

# Captain Jack:

## New Variable Selection Heuristics in Local Search for SAT

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<http://www.cs.ubc.ca/research/captain-jack>

# Key Contribution:

- **Captain Jack** is a *highly parametric* algorithm

Incorporates elements from  
**Sparrow**

[Balint, Fröhlich, SAT 2010]

& **VE-Sampler**

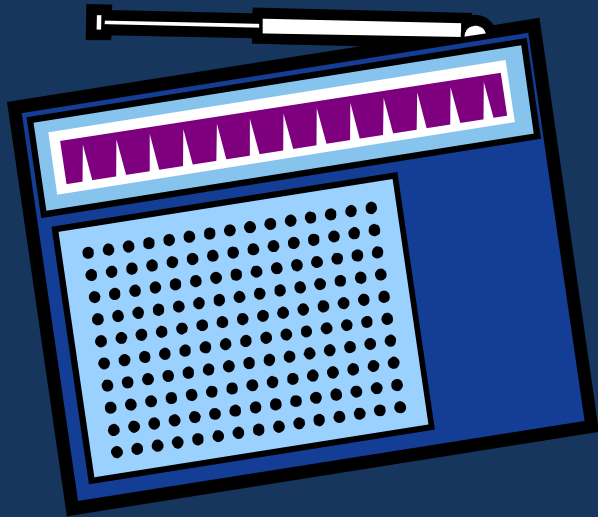
[Tompkins, Hoos, SAT 2010]



good performance  
"jack of all trades"

