# Strategic interactions and uncertainty in decisions to curb greenhouse gas emissions II: Numerics

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Vienna, Thursday, July 5, 2018
2:00-3:40pm
Session 56

# Overview

Temperature $X$ (relative to preindustrial) follows arithmetic Brownian motion ($d\mathcal{Z}$ = increment of a Wiener process)

$$dX(t) = \eta(t)\left[\bar{X}(S,t) - X(t)\right]dt + \sigma d\mathcal{Z}$$

mean reversion level $\bar{X}(S,t)$ depends on carbon stock $S$

Global carbon stock $S$ follows[1]

$$\begin{aligned} \frac{dS(t)}{dt} &= E_1 + E_2 + (\bar{S} - S(t))\rho(X,S,t) \\ & E_p = \text{ carbon emission of player } p \text{ , } p = 1,2 \\ & \bar{S} = \text{ preindustrial carbon stock} \\ & \rho(X,S,t) = \text{ removal rate} \end{aligned} \tag{1}$$

Benefits (including negative effects of temperature damage) flowing to player $p$ : $\pi_p(E_1, E_2, X, S, t)$

[1] Current estimates are $1/\rho = 100$ years, decaying to $1/\rho = 3000$ years in 100 years. $1/\eta$ today is about 50 years. $\bar{X} \simeq 1.9$ today (above pre-industrial).

# Controls

Discrete decision times

$$\mathcal{T} = \{t_0 = 0 < t_1 < ...t_m... < t_M = T\}$$

At $t_m \in \mathcal{T}$, player $p$ chooses emission level $e_p^+$

$$\begin{aligned} e_p^+(E_1, E_2, X, S, t_m) &= \text{emission level of player } p = 1, 2 \\ &\quad \text{emission level applies } t \in (t_m, t_{m+1}) \end{aligned}$$

Admissible sets $Z_p$ are discrete $Z_p \in \{0, 3, 7, 10\}$ Gt/year.[2]

Define control set

$$K = \left\{ (e_1^+, e_2^+)_{t_0=0},\ (e_1^+, e_2^+)_{t_1=1},\ \dots, (e_1^+, e_2^+)_{t_M=T} \right\}$$

[2] US currently emits $\simeq 2$ Gt/year. Total world $\simeq 10$ Gt/year.

# Value functions

$$\begin{aligned}
\pi_p(E_1, E_2, X, S, t) &= \text{net benefits (including damages) of emissions} \\
\mathbb{E}_K[\cdot] &= \text{Expectation under control } K \\
r &= \text{discount rate} \\
V(0, 0, X(T), S(T), T) &= \text{stream of future benefits (decarbonized world)}
\end{aligned}$$

$$\begin{aligned}
V_p(e_1, e_2, x, s, t) = \; & \mathbb{E}_K \left[ \int_{t'=t}^{T} e^{-rt'} \pi_p(E_1(t'), E_2(t'), X(t'), S(t')) \; dt' \right. \\
& + e^{-r(T-t)} V(0, 0, X(T), S(T), T) \\
& \left. \Big| E_1(t) = e_1, E_2(t) = e_2, X(t) = x, \; S(t) = s \right]
\end{aligned}$$

# Dynamic Programming

Define ($t_m$ = decision time)

$$t_m^+ = t_m + \epsilon \quad ; \quad t_m^- = t_m - \epsilon \quad ; \quad \epsilon \to 0^+$$

Advance solution backward in time $t_{m+1}^- \to t_m^+$

$$\frac{\partial V_p}{\partial t} + \pi_p(e_1, e_2, x, s, t) + \mathcal{L} V_p = 0, \quad p = 1, 2$$

$$\mathcal{L} V_p \equiv \frac{(\sigma)^2}{2} \frac{\partial^2 V_p}{\partial x^2} + \eta(\bar{X} - x) \frac{\partial V_p}{\partial x} + [(e_1 + e_2)\rho(\bar{S} - s)] \frac{\partial V_p}{\partial s} - r V_p$$

- Two continuous state variables $(s, x)$ (carbon stock, temperature)
- Two discrete state variables $(e_1, e_2)$ (emissions)
- State of player p depends on $(\underbrace{e_1, e_2,}_{both\ controls} \; s, x, t)$

# Solution of linear PDE in $(t_{m+1}^-, t_m^+)$

Discretize in $x$ (temperature) direction

- Use positive coefficient discretization, fully implicit timestepping

Discretize in $s$ (carbon stock) direction

- Use semi-Lagrangian timestepping

# Optimal controls

At $t_m$, players choose optimal controls $e_p^+(\cdot); p = 1, 2$
Dynamic programming implies

$$\begin{aligned} V_1(e_1^-, e_2^-, s, x, t_m^-) &= V_1(e_1^+(\cdot), e_2^+(\cdot), s, x, t_m^+) \ , \\ V_2(e_1^-, e_2^-, s, x, t_m^-) &= V_2(e_1^+(\cdot), e_2^+(\cdot), s, x, t_m^+) \\ & \quad e_p^+(\cdot) = e_p^+(e_1^-, e_2^-, s, x, t_m^-) \end{aligned}$$

Note:

- $(s, x)$ do not change in $(t_m^-, t_m^+)$
- Applying controls causes state change

$$(e_1^-, e_2^-) \to (e_1^+, e_2^+) \ ; \ (t_m^- \to t_m^+)$$

- Both value functions effected by both controls.

