Fees for variable annuities: too high or too low?

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Background

General trend away from Defined Benefit (DB) pension plans

- Private sector and governments want to de-risk

Most of us now have to manage our own retirements

Alternative to DB:

- Traditional fixed rate annuity
  - With rates so low, who would buy a traditional annuity now?
  - Who wants to take on the inflation/credit risk?
- A Variable Annuity (VA) is an industry response to the reduction in DB plans
  - Allows the buyer to replicate a DB
  - More flexibility than a traditional annuity

Are you safe if you have a DB plan?

Many DB plans have limited (none?) inflation protection

If you are lucky to have a DB plan with inflation protection

- How good is this guarantee?

Solvency test

- In Ontario, indexation of liabilities is excluded from solvency test

Going Concern Valuation

- This test uses the risky discount rate to discount the certain liabilities
  - → i.e., this assumes that we get the *expected return* on our risky investments
  - Current plan members earn future risk premium with no risk[1]

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[1] Risk is accounted for by reducing the expected return slightly, to account for *adverse deviations*

Are you safe with a DB?

Closest actuarial test to *Mark to Market* → Wind-up valuation

- Wind-up valuation: receives scant attention in actuarial reports
- Many plans have significant deficits if measured on a *wind-up* basis

Conclusion:

- Even you have an indexed DB plan, it might be prudent to have a back-up plan

Some History

In Canada, variable annuities have a long history

Historically known as a *segregated fund* in Canada.

A typical segregated fund guarantee (15 years ago)

- Initial investment in insurance company mutual fund
- Guarantee takes form of 10-year European put, strike set at initial fund level
- However, holder can reset the strike to current fund level at any time.
- Upon reset, maturity extended to be 10 years after reset
- If investor dies, guarantee provided immediately (i.e. becomes American)

Segregated Fund

Some additional features

- No initial up-front fee
  - Guarantee paid for by withdrawing a *rider* fee from investment account
- Holder can lapse (i.e. redeem, surrender) contract with penalty
- Penalty typically declined to zero: five years after purchase
  - → Valuable lapse option (i.e. withdraw all funds) with no penalty now available to holder
- If the guarantee is worth less than the value of fees required to stay in the fund
  - → Investors who lapse when the guarantee is out of the money deprive the insurer of the future fee cash flows

Workshop: Toronto 2000 [2]

In 2000, a group of us at UofWaterloo organized a workshop (in Toronto) on segregated funds

Me: *"... and now we determine the no-arbitrage price by solving the following Partial Differential Equation (PDE)"*

Actuary from Insurance company X: *"But the market is not complete, and the no-arbitrage value is irrelevant."*

Me: *"But you have to hedge your exposure to these guarantees."*

Actuary from X: *"The risk to us is nothing. Everybody knows, the market is never down over any ten year period."*

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[2] Generously funded by the Royal Bank of Canada

We issued warning bells

From a paper we wrote in 2002, about complex products sold by insurance companies

> *"...in many cases these contracts appear to be significantly underpriced, ... current deferred fees being charged are insufficient to establish a dynamic hedge...This finding might raise concerns at institutions writing such contracts."*

Windcliff, Forsyth, LeRoux, Vetzal, North American Actuarial J., 6 (2002) 107-125

What happened?

From the Globe and Mail Streetwise Blog, November, 2008

> *"Concerns...sent* **XXX**[3] *shares reeling last month. Those concerns were a result of* **XXX***'s strategy of not fully hedging products such as annuities and segregated funds..."*

Globe and Mail Report on Business, December, 2008 "**XXX** in red, raises new equity,"

- **XXX** posted a large mark-to-market writedown to account for losses associated with segregated fund guarantees.

[3] A major Canadian financial institution

And the pain continued...

