Experiments with Pen-Based Math Systems:

MathBrush Case Study

George Labahn
Symbolic Computation Group
University of Waterloo

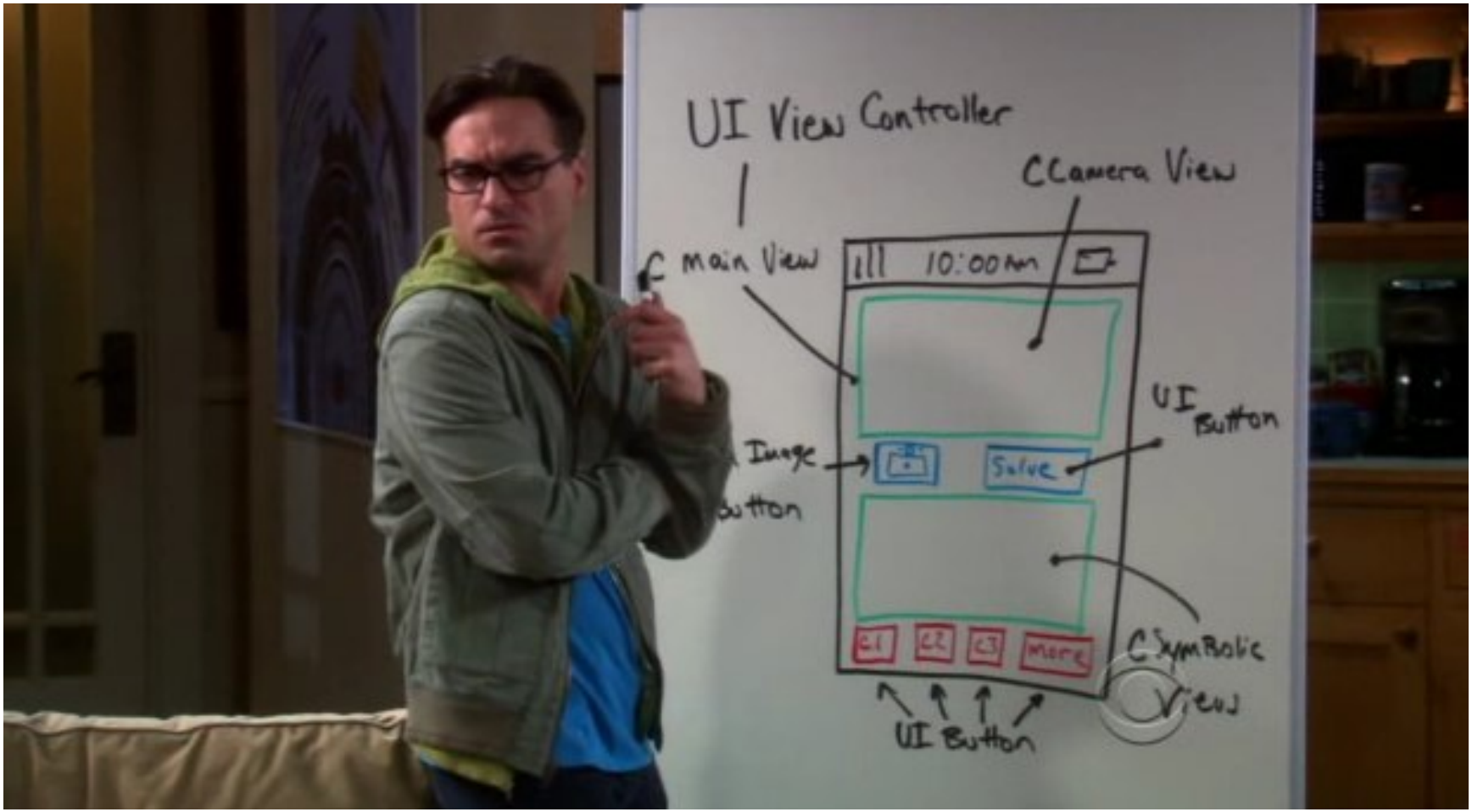
People Involved

- George Labahn
- Mirette Marzouk
- Scott Maclean

- Ed Lank
- David Tausky
- Winnie Lam
- Connor Flood

- Robert Amelard
- Mark Prosser
- Ian Rutherford

Lenowolooppali Differential Equation Solver



Talk Outline

- Pen math objectives, challenges and older systems
- MathBrush I
- MathBrush II
- MathBrush Recognizer
- Penmath editing
- Newer pen math systems

Objectives

- Investigate the use of pen-based devices for mathematical computation and exploration
- Study the key issues when combining pen-based interfaces with Computer Algebra Systems (CAS)
- Build an experimental pen-based math system to allow us to investigate the various components that make up a pen-math system e.g. ink input, editing, recognition, computation
  - MathBrush

Obvious Motivation : Which is easier?

$$\int \frac{(3x^2+2)\sin(x^3+2x-1)}{\cos(x^3+2x-1)^3 + 12\cos(x^3+2x-1)^2-3}\,dx$$

- Maple

```
Int((3*x^2+2)*sin(x^3+2*x-1)/(cos(x^3+2*x-1)^3 + 12*cos(x^3+2*x-1)^2-3),x);
```

- Mathematica

```
Integrate[(3*x^2+2)*Sin[x^3+2*x-1]/(Cos[x^3+2*x-1]^3 + 12*Cos[x^3+2*x-1]^2-
    3),x]
```

