

Mechanism
Design

Kate Larson

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Game Theory

- Given a game we are able to analyse the strategies agents will follow

Social Choice

- Given a set of agents' preferences we can choose some outcome

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Today **Mechanism Design**

- Game Theory + Social Choice
- Goal of Mechanism Design is to
 - Obtain some outcome (function of agents' preferences)
 - But agents are rational
 - They may lie about their preferences

Goal

Define the rules of a game so that in equilibrium the agents do what we want.

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- Set of possible outcomes O
- Set of agents N , $|N| = n$
 - Each agent i has type $\theta_i \in \Theta_i$
 - Type captures all private information that is relevant to the agent's decision making
- Utility $u_i(o, \theta_i)$ over outcome $o \in O$
- Recall: goal is to implement some system wide solution
 - Captured by a social choice function

$$f : \Theta_1 \times \dots \times \Theta_n \rightarrow O$$

where $f(\theta_1, \dots, \theta_n) = o$ is a collective choice

Examples of Social Choice Functions

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- **Voting:**
 - Choose a candidate among a group
- **Public project:**
 - Decide whether to build a swimming pool whose cost must be funded by the agents themselves
- **Allocation:**
 - Allocate a single, indivisible item to one agent in a group

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Recall that we want to implement a social choice function

- Need to know agents' preferences
- They may not reveal them to us truthfully

Example:

I like the
bear the
most!



No, I do!



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- By having agents interact through an institution we might be able to solve the problem
- Mechanism:

$$M = (S_1, \dots, S_n, g(\cdot))$$

where

- S_i is the strategy space of agent i
- $g : S_1 \times \dots \times S_n \rightarrow O$ is the outcome function

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Definition

A mechanism $M = (S_1, \dots, S_n, g(\cdot))$ **implements social choice function** $f(\Theta)$ if there is an equilibrium strategy profile

$$s^* = (s_1^*(\theta_1), \dots, s_n^*(\theta_n))$$

of the game induced by M such that

$$g(s_1^*(\theta_1), \dots, s_n^*(\theta_n)) = f(\theta_1, \dots, \theta_n)$$

for all

$$(\theta_1, \dots, \theta_n) \in \Theta_1 \times \dots \times \Theta_n$$

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We did not specify the type of equilibrium in the definition

- Nash

$$u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i) \geq u_i(g(s_i'(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i)$$

$$\forall i, \forall \theta_i, \forall s_i' \neq s_i^*$$

- Bayes-Nash

$$E[u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i)] \geq E[u_i(g(s_i'(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i)]$$

$$\forall i, \forall \theta_i, \forall s_i' \neq s_i^*$$

- Dominant

$$u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i) \geq u_i(g(s_i'(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i)$$

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Implementation

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Properties for Mechanisms

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Groves Mechanisms

- Efficiency
 - Select the outcome that maximizes total utility
- Fairness
 - Select outcome that minimizes the variance in utility
- Revenue maximization
 - Select outcome that maximizes revenue to a seller (or, utility to one of the agents)
- Budget-balanced
 - Implement outcomes that have balanced transfers across agents
- Pareto Optimal
 - Only implement outcomes o^* for which for all $o' \neq o^*$ either $u_i(o', \theta_i) = u_i(o^*, \theta_i) \forall i$ or $\exists i \in N$ with $u_i(o', \theta_i) < u_i(o^*, \theta_i)$

Participation Constraints

We can not force agents to participate in the mechanism.
Let $\hat{u}_i(\theta_i)$ denote the (expected) utility to agent i with type θ_i of its outside option.

- **ex ante individual-rationality:** agents choose to participate before they know their own type

$$E_{\theta \in \Theta} [u_i(f(\theta), \theta_i)] \geq E_{\theta_i \in \Theta_i} \hat{u}_i(\theta_i)$$

- **interim individual-rationality:** agents can withdraw once they know their own type

$$E_{\theta_{-i} \in \Theta_{-i}} [u_i(f(\theta_i, \theta_{-i}), \theta_i)] \geq \hat{u}_i(\theta_i)$$

- **ex-post individual-rationality:** agents can withdraw from the mechanism at the end

$$u_i(f(\theta), \theta_i) \geq \hat{u}_i(\theta_i)$$

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Definition

A direct mechanism *is a mechanism where*

$$S_i = \Theta_i \text{ for all } i$$

and

$$g(\theta) = f(\theta) \text{ for all } \theta \in \Theta_1 \times \dots \times \Theta_n$$

Incentive Compatibility

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Definition

A direct mechanism is **incentive compatible** if it has an equilibrium s^* where

$$s_i^*(\theta_i) = \theta_i$$

for all $\theta_i \in \Theta_i$ and for all i . That is, truth-telling by all agents is an equilibrium.