Financial Post, August 6, 2010

> "**XXX**[4] *continues to be plagued by market gyrations that contributed to a record loss of $2.4-billion in the second quarter. . .*
>
> **XXX** *is...hedging a greater proportion of the variable annuity businesses....*"

Globe and Mail, November 8, 2012

> "*...***XXX** *took a $1-billion charge that stemmed largely from a change in behaviour by its customers ...variable annuities...*"

[4] A major Canadian financial institution

What do we learn from this?

A good hedging plan for variable annuities is necessary

- The fees charged for guarantees should be based on the cost of hedging, not driven by marketing considerations

It is a bad idea to assume markets always go up!

It is dangerous to assume that retail investors' actions will never result in

$\rightarrow$ *worst case* hedging scenario for the seller

Cost of hedging

I am going to discuss the *cost of hedging* of a particular class of variable annuities (GLWBs)

- I am not assuming a complete market[5]

- Separate the *cost of hedging* from retail consumer behaviour
- Worst case for the hedger
  → Holder carries out *loss maximizing withdrawal strategy*
- Unfortunately referred to as the *optimal* withdrawal strategy
- But it may not be optimal for anyone.

[5] In 2001, we were approached by a hedge fund. Apparently, in Saskatchewan, a retail customer can sell her variable annuity to a 3rd party. The idea was to incorporate the fund in Saskatchewan and then buy variable annuities at less than the no-arbitrage price, delta hedge, and make millions.

Guaranteed Lifelong Withdrawal and Death Benefits (GLWDB)

GLWDBs attempt to replicate a DB plan (i.e. lifelong guaranteed cash flows, with possible increase if market does well).

→ Important feature: contract holder retains option to withdraw all funds from contract

Contract bootstrapped by initial payment to insurance company, S_0

- Virtual withdrawal account $W(t)$ and death benefit account $D(t)$ set to S_0
- S_0 invested in risky assets, value $S(t)$.
- Fund management fee and guarantee fee withdrawn from risky asset account $S(t)$
- At a series of event times, t_i (usually yearly) various actions can be triggered.

Event actions at t_i

Withdrawal Event Holder can withdraw

$$\begin{aligned} \text{withdrawal amount} \quad &\in \quad [0, G * W(t_i^-)] \\ & \quad G = \text{ spec'd contract rate} \\ & \quad W = \text{ Withdrawal account} \end{aligned}$$

Death benefit account D and risky asset account S reduced by withdrawal amount.
Note: Contract amount can be withdrawn even if $S = 0$.

Surrender Event Holder withdraws an amount $> G * W(t_i^-)$

- Penalty charged as fraction of withdrawal
- $W(t_i^+), D(t_i^+)$ reduced proportionately
- Total amount withdrawn cannot exceed $S(t_i^-)$

Events c't'd

Ratchet Event Withdrawal/Death benefit account can ratchet up, i.e.

$$\begin{aligned} W(t_i^+) &= \max(S(t_i^-), W(t_i^-)) \\ D(t_i^+) &= \max(S(t_i^-), D(t_i^-)) \end{aligned}$$

Note: W can never decrease[6], even if market crashes.

Bonus Event If holder does not withdraw, withdrawal account increased

$$\begin{aligned} W(t_i^+) &= (1+B)W(t_i^-) \\ & B = \text{ bonus rate} \end{aligned}$$

[6] except if the holder surrenders

Death Benefits, Assumptions

If you die, then your estate gets

$$\max(D(t), S(t)) \tag{1}$$

Estate guaranteed to get back initial payment (less withdrawals)

We assume

- Mortality risk is diversifiable, i.e. determine cost of hedging for a large number of contracts of similarly aged clients.
- Risky asset follows a *regime switching* process
  - Contracts are long-term (30 years)
  - Can impose views on possible future states of the economy

Key Idea: Separate *the cost of hedging* from *retail consumer behaviour*

Fees

We assume that two classes of fees are withdrawn continuously from the investment account $S(t)$

α_M is the MER for the underlying mutual fund

α_R is the *rider* which pays for the guarantee

$\alpha_{tot} = \alpha_M + \alpha_R$ is the total proportional fee

- **Note:** retail customer sees α_{tot}.
- Large α_{tot} often criticized by financial planners[7]

[7] I'll have more to say about this later.