- Latex

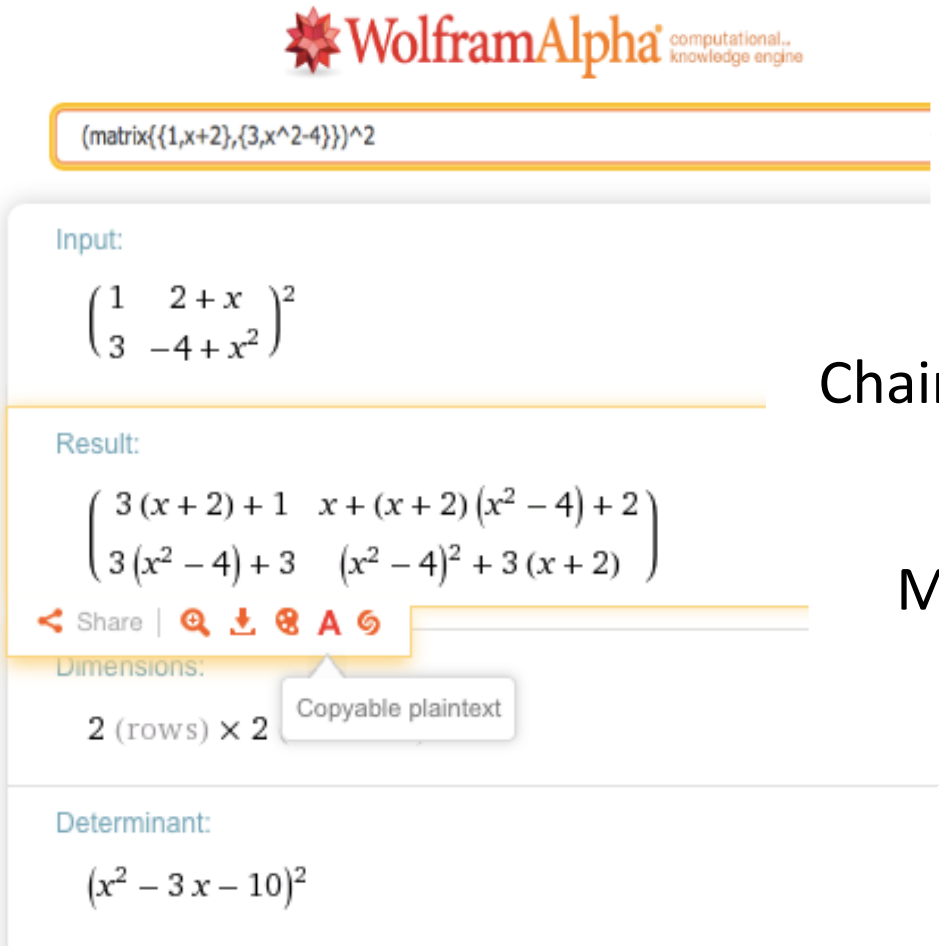
```
\begin{equation}
    \int {\frac { \left( 3\,{x}^{2}+2 \right) \sin \left( {x}^{3}+2\,x-1 \right) } { \left(
    \cos \left( {x}^{3}+2\,x-1 \right) \right) ^{3}+12\, \left( \cos \left( {x}^{3}+2\,x-
    1 \right) \right) ^{2}-3}} dx
\end{equation}
```

Additional Motivation

- Entering mathematical concepts on a computer for presentation is tedious and difficult.
- No 10 year old can do math on a computer. But 10 year olds can produce creative work on a computer involving images and words.
- Current interfaces of CAS, although improved, constraints the user to express their thoughts in a command line form
- Different systems have different languages and commands.
- Manipulation and transformation of expressions in CAS lack intuitiveness when using only keyboard and mouse

Math for the Masses

E.g. Wolfram Alpha.



Knowledge of syntax required:
Different CAS different syntax

Chains of operations not supported

Manipulating results requires…
More syntax

Sharing requires subscription

Challenges I

- Text recognizers :
  - work with ASCII Characters
  - depend on language specific dictionaries
  - assume input is one dimensional
  - not suitable for math

- Math symbols more ambiguous than regular text. Eg.

1+1 or |t| ?

SyZ or 5x2 ?

Challenges II

- Mathematical symbols appear with wide variation in size and do not necessarily follow baselines

  Ex. superscripts and subscripts

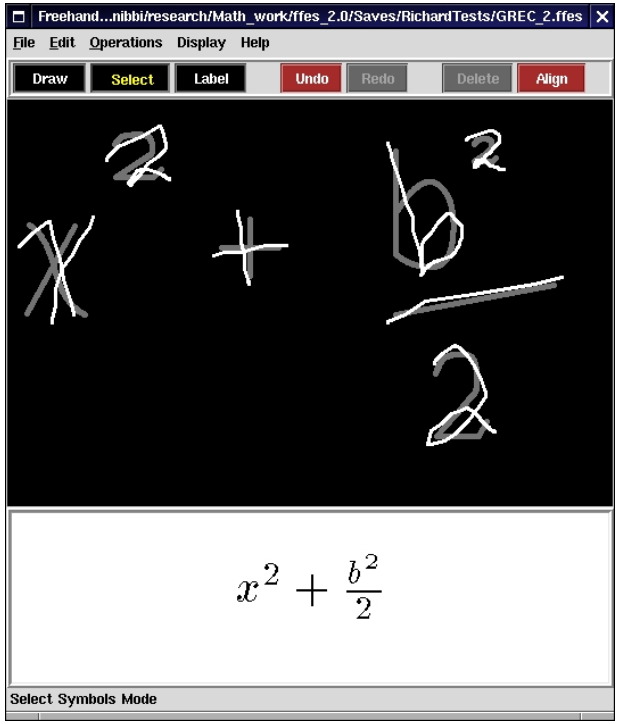
- Correct symbols recognition does not lead to a unique math expression.

  Ex. u(x+y) u times x+y or u applied to argument x+y ?

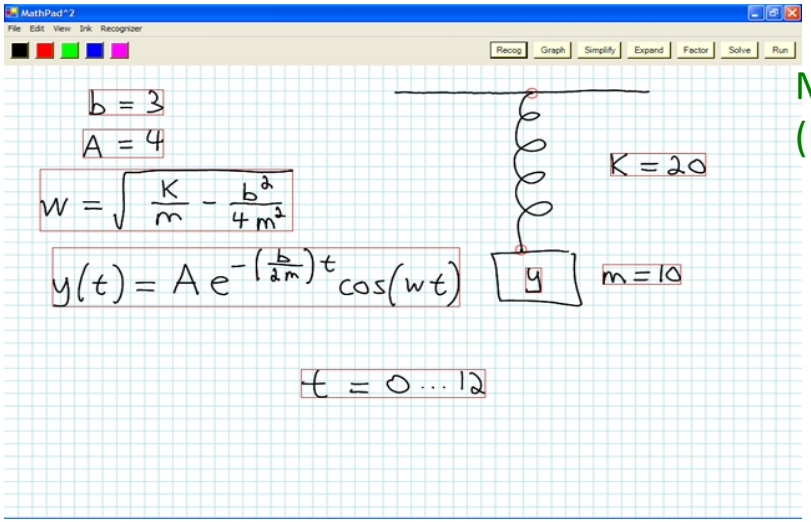
- Rendering 2 dimensional math expressions with line-breaking.

- Users require interactivity: gestures, sub-expressions, editing

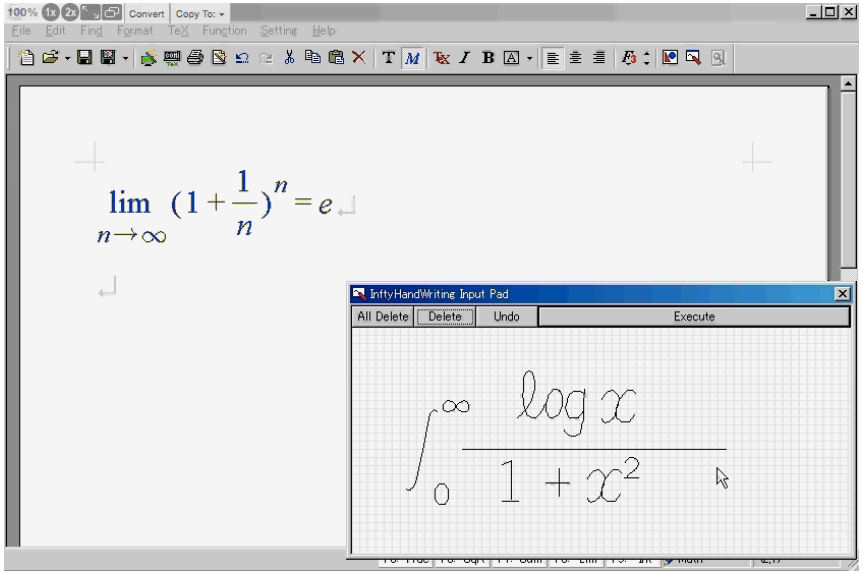
Early Penmath Attempts



FFES (Zanibbi, et al)



