Decumulation of Retirement Savings: *The Nastiest, Hardest Problem in Finance*

Peter Forsyth[1]

[1]Cheriton School of Computer Science
University of Waterloo

CAIMS
Thursday, June 16, 2022
6:15 PM (Waterloo time)

Motivation

Defined Benefit Plans (DB) are disappearing

$\rightarrow$ Corporations/governments no longer willing to take risk of DB plans

A retiree with savings in a DC plan (i.e. an RRSP) has to decide on

- An investment strategy (stocks vs. bonds)
- A decumulation schedule

The retiree now has two major sources of risk

- Investment risk
- Longevity risk (running out of cash before death)

William Sharpe (Nobel Laureate in Economics) calls this

"The nastiest hardest problem in finance"

The Four per Cent Rule

Based on rolling 30-year historical periods, Bengen (1994) showed:

A retiree who

- Invested in a portfolio of 50% bonds, 50% stocks (US), rebalanced annually
- Withdrew 4% of initial capital (adjusted for inflation) annually

  → Would never have run out of cash, over any rolling 30-year period (from 1926)

Criticism

- Simplistic asset allocation strategy
- Simplistic withdrawal strategy
- Rolling 30 year periods contain large overlaps
  → Underestimates risk of portfolio depletion

Objective of this talk

Determine a decumulation strategy which has

- Variable withdrawals (minimum and maximum constraints)
- Minimizes risk of portfolio depletion
- Maximizes total expected withdrawals
- Allows for dynamic, non-deterministic asset allocation

We will treat this as a problem in optimal stochastic control

Formulation

Investor has access to two funds

- A broad stock market index fund
  - *Amount* in stock index S_t
- A constant maturity bond index fund
  - *Amount* in bond index B_t

$$\text{Total Wealth } W_t = S_t + B_t \quad (1)$$

Model the returns of both indexes

- Parametric, jump diffusion
- Non-zero stock-bond correlation

• Fit parameters to market data 1926:1-2019:12
$\hookrightarrow$ All returns adjusted for inflation

Notation

Withdraw/rebalance at discrete times $t_i \in [0, T]$

The investor has two controls at each rebalancing time

$$\begin{aligned} q_i &= \text{Amount of withdrawal} \\ p_i &= \text{Fraction in stocks after withdrawal} \end{aligned} \tag{2}$$

At t_i, the investor withdraws q_i

$$\begin{aligned} W_i^- &= \overbrace{S_i^- + B_i^-}^{\textit{wealth before withdrawal}} \\ W_i^+ &= W_i^- - q_i \end{aligned} \tag{3}$$

Then, the investor rebalances the portfolio

$$\begin{aligned} S_i^+ &= p_i W_i^+ \\ B_i^+ &= (1 - p_i) W_i^+ \end{aligned} \tag{4}$$

Can show that

$$q_i = q_i(W_i^-) \quad ; \quad p_i = p_i(W_i^+)$$

Controls

Constraints on controls

$$
\begin{aligned}
q_i &\in [q_{\min}, q_{\max}] \quad ; \quad \text{withdrawal amount} \\
p_i &\in [0, 1] \quad ; \quad \text{fraction in stocks} \\
&\quad \text{no shorting, no leverage}
\end{aligned}
$$

Set of controls

$$
\mathcal{P} = \{(q_i(\cdot), p_i(\cdot))) : i = 0, \ldots, M\} \tag{5}
$$

Reward and Risk

Reward: Expected total Withdrawals (EW)

$$\text{EW} = E\Big[\overbrace{\sum_i q_i}^{\textit{total withdrawals}} \Big]$$

$$E[\cdot] = \text{ Expectation}$$

Risk measure: Expected Shortfall ES

$$ES(5\%) \equiv \Big\{ \text{Mean of worst 5\% of } W_T \Big\}$$

$$W_T = \text{ terminal wealth at } t = T$$

ES defined in terms of final wealth, *not losses*[1]