Risk Neutral Regime Switching Process

$$dS = (r^j - \alpha_{tot})S\ dt + \sigma^j S\ dZ$$

$$r^j = \text{ interest rate in regime } j$$
$$\sigma^j = \text{ volatility in regime } j$$
$$dZ = \text{ increment of a Wiener process}$$

Probability of switching: Markov chain

$$Prob(i \rightarrow j) = q_{i,j}\ dt$$
$$Prob(stay\ in\ i) = 1 - \sum_{k \neq i} q_{i,k}\ dt$$

Why regime switching?

For long time periods (i.e. $20 - 30$ years)

- Reasonable fit to market[8]
- Parsimonious stochastic volatility and stochastic interest rate model
- Can easily interpret parameters and impose economic reasoning

Alternatives:

- Full stochastic volatility model (i.e. Heston)
  - → Does it make sense to calibrate to today's short term (i.e. max 5 year) options and project forward 30 years?
- Full stochastic interest rate model
  - → Same calibration problem

[8] Hardy, North Amer. Act. J. (2001)

Computational Procedure

Let $V(S, W, D, t)$[9] be the hedged value of this guarantee.

Assume that no contract holders will be alive at $t = T$

$$V(S, W, D, T) = 0$$

Usual dynamic programming approach: work backwards to today ($t = 0$).

- $t_{i+1}^{-} \rightarrow t_i^{+}$: solve regime switching PDE
  - Include fee withdrawals and death benefits
  - Cost of hedging $\rightarrow \mathbb{Q}$ measure.

Advance solution (backwards in time) across the event time

$$V(S^{-}, W^{-}, D^{-}, t_i^{-}) \;=\; V(S^{+}, W^{+}, D^{+}, t_i^{+}) + \text{ cash flows}$$

Then, solve PDE $t_i^{-} \rightarrow t_{i-1}^{+}$, etc.

[9] Assume single regime for ease of exposition

Across Event Times

Let γ be the impulse control applied to the system at t_i.

- Action due to the holder (e.g. surrender) or contract (e.g. ratchet)

Let

$$\begin{aligned}
\mathbf{x} &= (S, W, D) = \text{ state} \\
\mathbf{x}^+(\mathbf{x}(t_i^-), \gamma(\mathbf{x}(t_i^-)) &= \text{ state after control is applied} \\
&\quad\ \text{conditional on } \mathbf{x} = \mathbf{x}(t_i^-) \\
C(\mathbf{x}(t_i^-), \gamma(\mathbf{x}(t_i^-)) &= \text{ cash flow after control is applied} \\
&\quad\ \text{conditional on } \mathbf{x} = \mathbf{x}(t_i^-)
\end{aligned}$$

Move solution across event times

$$V(\mathbf{x}, t_i^-) = V(\mathbf{x}^+(\mathbf{x}, \gamma), t_i^+) + C(\mathbf{x}, \gamma(\mathbf{x}))$$

Hedging fee

Let α_R be the fee for this guarantee[10]

We can parameterize the solution as a function of this fee, i.e.

$$V = V(\mathbf{x}, t; \alpha_R)$$

The fee α_R^* which covers the cost of hedging can be determined by solving

$$V(S_0, S_0, S_0, 0; \alpha_R^*) = S_0$$

since no up-front fee is charged.[11]

[10] Recall that the total fee withdrawn $\alpha_{tot} = \alpha_R + \alpha_M$

[11] α_R^* found by a Newton iteration, each iteration requires a PDE solve.

Cost of hedging

Once the control γ is given

- Cost of hedging completely determined
- E.g. delta hedging can be carried out, delta determined from PDE solve under $\mathbb{Q}$ measure

Note: we have made no assumptions (up to now) about how the control γ is determined.

