CS 886: Multiagent Systems

Fall 2016

Kate Larson

Review: Introduction to Social Choice

- Social choice is a mathematical theory which studies how to aggregate individual preferences

- **Voting Model**
  - Set of **voters** N={1,...,n}
  - Set of **alternatives** A, |A|=m
  - Each voter has a **ranking** over the alternatives (**preferences**)
  - **Preference profile** is a collection of voters’ rankings

1	2	3
a	b	c
b	a	a
c	c	b

Arrow’s Theorem (1951)

If there are **at least three alternatives**, then any **universal social welfare function** that satisfies the **Pareto condition** and is **IIA** must be a **dictatorship**.

Manipulation of Voting Rules

- So far we have assumed that voters truthfully report their preferences

True Preferences

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

Reported Preferences

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Example from A. Procaccia

Strategyproofness

A voting rule is **strategyproof (SP)** if no voter can ever benefit by lying about its preferences.

Formally, let

- f be the voting rule
- $\succ=(\succ_1, \succ_2,\ldots, \succ_n)=(\succ_i, \succ_{-i})$ be a preference profile

Then f is SP if

$$\forall \succ, \forall i \in N, \forall \succ_i', f(\succ) \succeq_i f(\succ_i', \succ_{-i})$$

Examples of Strategyproof Voting Rules

- **Dictatorship**
  - There is a voter that always gets its most preferred alternative

- **Constant function**
  - The same outcome is chosen no matter how voters vote

Gibbard-Sattherthwaite Theorem

- A voting rule is **onto** if any alternative can be chosen.

If there are **at least three alternatives**, then any **universal** and **onto** **social welfare function** that is **strategyproof** must be a **dictatorship**.

Now what?

- Restrict to two alternatives
- Restrict the preferences
- Use computational complexity as a barrier to manipulation

Single-peaked preferences

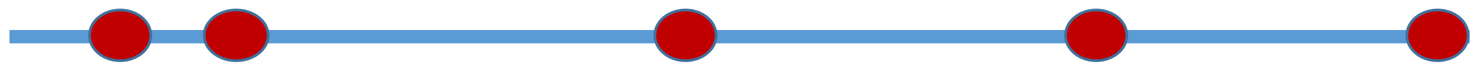
- Assume there is a linear ordering L over alternatives. Then for any three candidates a,b,c

$$(aLbLc \vee cLbLa) \Rightarrow (\forall v \in V)[a \succ_v b \Rightarrow b \succ_v c]$$

Single-peaked preferences

- **Right-most peak rule**: return the right-most peak

- **Mid-peak rule**: return the average of the leftmost and rightmost peaks



Single-peaked preferences

- **Median rule**: return the median peak

- The median rule is
  - Onto
  - Non-dictatorial
  - Selects a Condorcet winner
  - Is strategy-proof

Now what?

- Restrict to two alternatives
- Restrict the preferences
- **Use computational complexity as a barrier to manipulation**

Complexity and Manipulation

- While manipulation is always possible in theory, what about in practice?

- Are there reasonable voting rules where manipulation is a hard computational problem? [Bartholdi, Tovey and Trick, 1989]

The Manipulation Problem

- Given
  - A profile of votes cast by everyone but the manipulator
  - A preferred alternative *p*
- Question
  - Is there a vote that the manipulator can cast so that *p* wins?

1	2	3
b	b	
a	a	
c	c	
d	d	

p=a?

Greedy Algorithm for Manipulation

- Place *p* at the top of the ranking

- While there are unranked alternatives
  - Select alternative *a* such that it can be put into the next spot in the ranking while still ensuring that *p* wins
  - If no such *a* exists, return false

Manipulating Borda (p=a)?

1	2	3
b	b	
a	a	
c	c	
d	d	

Manipulating Copeland (p=a)?

1	2	3	4	5
a	b	e	e	
b	a	c	c	
c	d	b	b	
d	e	a	a	
e	c	d	d	

BTT Conditions

- A voting rule satisfies **BTT conditions** if
  - It can run in polynomial time.
  - For every profile $\succ$ and for every alternative a, the rule assigns a score S($\succ$,a).
  - For every profile $\succ$, the alternative with the maximum score wins.
  - The following monotonicity condition holds

$$\forall v, \forall \succ_v, \succ_v' \ \text{ if } \{b|a \succ_v b\} \subseteq \{b|a \succ_v' b\} \text{ then } S(\succ_v, a) \leq S(\succ_v', a)$$

Bartholdi et al, (1989)

- **Theorem**: The manipulation problem can be solved in polynomial time for any rule satisfying the BTT conditions.

- Many voting rules are easy to manipulate:
  - Plurality, Plurality with runoff, Borda, Veto, Copeland, Maximin,…

What is Hard to Manipulate?

- **STV** is hard to manipulate

- Also
  - **Nanson**: Borda with elimination where in each round you eliminate all alternatives with less than the average Borda score
  - **Baldwin**: Borda with elimination where in each round you eliminate the alternative with the lowest Borda score
  - “Tweaked” versions of many voting rules (Conitzer and Sandholm, 2003)