Definition

A direct mechanism is **strategy-proof** if it is incentive compatible and the equilibrium is a dominant strategy equilibrium.

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Theorem

Suppose there exists a mechanism $M = (S_1, \dots, S_n, g(\cdot))$ that implements social choice function f in dominant strategies. Then there is a direct strategy-proof mechanism M' which also implements f .

[Gibbard 73; Green & Laffont 77; Myerson 79]

“The computations that go on within the mind of any bidder in the nondirect mechanism are shifted to become part of the mechanism in the direct mechanism.”

[McAfee & McMillan 87]

Revelation Principle: Intuition

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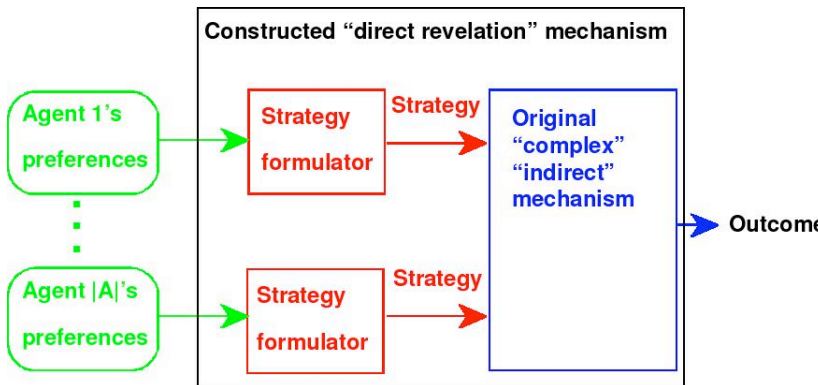
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- **Literal interpretation:** Need only study direct mechanisms
 - A modeler can limit the search for an optimal mechanism to the class of direct IC mechanisms
 - If no direct mechanism can implement social choice function f then no mechanism can
 - Useful because the space of possible mechanisms is huge

Practical Implications

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Groves Mechanisms

- Incentive-compatibility is “free”
 - Any outcome implemented by mechanism M can be implemented by incentive-compatible mechanism M'
- “Fancy” mechanisms are unnecessary
 - Any outcome implemented by a mechanism with complex strategy space S can be implemented by a direct mechanism

BUT Lots of mechanisms used in practice are not direct and incentive-compatible!

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Quick Review

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We now know

- What a mechanism is
- What it means for a SCF to be dominant-strategy implementable
- Revelation Principle

We do not yet know

- What types of SCF are dominant-strategy implementable

Gibbard-Satterthwaite Impossibility

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Theorem

Assume that

- O is finite and $|O| \geq 3$,
- each $o \in O$ can be achieved by SCF f for some θ , and
- Θ includes all possible strict orderings over O .

Then f is implementable in dominant strategies (strategy-proof) if and only if it is dictatorial.

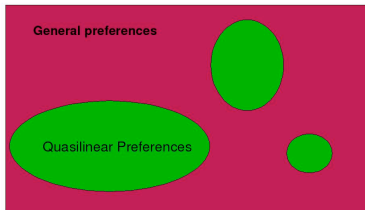
Definition

SCF f is **dictatorial** if there is an agent i such that for all θ

$$f(\theta) \in \{o \in O \mid u_i(o, \theta_i) \geq u_i(o', \theta_i) \forall o' \in O\}$$

Circumventing Gibbard-Satterthwaite

- Use a weaker equilibrium concept
- Design mechanisms where computing a beneficial manipulation is hard
- Randomization
- Restrict the structure of agents' preferences



Single-Peaked Preferences

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- Define $A = [0, 1]$ be the outcome space
- Each agent $i \in N$ has a preference \succeq_i over A such that $\exists p_i \in A$ such that for all $\{x\} \in A \setminus \{p_i\}$ and for all $\lambda \in [0, 1)$, $(\lambda x + (1 - \lambda)p_i) \succeq_i x$.
 - political decisions
 - facility location
 - temperature settings
- The Median-Voter rule is strategy-proof.