We have decoupled the specification of the control from the cost of hedging.

Worst Case Cost of Hedging

Under a worst case scenario, the cost of hedging is given by

$$V(\mathbf{x}, t_i^-) = \max_{\gamma}\left\{ V(\mathbf{x}^+(\mathbf{x},\gamma), t_i^+) + C(\mathbf{x}, \gamma(\mathbf{x}))\right\}$$

No-arbitrage price if retail customers could buy/sell annuities.

But, the market is not complete

- Upper bound to the cost of hedging these annuities
- Commonly argued that a retail customer would not choose to follow this strategy[12]

[12] However, empirical studies in Japanese market show moneyness of guarantee explains much policy holder behaviour (Knoller et al (2013))

More General Approach

Assume control is determined by a completely separate process. Example:

- Assume policy holder acts so as to maximize
  - After tax cash flows (e.g. Moenig and Bauer (2013))
  - A utility function of the cash flows
  - etc.

In a PDE context

- We solve a completely separate PDE system (under the $\mathbb{P}$ measure)
- This PDE system represents the value function being maximized by the policy holder, $\bar{V}(\mathbf{x}, t)$[13]
- Solve backwards in time $\rightarrow$ optimal control

[13] This is **not** the cost of hedging

Optimal control: consumption utility

Let $\mathbb{U}(\cdot)$ be a consumption utility function.

The control $\bar{\gamma}$ is determined by maximizing the policy holder value function $\bar{V}(\cdot)$

$$
\begin{aligned}
\bar{V}(\mathbf{x}, t_i^-) &= \bar{V}(\mathbf{x}^+(\mathbf{x}, \bar{\gamma}), t_i^+) + \mathbb{U}(C(\mathbf{x}, \bar{\gamma}(\mathbf{x}))) \\
\bar{\gamma} &= \arg\max_{\gamma} \left\{ \bar{V}(\mathbf{x}^+(\mathbf{x}, \gamma), t_i^+) + \mathbb{U}(C(\mathbf{x}, \gamma(\mathbf{x}))) \right\}
\end{aligned}
$$

This control is then fed into the cost of hedging $V(\cdot)$

$$
V(\mathbf{x}, t_i^-) = V(\mathbf{x}^+(\mathbf{x}, \bar{\gamma}), t_i^+) + C(\mathbf{x}, \bar{\gamma}(\mathbf{x}))
$$

Numerical Example: $\mathbb{Q}$ measure regime switching[14]

Parameter			Value	
Volatility	σ_1	σ_2	0.0832	0.2141
Risk-free rate	r_1	r_2	0.0521	0.0521
Rate of transition	$q^{\mathbb{Q}}_{1\to 2}$	$q^{\mathbb{Q}}_{2\to 1}$	0.0525	0.1364
Initial regime	I			1
Initial investment	$S(0)$			100
MER for mutual fund	α_M			100 bps
Contract rate	G			0.05
Bonus rate	B			0.05
Initial age	x_0			65
Expiry time	T			57
Mortality data				Padiska et al (2005)
Ratchets				Triennial
Withdrawals				Annual

[14] Parameters from O'Sullivan and Moloney (2010), calibrated to FTSE options, January, 2007

Hedging Costs: Worst Case and Contract Rate

Case	Hedging fee (bps)			
	Worst	Contract	Worst	Contract
	Death Benefit		No Death Benefit	
Initial Regime Low Vol	54	48	27	19
Initial Regime High Vol	158	113	86	52

Table: Hedging fee α_R: regime switching[15] [16] [17]

- **Worst:** assume holder's strategy produces highest possible hedging cost
- **Contract:** assume holder always withdraws at rate $G * W$, i.e. no surrender, no bonus

[15] Recall that $\alpha_{tot} = \alpha_R + \alpha_M$.