MathPad^2
(LaViola, et al)



Infty
(Suzuki, Tokyo)

Our Starting Point

- At start pen based devices were Tablet PCs
  - Tablet PCs were solutions looking for a problem to solve
  - They did generate a great deal of interest.
  - End of day : never really caught on
- Mathematics and CAS were thought to be the killer apps for Tablet PC's.
- Quick lesson : Math on tablet is difficult and complex

  (In particular if you want to build a real system)

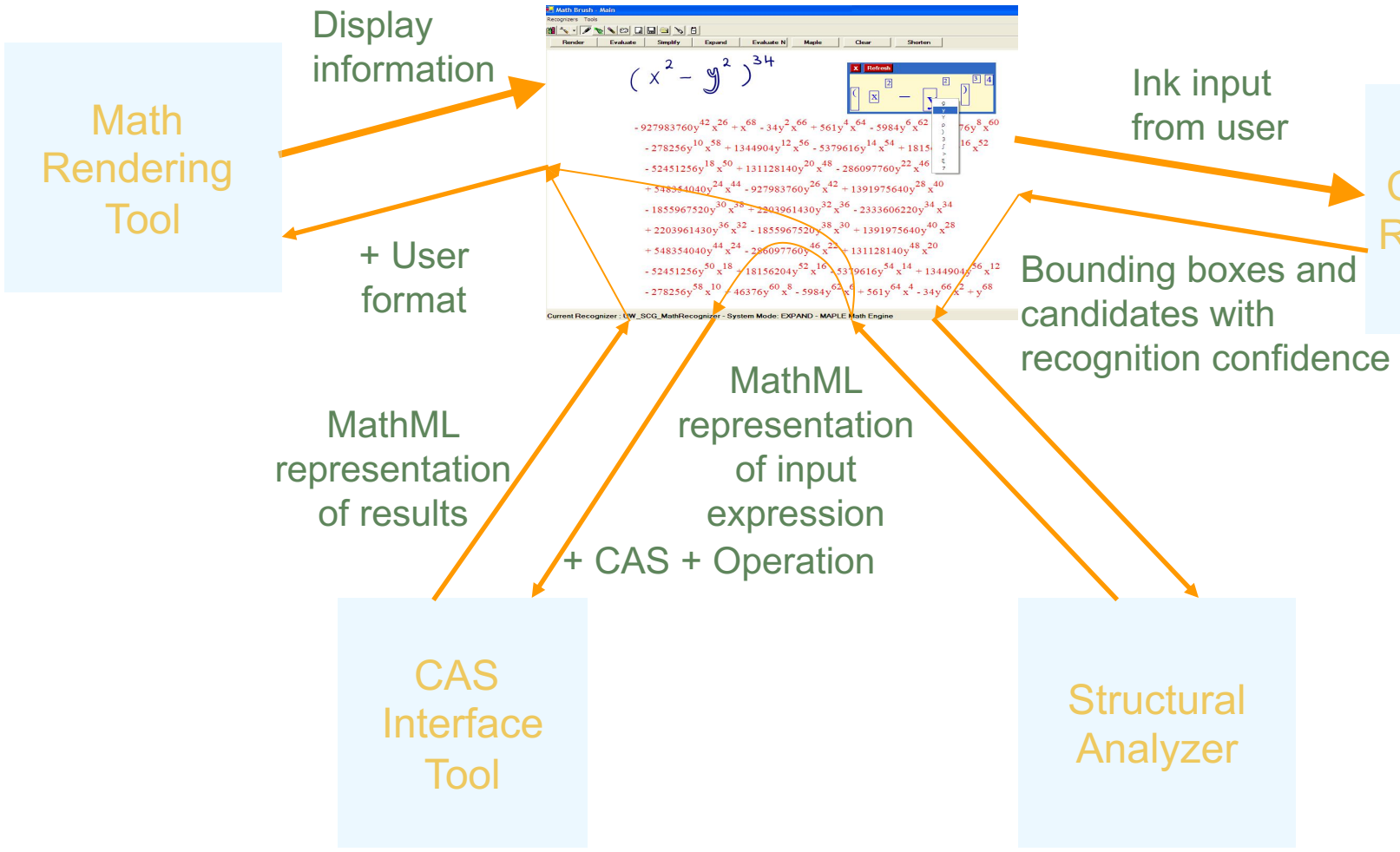
MathBrush I

- First pancake. Tablet PC version
- Something to experiment with – not a complete interface
- Math computations done via connections to existing CAS
  - Maple and Mathematica
- Connections done via MathML
- Math operations done via context menus
- Ultimately modelling (invisible) pen based math windows which sit inside existing CAS

MathML

- Standard for math communication
  - Presentation MathML
  - Content MathML
- Unfortunately very verbose
  - Not good for external communication
- Unfortunately MathML has not really caught on
  - Maple version of MathML was quite buggy
  - Mathematica thinks it is the standard
  - Microsoft had its own proprietary version
  - SAGE, Matlab do not support MathML