Single-Peaked Preferences

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 - political decisions
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Quasi-linear preferences

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- Outcome $o = (x, t_1, \dots, t_n)$
 - x is a “project choice”
 - $t_i \in \mathbb{R}$ are transfers (money)
- Utility function of agent i

$$u_i(o, \theta_i) = v_i(x, \theta_i) - t_i$$

- Quasi-linear mechanism

$$M = (S_1, \dots, S_n, g(\cdot))$$

where

$$g(\cdot) = (x(\cdot), t_1(\cdot), \dots, t_n(\cdot))$$

Social Choice Functions and Quasi-linearity

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- SCF is **efficient** if for all θ

$$\sum_{i=1}^n v_i(x(\theta), \theta_i) \geq \sum_{i=1}^n v_i(x'(\theta), \theta_i) \forall x'(\theta)$$

This is also known as **social welfare maximizing**

- SCF is **budget-balanced** if

$$\sum_{i=1}^n t_i(\theta) = 0$$

Weakly budget-balanced if

$$\sum_{i=1}^n t_i(\theta) \geq 0$$

Groves Mechanisms [Groves 73]

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A **Groves mechanism** $M = (S_1, \dots, S_n, (x, t_1, \dots, t_n))$ is defined by

- Choice rule

$$x^*(\theta) = \arg \max_x \sum_i v_i(x, \theta_i)$$

- Transfer rules

$$t_i(\theta) = h_i(\theta_{-i}) - \sum_{j \neq i} v_j(x^*(\theta), \theta_j)$$

where $h_i(\cdot)$ is an (arbitrary) function that does not depend on the reported type θ'_i of agent i .

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Theorem

Groves mechanisms are strategy-proof and efficient.

We have gotten around Gibbard-Satterthwaite.

Proof

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Agent i 's utility for strategy $\hat{\theta}_i$, given $\hat{\theta}_{-i}$ from agents $j \neq i$ is

$$\begin{aligned} u_i(\hat{\theta}_i) &= v_i(x^*(\hat{\theta}, \theta_i) - t_i(\hat{\theta})) \\ &= v_i(x^*(\hat{\theta}, \theta_i) + \sum_{j \neq i} v_j(x^*(\hat{\theta}, \hat{\theta}_j) - h_j(\hat{\theta}_{-i})) \end{aligned}$$

Ignore $h_j(\hat{\theta}_{-i})$ and notice $x^*(\hat{\theta}) = \arg \max_x \sum_j v_j(x, \hat{\theta}_j)$
i.e it maximizes the sum of reported values. Therefore, agent i should announce $\hat{\theta}_i = \theta_i$ to maximize its own payoff.

Thm: Groves mechanisms are unique (up to $h_j(\theta_{-i})$).

Vickrey-Clarke-Groves Mechanism

aka Clarke mechanism, aka Pivotal mechanism

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- Implement efficient outcome

$$x^* = \arg \max_x \sum_i v_i(x, \theta_i)$$

- Compute transfers

$$t_i(\theta) = \sum_{j \neq i} v_j(x^{-i}, \theta_j) - \sum_{j \neq i} v_j(x^*, \theta_j)$$

where $x^{-i} = \arg \max_x \sum_{j \neq i} v_j(x, \theta_j)$

VCG are efficient and strategy-proof.

VCG Mechanism

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Agent's equilibrium utility is

$$\begin{aligned}u_i((x^*, t), \theta_i) &= v_i(x^*, \theta_i) - \left[\sum_{j \neq i} v_j(x^{-i}, \theta_j) - \sum_{j \neq i} v_j(x^*, \theta_j) \right] \\&= \sum_{j=1}^n v_j(x^*, \theta_j) - \sum_{j \neq i} v_j(x^{-i}, \theta_j) \\&= \text{marginal contribution to the welfare of the system}\end{aligned}$$

Examples

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