# Stackelberg Game

Conceptually, player 1 goes first, followed immediately by player 2

## Definition 1 (Response function of player 2)

The best response function of player 2, $R_2(\omega_1; e_1^-, e_2^-, s, x, t_m)$ is defined to be the best response of player 2 to a control $\omega_1$ of player 1.

$$R_2(\omega_1; e_1^-, e_2^-, s, x, t_m) \quad = \quad \arg\max_{e_2'} V_2(\underbrace{\omega_1}_{\substack{player 1\\control}}, e_2', s, x, t_m^+) \quad (2)$$

## Remark 1 (Tie breaking)

*We break ties by (i) staying at the current emission level if possible, or (ii) choosing the lowest emission level. Rule (i) has priority over rule (ii). The notation $R_2(\cdot; e_1^-, e_2^-, \cdot)$ shows dependence on the states $(e_1^-, e_2^-)$ due to the tie breaking rule.*

# Response function of player 1

Similarly, we define the best response function of player 1.

Definition 2 (Response function of player 1)

The best response function of player 1, $R_1(\omega_2; e_1^-, e_2^-, s, x, t_m)$ is defined to be the best response of player 1 to a control $\omega_2$ of player 2.

$$R_1(\omega_2; e_1^-, e_2^-, s, x, t_m) \quad = \quad \arg\max_{e_1'} V_1(e_1', \underbrace{\omega_2}_{\substack{player2 \\ control}}, s, x, t_m^+). \quad (3)$$

To avoid notational clutter, fix $(e_1^-, e_2^-, s, x, t_m)$

$$R_1(\underbrace{\omega_2}_{\substack{player2 \\ control}}) \equiv R_1(\omega_2; e_1^-, e_2^-, s, x, t_m) \quad ; \quad R_2(\underbrace{\omega_1}_{\substack{player1 \\ control}}) \equiv R_2(\omega_1; e_1^-, e_2^-, s, x, t_m)$$

# Stackelberg Game II

## Definition 3 (Stackelberg Game: Player 1 first)

The optimal controls $(e_1^+, e_2^+)$ assuming player 1 goes first are given by

$$\begin{aligned} e_1^+ &= \arg\max_{\omega_1'} V_1(\omega_1', R_2(\omega_1'), s, x, t_m^+) \\ e_2^+ &= R_2(e_1^+) \ . \end{aligned} \tag{4}$$

## Definition 4 (Nash Equilibrium[3] )

Given the best response sets $R_2(\omega_1)$, $R_1(\omega_2)$ defined in Equations (2)-(3), then the pair $(e_1^+, e_2^+)$ is a Nash equilibrium point if and only if

$$e_1^+ = R_1(e_2^+) \quad ; \quad e_2^+ = R_2(e_1^+) \ . \tag{5}$$

[3] In our numerical tests, we find that a Nash equilibrium does not always exist

# Leader-Leader (Trumpian) Game

Each player assumes that they are the leader in a Stackelberg game

→ This, of course, cannot be true

The Trumpian controls are determined from[4] [5]

$$
\begin{aligned}
e_1^+ &= \arg\max_{\omega_1'} V_1(\omega_1', R_2(\omega_1'), s, x, t_m^+) \\
e_2^+ &= \arg\max_{\omega_2'} V_2(R_1(\omega_2'), \omega_2', s, x, t_m^+)\ .
\end{aligned} \tag{6}
$$

[4]Each player assumes (i) that they get to go first and (ii) that the other player will reply with their best response function.

[5]Usual tie breaking rules.

# Interleave Game

Decision times $t_{2i}; i = 0, 1, \ldots$ [6]

$$\begin{aligned} e_1^{(2i)+} &= \text{optimal control for player 1}, \\ e_2^{(2i)+} &= e_2^{(2i)-} \; ; \; \text{player 2 control fixed} . \end{aligned} \tag{7}$$

At times $t_{2i+1}; i = 0, 1, \ldots$[7]

$$\begin{aligned} e_1^{(2i+1)+} &= e_1^{(2i+1)-} \; ; \; \text{player 1 control fixed}, \\ e_2^{(2i+1)+} &= \text{optimal control for player 2} \end{aligned} \tag{8}$$

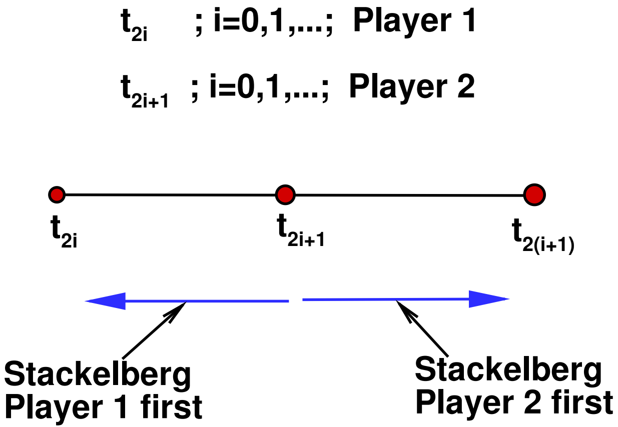
Usual tie breaking rules

[6] Player one chooses control at $t_i$, $i$ even.