MathBrush I System Architecture



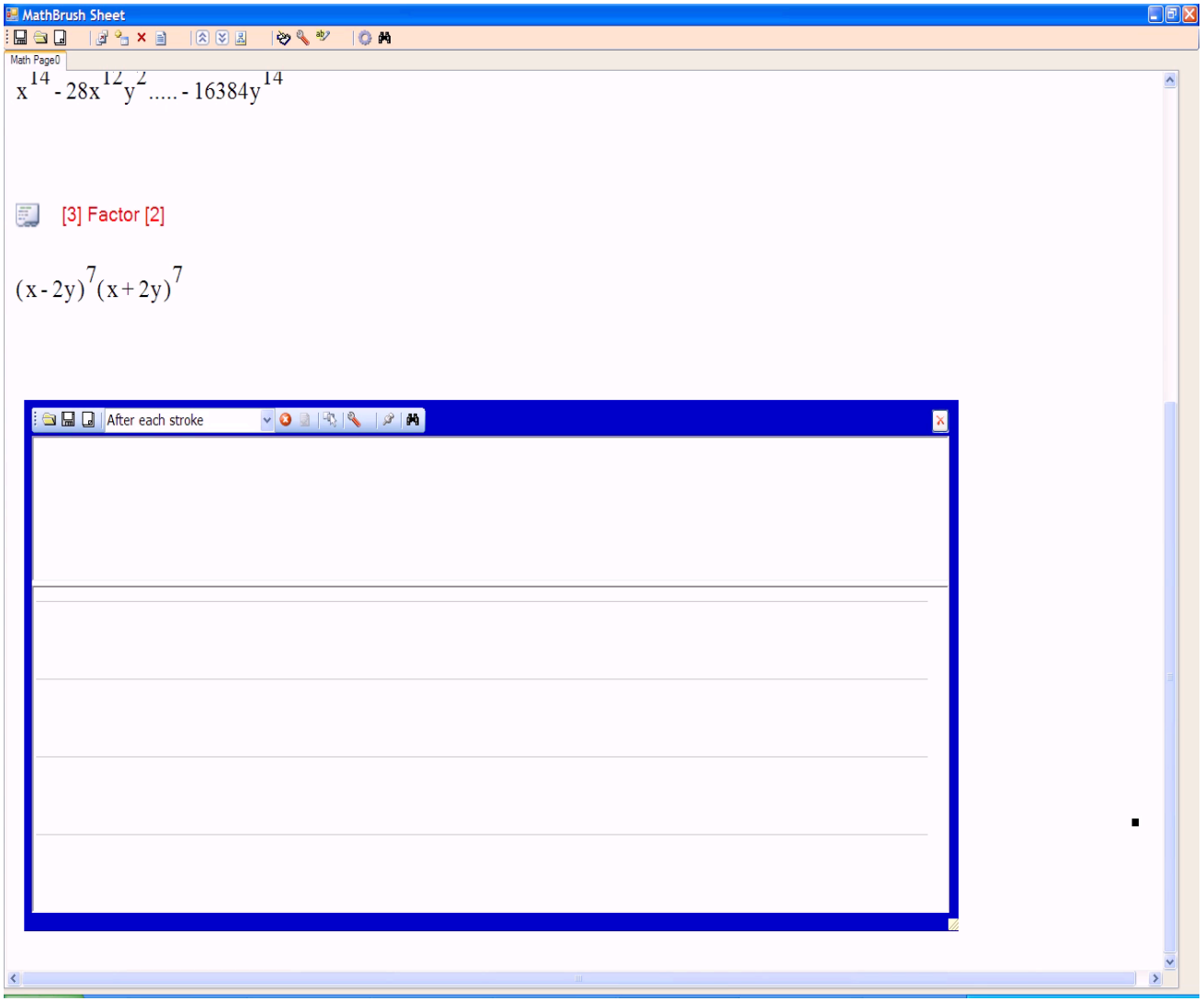
MathBrush II

- Math recognizer much better; approach smarter
  - Recognition is a ``fuzzy'' process
  - Return a `best bet' along with alternatives
    - Easy then to do corrections
  - Ability to see recognition as user writes
  - Merges character recognition and semantic analysis
  - Recognition of partial expressions
- Ability to edit expressions
- Ability for users to train their symbols
- Still based on use with Tablet PCs
- Still use MathML for communication with CAS
- Still use context menus to do math

MathBrush II (a bit later)

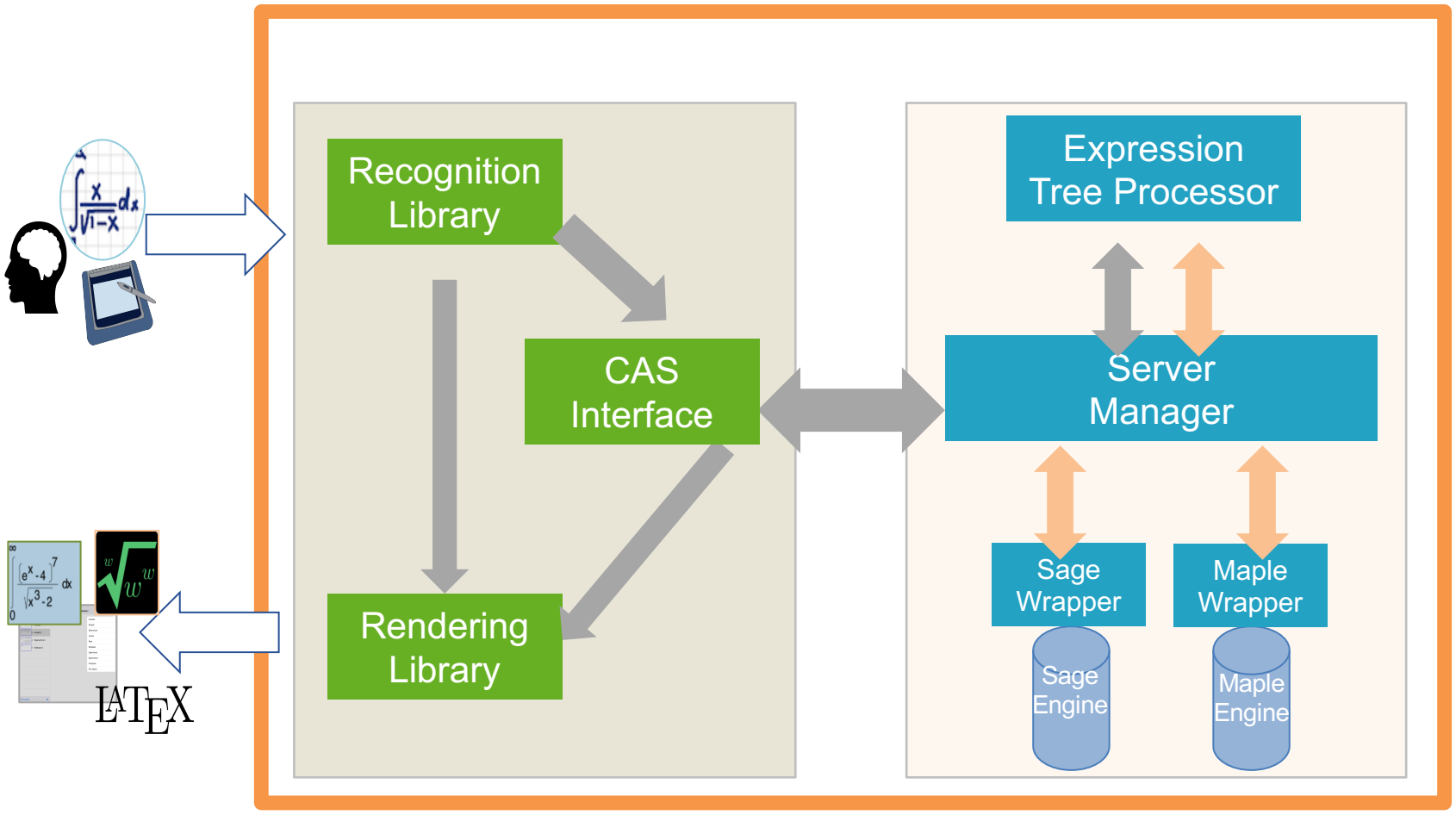
- Works on windows version
  - Tablet PC version evolved to Surface Pro
- iPad version
  - Initially convert math to Latex, MathML, etc
  - Later connect with CAS system - client server architecture
    - CAS now SAGE (no licensing issues)
  - Quick lesson : iPad not great for math
    - Interface was quick enough (eventually)
    - Lack of a (real) pen an issue
      - E.g. Fat finger issue
    - User studies useful