[16] Note high value of Death Benefit.

[17] Always withdrawing at contract rate is still quite valuable.

Perturb parameters from O'Sullivan and Moloney (2010)

Case	Hedging fee (bps)			
	Worst	Contract	Worst	Contract
	Death Benefit		No Death Benefit	
Base $(\sigma_1, \sigma_2) = (.08, .21)$ $(r_1, r_2) = (.05, .05)$	54	48	27	19
$(r_1, r_2) = (.02, .08)$	239	212	129	104
$(\sigma_1, \sigma_2) = (0.15, 0.25)$	133	123	70	51

Table: Hedging fee α_R: regime switching[18] Initial regime: low volatility.
[19]

- **Worst:** assume holder's strategy produces highest possible hedging cost
- **Contract:** assume holder always withdraws at rate $G * W$, i.e. no surrender, no bonus

[18] Recall that $\alpha_{tot} = \alpha_R + \alpha_M$.

[19] Fee very sensitive to initial low r regimes, volatility.

The best things in life aren't fees

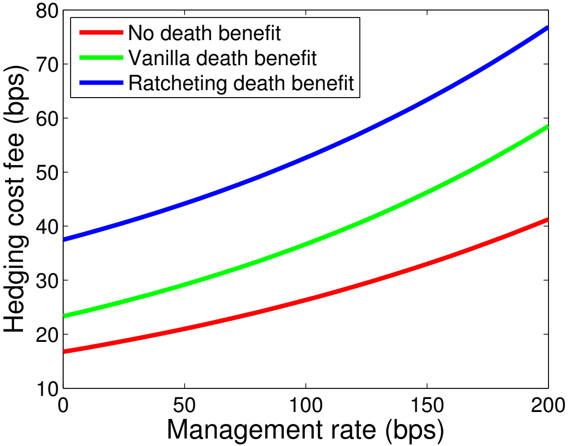

Figure: Loss maximizing. α_R is a superlinear function of α_M. Recall that $\alpha_{tot} = \alpha_M + \alpha_R$.[20]

[20] Use of underlying asset mutual fund with high MER makes guarantee very expensive

Loss Maximizing Withdrawal Strategies: $t = 1, 2, \ldots 6$

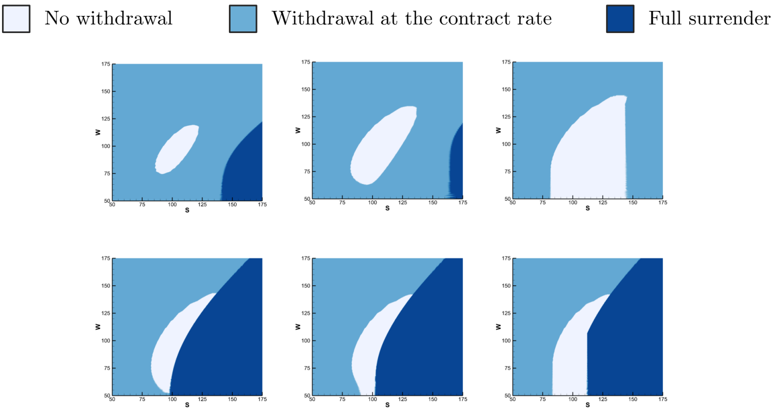

Figure: Loss-maximizing strategies at $D = 100$ under regime 2 (high vol). X-axis: risky asset account S. Y-axis: withdrawal account W. Note: loss-maximizing control: no partial withdrawals.

Alternate assumption: control determined by utility consumption model

Assume HARA utility of consumption

$$\mathbb{U}(X) = \begin{cases} \log(aX + b) & p = 0 \\ \frac{1-p}{p}\left(\frac{aX}{1-p} + b\right)^p & 0 < p < 1 \\ aX & p = 1 \end{cases}$$

p, a, b are parameters.