[7] Player two chooses control at $t_i$, $i$ odd.

# Limit of Interleave Game

- Fix player 1 times $t_{2i}, i = 0, 1, \ldots$.
- Move player 2 times $t_{2i+1}, i = 0, 1, \ldots$.



# Social Planner

Social planner maximizes total welfare

$$(e_1^+, e_2^+) \quad = \quad \arg\max_{\substack{\omega_1 \\ \omega_2}} \left\{ V_1(\omega_1, \omega_2, s, x, t_m^+) + V_2(\omega_1, \omega_2, s, x, t_m^+) \right\} .$$

Break ties

(i) Minimize $|V_1(e_1^+, e_2^+, s, x, t_m^+) - V_2(e_1^+, e_2^+, s, x, t_m^+)|$

(ii) Choose the lowest emission level

- Rule (i) has priority over Rule(ii)

Gives the most equal distribution of welfare amongst players.

# Numerical Examples

We will assume two players

- Players are identical, with identical damage/benefit functions
- Players are symmetric
  - $\rightarrow$ Differences in results for player 1,2 will only be determined by the type of game
- We show utilities at $t = 0$ for a range of temperatures, and carbon stock

Decision intervals: 2 years; $T = 150$ years.

Current temperature: 1 deg C (above pre-industrial). Current carbon stock: 900 Gt.

# Base Case: Stackelberg vs. Social Planner

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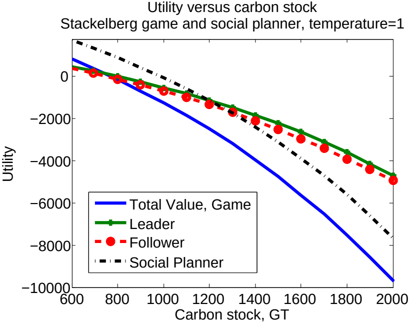


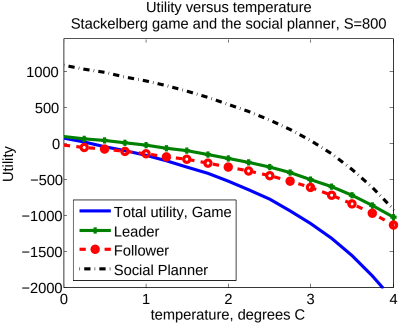


Utilities versus carbon stock and temperature for base Stackelberg game and social planner, time $= 0$, state variables $E1 = 10$, $E2 = 10$. Temperature is in degrees C above preindustrial levels.

•Total utility social planner $>$ total utility game.
•Follower worse off than leader

[8] Nash equilibria exist at about 65% of nodes, averaged over time and space.

# Trumpian Game

- Recall that players are symmetric
- Trump controls are also symmetric

$\hookrightarrow$ Player 1 and player 2 have same utilities

- Total utility smaller than base (Stackelberg) game.
- Player 1 (former leader in Stackelberg game) now much worse off.
- Player 2 (former follower in Stackelberg game) almost same

When faced with Trump

$\Rightarrow$ Play Trump

# Interleaved Game

Decisions made every two years

- Each individual player makes decisions every 4 years
- Player 1 chooses emission level at time zero
- Player 2 sticks with initial emission level for first two years.

Total utilities

- Total utility higher than Trump or base (Stackelberg) game

Individual player utilities

- Player 2 (Interleave follower) chooses $e_2 = 10$ at $t = 0$
  - Player 2 utility very close to Player 1
  - Player 2 better off than Stackelberg (flwr), Player 1 worse off than Stackelberg (leader)
- Player 2 (Interleave follower) chooses $e_2 = 0$ at $t = 0$

$\rightarrow$ Both players better off than Stackelberg

# Conclusions

- If you have to react immediately to the other player's emission levels
  - ↪ You might as well play Trump: very little advantage to being a Stackelberg follower[9]
  - ↪ And you have the satisfaction of causing damage to the other player
- You are better off playing an Interleave game if possible
  - ↪ Some choices of initial emission levels make both players better off (compared with Stackelberg)
  - ↪ Stackelberg follower is always better off playing Interleave (assuming decision time intervals same)
- Of course, it is better if everyone cooperates

[9] When faced with Trump, play Trump