MathBrush II on Tablet PC



MathBrush II on iPad

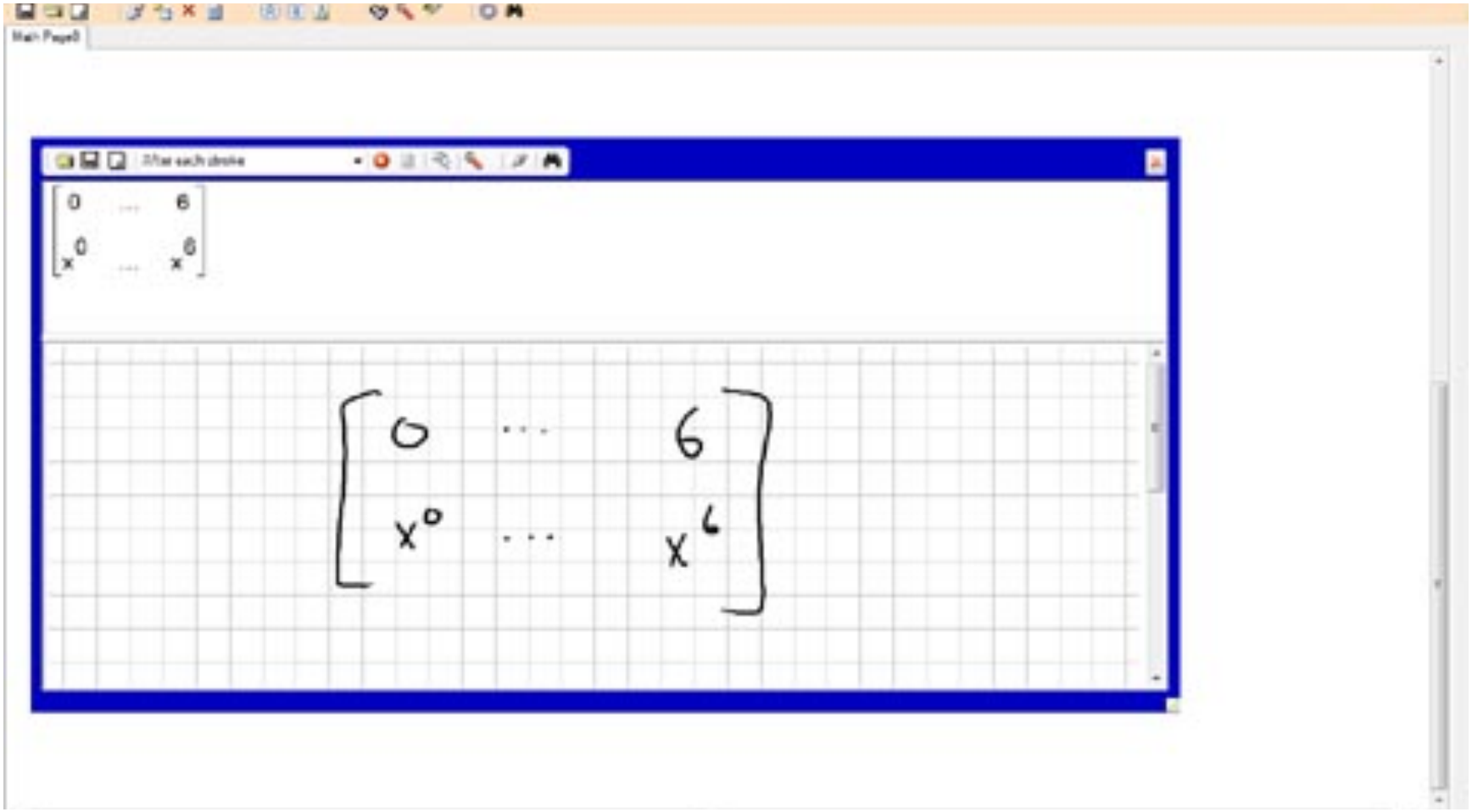
MathBrush II System Architecture



MathBrush II Math Recognizer

- Based on relational grammars and fuzzy sets
  - S. Maclean and G. Labahn, A new approach for recognizing handwritten mathematics using relational grammars and fuzzy sets, International Journal of Document Analysis and Recognition, 16(2) (2013) 139-163
- Three step process : symbolic recognition, parsing and tree extraction
- Parsing step uses a relational grammar with some assumptions to reduce complexity
- Returns ranked alternatives for symbols and sub-expressions.
- Same approach used by Microsoft (simultaneously) with math recognizer in Windows 7.
- Ability to train recognizer for symbols
- Recognition of ``underspecified matrices''
- Probabilistic model for recognition :
  - S. MacLean and G. Labahn, A Bayesian model for recognizing handwritten mathematical expressions, Pattern Recognition, 48 (2015) 2433--2445

Underspecified Matrices



CROHME Competition

- Competition on Recognition of Online Handwritten Mathematical Expressions (CROHME)

- Run in 2011, 2012, 2013, 2014
  - Currently organized by R. Zanibbi at Rochester Tech.
  - MathBrush finished second in 2012. First place went to Vision Objects, a commercial outfit (myscript math recognizer)
  - Three main recognition tasks in 2014.
    - Isolated symbols
    - Full expressions
    - Matrices (new in 2014 – because of MathBrush)

CROHME results

2012 CROHME data

Recognizer	Stroke reco.	Symbol seg.	Symbol reco.	Expression reco.
MathBrush prob.	91.54	96.56	94.83	66.36
MathBrush fuzzy	90.72	95.56	93.96	56.07
MathBrush 2012	89.00	97.39	91.72	51.85
Vision Objects	97.01	99.24	97.80	81.48
University of Nantes	90.05	94.44	95.96	57.41