$\rightarrow$ Larger is better

[1]ES is basically the negative of CVAR

Objective Function

Multi-objective problem $\rightarrow$ scalarization approach for Pareto points

Find controls $\mathcal{P}$ which maximize (scalarization parameter $\kappa > 0$)

$$\sup_{\mathcal{P}} \Big\{ EW + \kappa \; ES \Big\}$$

$$\sup_{\mathcal{P}} \Bigg\{ \overbrace{E_{\mathcal{P}}[\sum_i q_i]}^{total\ \ withdrawals} + \kappa \overbrace{\left(\frac{E_{\mathcal{P}}[W_T \; \mathbf{1}_{W_T \le W^*}]}{.05} \right)}^{mean\ \ worst\ \ 5\%\ \ outcomes} \Bigg\}$$

$$\text{s.t.} \qquad Prob[W_T \le W^*] = .05$$

Varying κ traces out the efficient frontier in the (EW, ES) plane[2]

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[2]ES is not formally time-consistent. We assume that the investor follows the *induced time consistent* policy. See (Forsyth, SIFIN, 2019). The induced time consistent control is identical to the pre-commitment control at $t = 0$.

Scenario: all amounts indexed to inflation

- DC account at $t = 0$ (age 65) \$1,000K (one million)
- Minimum withdrawal from DC account \$35K per year[3]
- Maximum withdrawal from DC \$60K per year
- Annual rebalancing/withdrawals
- Owns mortgage-free real estate worth \$400K

Investment Horizon

- $T = 30$ years, i.e. from age 65 to 95

[3] Assume gov't benefits of 22K/year. Minimum income $\simeq 22K + 35K = 57K$/year.

Scenario II

Why do we include real estate in the scenario?

Since $q_{\min} = 35K$ per year, W_t can become negative

- When $W_t < 0$, the retiree is borrowing, using a reverse mortgage
  - Reverse mortgages allow borrowing of 50% of home value
  - In our case: $200K
- Once $W_t < 0$
  - All stocks are liquidated
  - Debt accumulates at borrowing rate
- If $W_T > 0$, then real-estate is a bequest
- Real estate is a hedge of last resort: not fungible with other wealth
  - This mental bucketing of real estate is a well-known behavioural finance result. I also observe this with my fellow retirees.

Numerical Method

Dynamic programming

- Conditional expectations at t_i^+
  - Solve linear 2-d PIDE
  - Use ϵ-monotone Fourier method (Forsyth and Labahn (2019))
- Optimal controls at each rebalancing time
  - Discretize controls
  - Find maximum by exhaustive search
- Guaranteed to converge to the solution as discretization parameters $\rightarrow 0$

Data

Center for Research in Security Prices (CRSP) US

- Cap weighted index, all stocks on all major US exchanges 1926:1-2019:12
- US 10 year Treasury index
- Monthly data, inflation adjusted by CPI

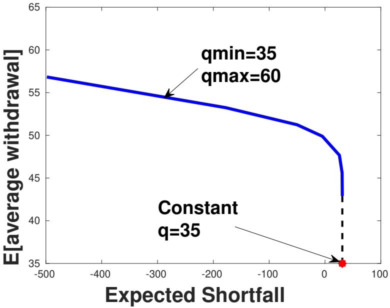
Synthetic Market

- Stock/bond returns driven by parametric jump-diffusion model, calibrated to data
- Optimal controls computed in the *synthetic* market

Historical market

- Stock/bond returns from stationary block bootstrap resampling of actual data
- No assumptions about stock/bond processes
- Used to test controls computed in the synthetic market

EW-ES efficient frontier (Units: thousands)



- ES is the mean of the worst 5% of outcomes
- Each pt on curve, different κ
- Reverse mortgage hedge
  - $\rightarrow$ Any point $ES > -200K$ is acceptable