Now, determine hedging fee, solve two systems of PDEs

A PDE for $\bar{V}$ determines the withdrawal strategy (holder utility under $\mathbb{P}$ measure)

B PDE for V determines the hedging cost, uses strategy from (A) ($\mathbb{Q}$ measure cash flows)

Utility based control: cost of hedging

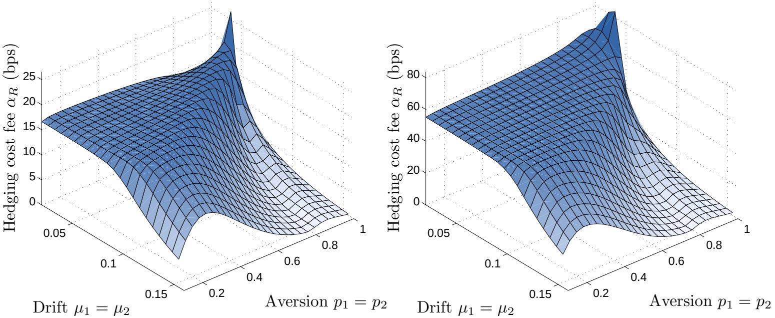

Figure: Left: initial regime low vol. Right: initial regime high vol. Effects of varying drift and risk-aversion on the hedging cost fee. No death benefit.

- Upper right maximum: parameters reduce to worst case hedging cost.
- Lower right corner: unrealistically large $\mathbb{P}$ measure drift.
- Flat region: always withdraw at contract rate G

What is a good assumption for retail customer behaviour?

- Utility maximization
  - $\rightarrow$ Suggests that the typical assumption *always withdraw at contract rate* is reasonable
- But we know from
  - Japanese studies[21]
  - $\rightarrow$ Consumers will surrender (maximize hedger's losses) if

$$(\text{ surrender value }) \gg (\text{ continue to hold value })$$

[21] Knoller et al (2013), "On the Propensity to Surrender a Variable Annuity Contract - An Empirical Analysis of Dynamic Policyholder Behavior,"

Ho et al (2005) model

Simple one parameter model $\alpha \in [0, \infty]$ (determined empirically)

- Withdraw at contract rate unless

$$
\begin{aligned}
&(V_{loss\ maximizing} - V_{contract\ rate}) \\
&> \alpha \text{ (contract rate withdrawal)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha &= 0 \;;\quad \text{loss-maximizing} \\
&= \infty \;;\quad \text{withdraw at contract rate}
\end{aligned}
$$

- With this model

$$
(fee:\ contract\ rate) \leq (actual\ fee) \leq (fee:\ worst\ case)
$$

Summary: GLWBs

Separate cost of hedging from retail consumer behaviour

- Two PDE systems: hedging cost, control strategy of consumer
- Control strategies of consumer
  - Worst case for hedger
  - Maximize utility
  - Maximize after tax expected value
  - Function of moneyness of guarantee

Surprising result

- For a large range of utility parameters $\rightarrow$ retail customer always withdraws at contract rate
- But we need to be cognizant of worst case hedging cost

Note: even withdrawing always at contract rate $\rightarrow$ guarantee still quite valuable

$\rightarrow$ Sensitive to interest rates and volatility assumptions

Conclusions: Fees too high or too low?

- These products are in high demand from retail customers.
- Typical fees $\alpha_R = 50 - 75$ bps seem to be low considering interest rate, volatility risk, drag from MER of underlying fund
- **But** the total fee $\alpha_R + \alpha_M$ seems large to customers
- Solution:
  - → Use cheap index fund as underlying asset[22]
  - → Carefully engineer product to eliminate high cost options, but still produce a useful product. [23]

GLWDBs are socially useful products, that people want

- Why not manufacture a product at the lowest possible cost to satisfy this demand?

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[22] Vanguard now does this.

[23] A bad idea: a well known pension consultant suggests *managing volatility*. This reduces the value of the guarantee by having a high bond allocation.