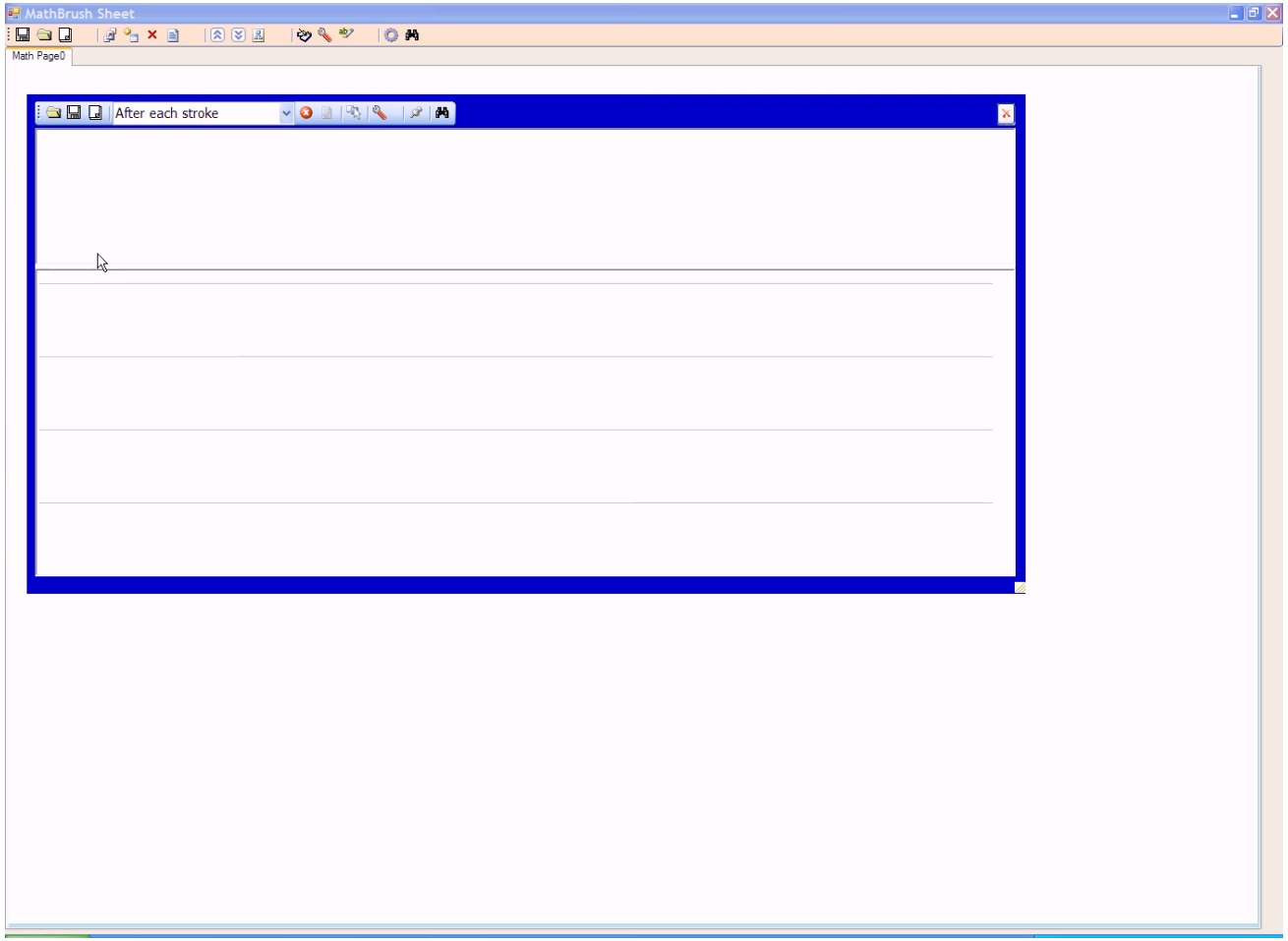
Recognizer	Stroke reco.	Symbol seg.	Symbol reco.	Expression reco.
MathBrush prob.	92.74	95.57	97.05	55.18
MathBrush fuzzy	89.82	94.49	95.73	42.47
MathBrush 2012	90.71	96.67	94.57	49.17
Vision Objects	96.85	98.71	98.06	75.08
University of Nantes	82.28	88.51	94.43	38.87

Editing ?

Working with Sub-expressions

- Simplification very tricky
  - `Simple' often ill defined
  - Better to do interactively
- Lack of easy to use manipulation tools one of the main difficulties for working with CAS
  - What command to use? e.g. in Maple
    - op(2, op(1,term)) gives the first part of the second part of term.
  - Better with natural gestures e.g.
    - crossing out terms from both sides of the equation
    - move terms from one side to the other.
    - select partial sub-expression
    - move and reorder terms, etc

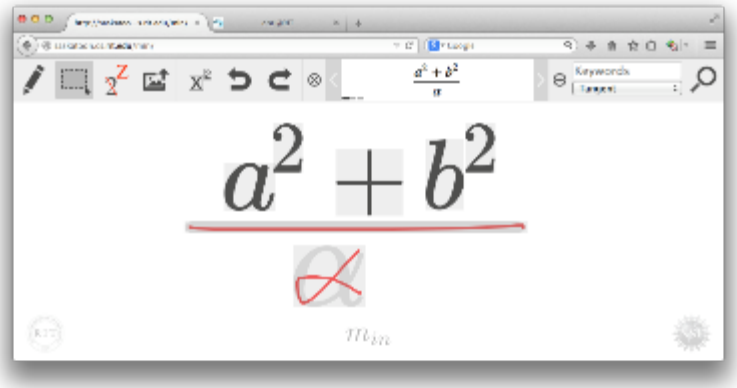
Editing in Action:



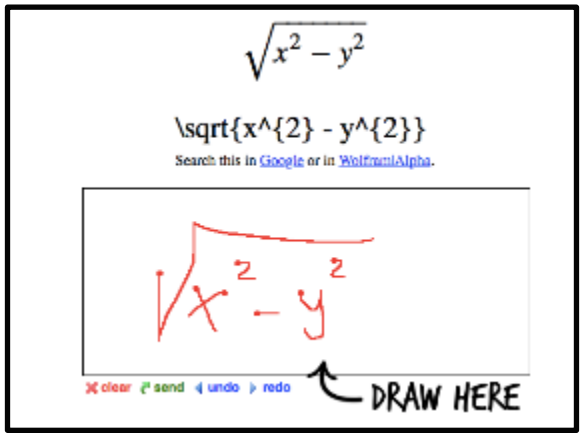
Competitive PenMath Systems

MyScript - MathPad

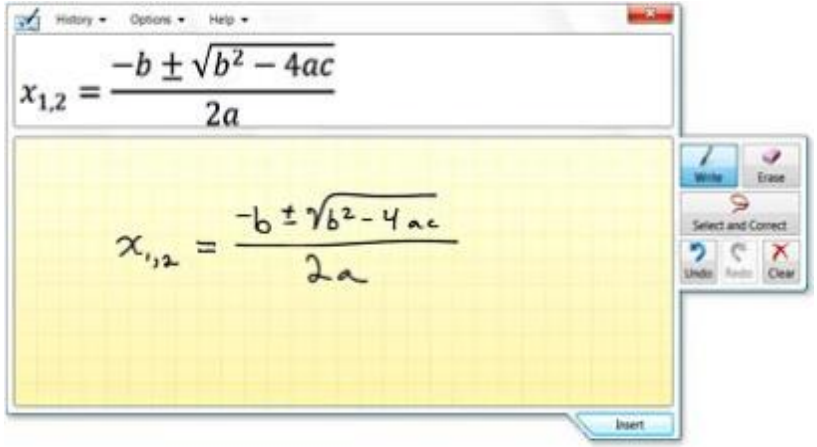
m_{in}



PRHLT



Math Input Panel

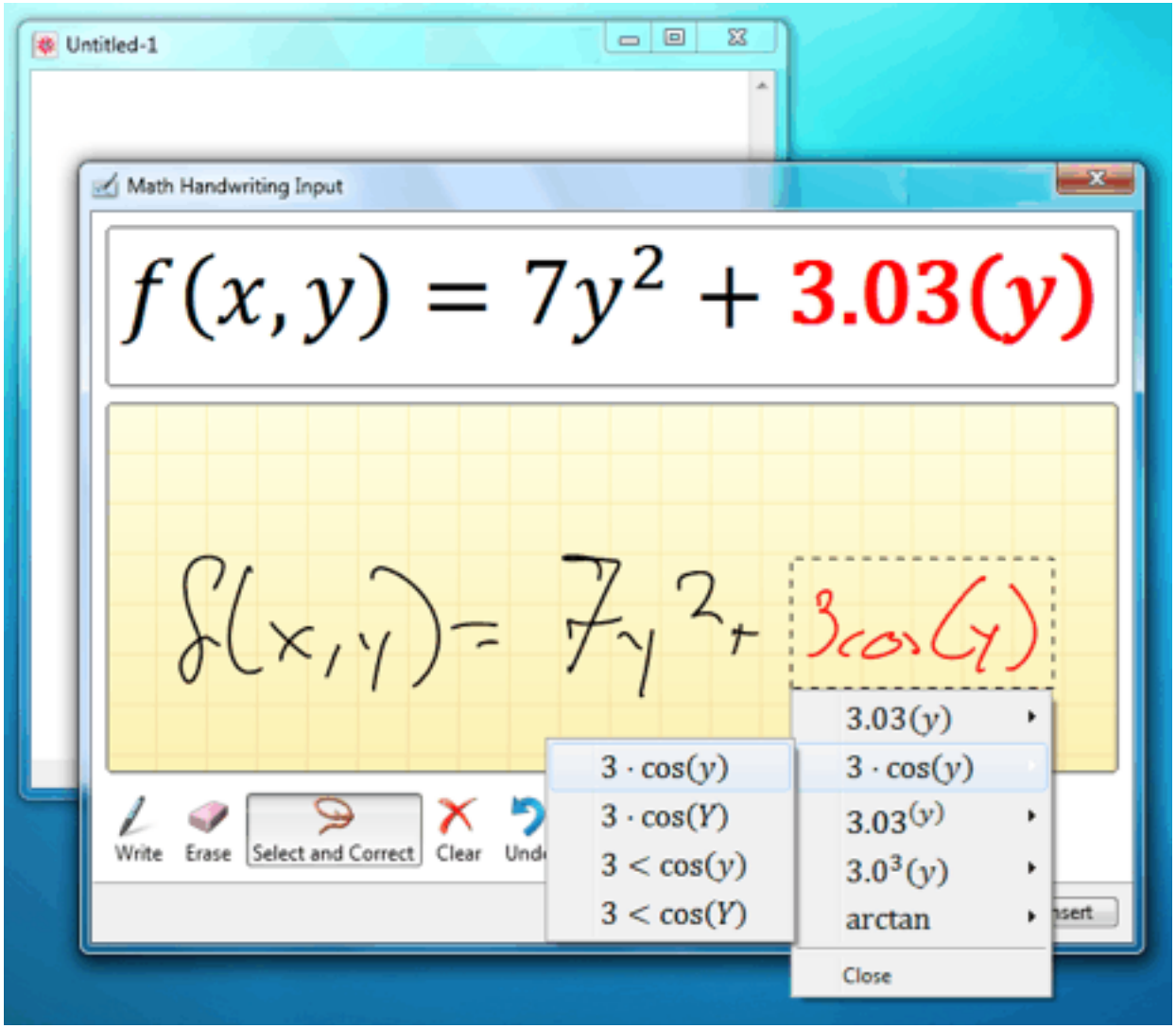


Competitive PenMath Systems

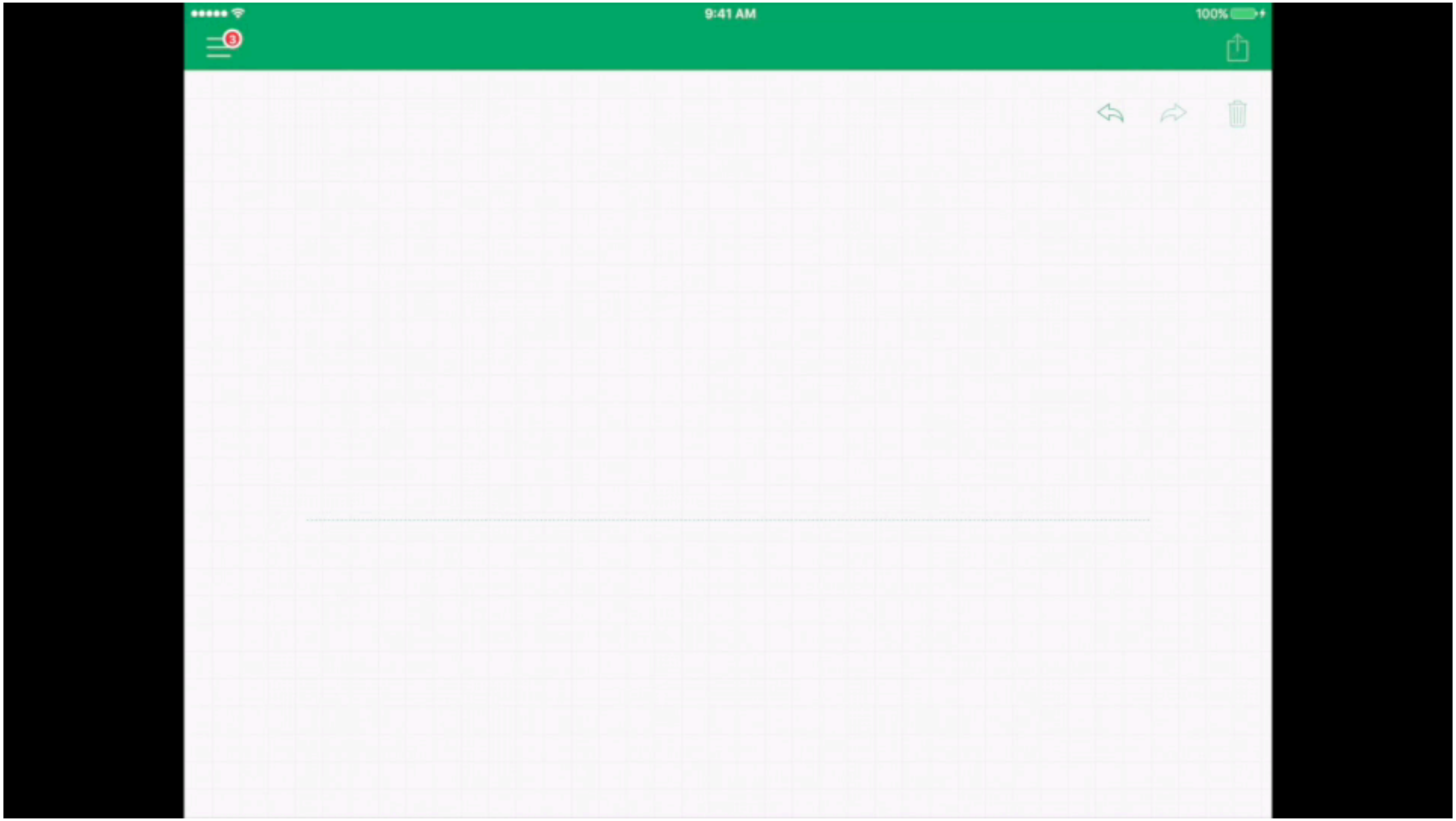
	MathBrush	MathPad	Min	PRHLT	MS Math Input Panel
Recognition**	Good	Very Good*	FFES	Good/small	Good
Gestures	Zoom, move, add space, erase	Erase	-	-	-
Editing**	Easy	Medium	Hard	Hard	Medium
Expr. Size	Large	Large	Small	Small	Large
Multi-expr					
Matrices					
Manipulation					
OS	iOS, Windows	iOS, Web	Web***	Web***	Windows 7+
Widgets	Windows	iOS			
Objective	Do/Share Math	Share and interface	Search	Search/Share	Windows programs