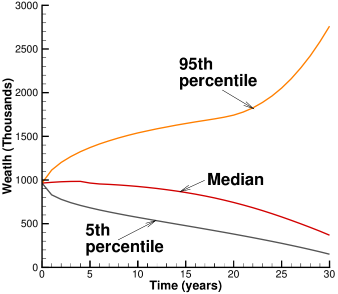
Note Efficient Frontier almost vertical at right hand end

- Base case: constant withdrawal 35K/year
- Tiny increase in risk (smaller ES)

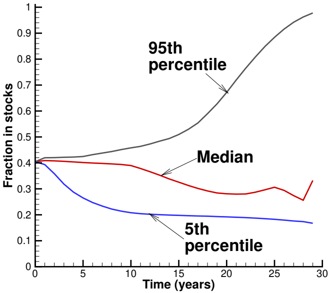
$\rightarrow$ Average withdrawal increases to 50K per year (never less than 35K)

Point on Frontier: Expected average withdrawals = 51K/year

Percentiles: wealth



Percentiles: fraction in equities



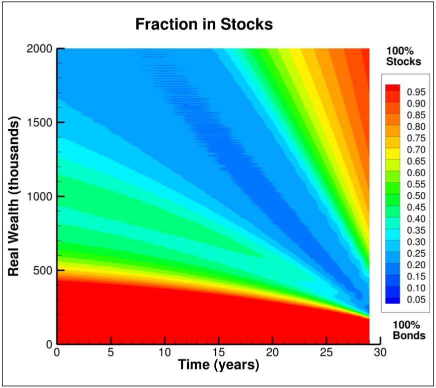
Bootstrap resamples of optimal strategy (5% of initial capital on average per year, inflation adjusted)

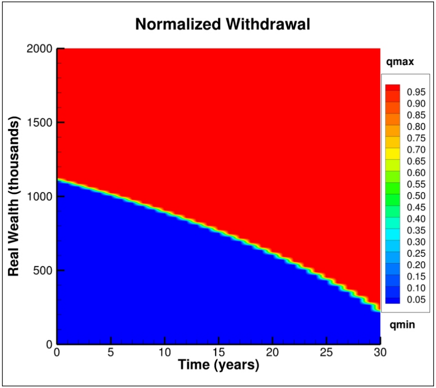
$\rightarrow$ $ES \simeq -17K$

Bootstrap resamples of Bengen 4% rule (4% of initial capital per year)

$\rightarrow$ $ES \simeq -300K$

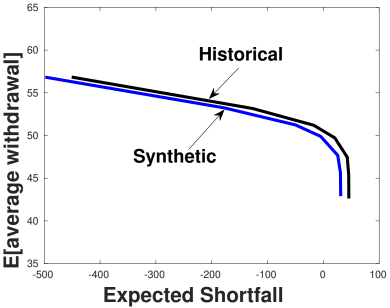
Expected Average Withdrawals: 51K/year





- Withdrawal controls $\simeq$ *bang-bang*, i.e. only withdraw either $q_{\min}$ or $q_{\max}$.
- Median $W_t \simeq 1000K \rightarrow 300K$

Robustness Check: Efficient Frontier (Units: thousands)



Controls computed and stored in the *synthetic* market

- Parametric model calibrated to historical data

Controls tested[4] in the bootstrapped historical market

$\rightarrow$ Controls are robust to parametric model misspecification

[4] "Out-of-sample" test.

Conclusions

- Optimal strategy: flexible withdrawals, dynamic stock-bond allocation
  - → Less risk, higher average withdrawals compared to 4% rule
  - → Bootstrap resampling ⇒ controls are robust
- In the continuous withdrawal limit
  - → Optimal withdrawals are *bang-bang*, i.e. only withdraw at either maximum or minimum rate[5]
- Discrete rebalancing: withdrawal controls are very close to bang-bang
- Intuition: if you are lucky, and make money in stocks, take money off the table and enjoy
  - → Otherwise: sit tight

[5] Proof: Forsyth (North American Actuarial Journal, 2022), independent of risk measure.