Efficient Algorithms for Integer and Polynomial Matrices

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The Plan

- Set of 8 lectures (or modules). Likely 2 per day.
- Focus is (mostly) on linear algebra for matrices of integers and matrices of polynomials

Slides

- Slides are available at

https://cs.uwaterloo.ca/~glabahn/talks.html

Topics I and 2

Topic 1: Linear Algebra over a field

- What do we want to compute (e.g. rank, solve, etc)?
- What are the costs and how do we measure this. Notation.
- Incorporating matrix multiplication: better performance and better complexity

Topic 2: Linear Algebra over over $\mathbb{Z}$ and $\mathbb{K}[z]$

- Issues when computing over domains such as $\mathbb{Z}$ and $\mathbb{K}[z]$
- Three cost models

Topics 3 and 4

Topic 3: Linear Solving and Tools

- Fraction-free arithmetic, Chinese remaindering, p-adic lifting
- Higher-order lifting
- Integrality certification

Topic 4: Matrix Normal Forms

- Domains : $\mathbb{Z}$, $\mathbb{K}[x]$, $\mathbb{K}[x][D_x]$
- Hermite,Smith, Popov normal forms
- Hermite vs Popov (via Gröbner)

Topics 5 and 6

Topic 5: More on Hermite Form

- Classical elimination procedures for integer matrices and polynomial matrices
- Computing transforming equation

Topic 6: Fast Hermite, Polynomial Matrices

- Polynomial matrix algebraic objects
- Modules: Kernels, Row/Column spaces, etc
- Fast Hermite normal form

Topics 7 and 8

Topic 7: Fast Hermite, Integer Matrices

- Minimal denominators
- Incorporating Smith normal forms
- Computing modulo s_n and Duality (s_n largest invariant factor)
- Column Howell form

Topic 8: Fast Smith, Integer Matrices

- Fast Smith form
- Smith Massagers
- Fast Smith form and multipliers

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