*In the CHROME competition 2012 MathBrush recognizer 91.27% MathPad Recognizer 96.91%
**Basd on internal feedback when using the systems
*** Not always available/working

Mathematica and Math Panel



Nice use of space : myscript/MathPad^2



Myscript Recognizer

- The overall recognition system is built on the principle that segmentation, recognition and interpretation have to be handled concurrently and at the same level in order to produce the best candidate interpretation.
- The equation recognition engine analyzes spatial relationships between all the parts of the equation, in conformance with a grammar, to determine the segmentation of all its parts. Each grammar rule is associated with a specific spatial relationship.
  - e.g., a fraction rule defines a vertical relationship between a numerator, a fraction bar and a denominator.
- The system has also a symbol expert that estimates the probabilities for all the parts in the suggested segmentation. This expert is based on feature extraction stages, where different sets of features are computed, using a combination of on-line and off-line information. The feature sets are processed by a set of character classifiers, which use Neural Networks and other pattern recognition paradigms.
- The equation recognition engine includes a statistical language model that uses context information between the different symbols depending on their spatial relationships in the equation. Statistics have been estimated on hundreds of thousands of equations.
- A global discriminant training scheme on the equation level with automatic learning of all classifier parameters and meta-parameters of the recognizer is employed for the overall training of the recognizer. The recognizer has been trained on writing samples that have been collected from writers in several countries.
- For CROHME the system was trained using roughly 30000 math expressions in a private corpus.

Ongoing/Future I

- Recognition
  - Data collection for recognizer training
    - capture math expressions when sent to SAGE and Maple
    - such expressions likely ground-truthed
    - use to improve recognizer

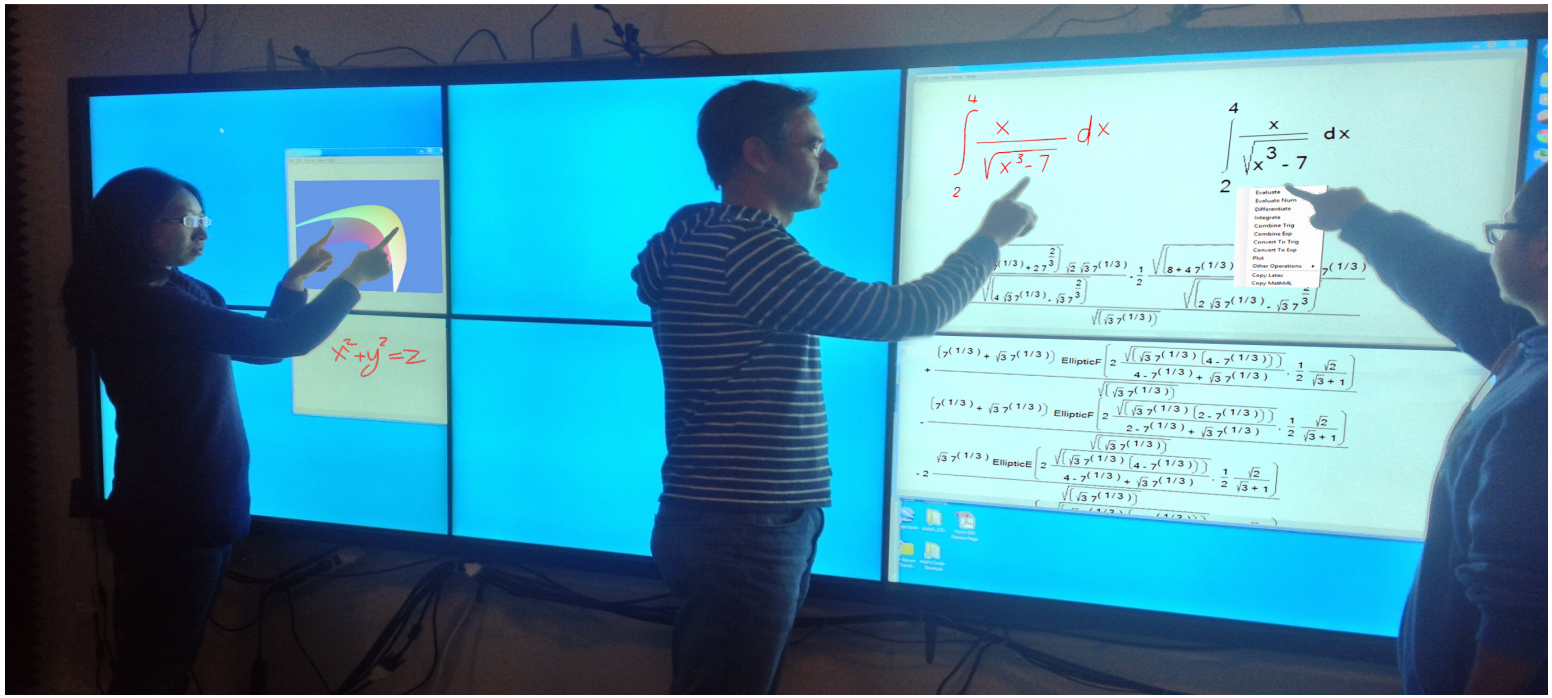
      anyone who has data wins battle
  - Support for auto completion.
  - Better use of timing and `in air' information

Ongoing/Future II

- Interface and Platforms
  - Natural gestures for doing math operations
  - Morphing of handwritten math expression and editing
  - Math dictionary and support for auto completion
  - Domain specific applications.
    - Linear algebra
    - Trig functions
    - Solving equations
    - Differential equations
- Networking and communication
  - Best practices for high demand queries
  - Fair usage and distribution of loads
  - Real system for investigations

Ongoing/Future III

- Adding math to Ed Lank's Powerwall
  - Would let students do math in the Great Hall



Thanks for listening