Rules

  $\longrightarrow$ Are actually a complex contingent claim.

- A conservative estimate of the unfunded liability of University Endowments following typical spending rules

  $\longrightarrow$ 25% of value of endowments.

- Incentive structures: Banks and Universities

  $\longrightarrow$ Rewards for short-term gains

  $\longrightarrow$ Rewards for ignoring long-term risks

Conclusions II

- University Endowments have many of the characteristics of variable annuity products sold by insurance companies

  $\longrightarrow$ Investments in risky assets is used to provide certain cash flows to retirees (GMWB, GMDB)

  $\longrightarrow$ Insurance companies have undercharged for these guarantees and are facing large mark-to-market losses (Chen, Forsyth, Vetzal, *Insurance: Mathematics and Economics* (2008))