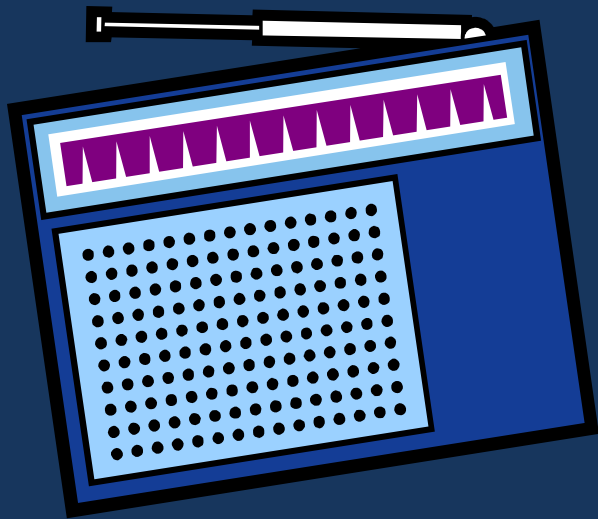
"interesting"  
configuration  
space

# Algorithm Design Philosophy

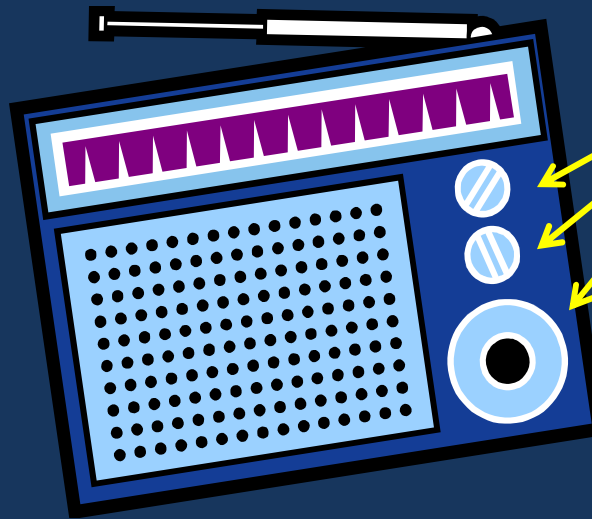


Parameterless  
Algorithm

# Algorithm Design Philosophy

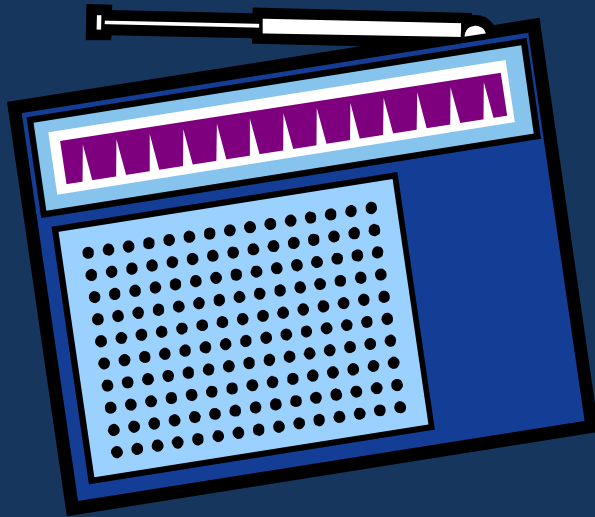


Parameterless  
Algorithm

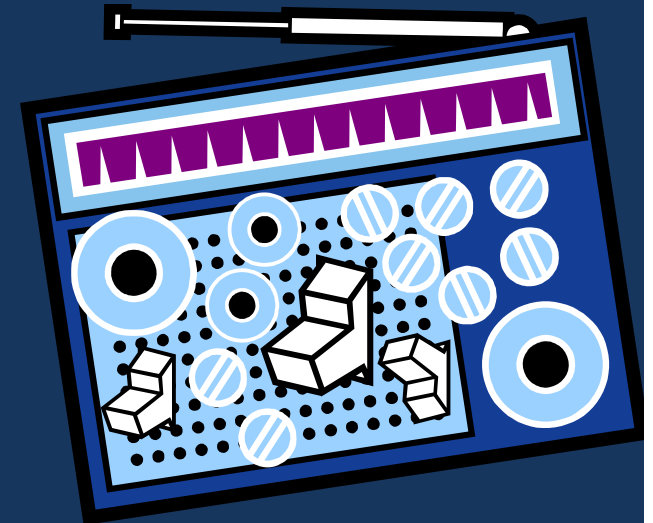
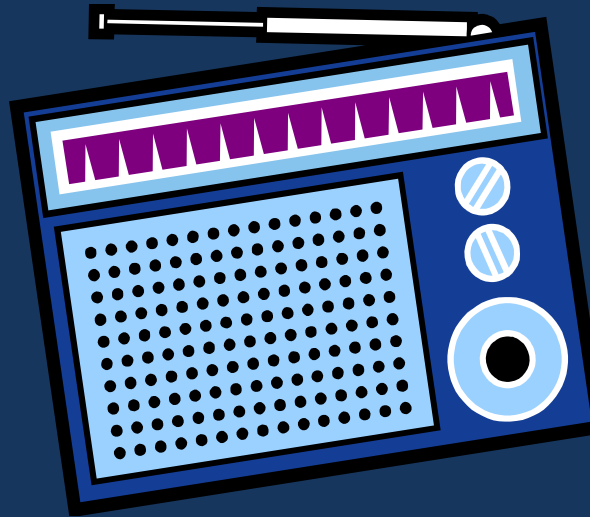


Algorithm  
Parameters

# Algorithm Design Philosophy



Parameterless  
Algorithm



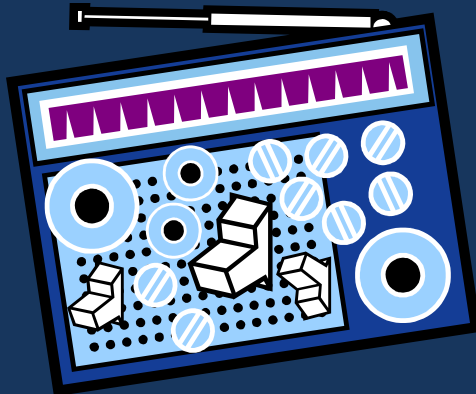
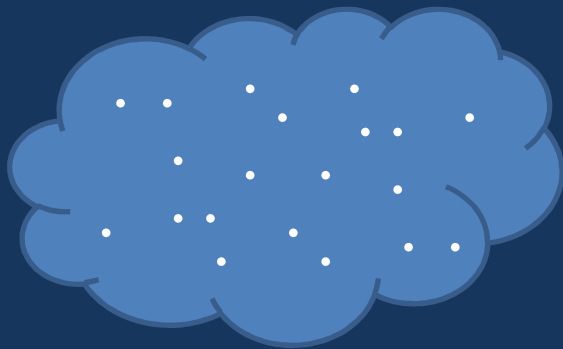
Highly  
Parametric  
Algorithm

# Automated Configuration

- We can use **automated configurators** to determine the optimal algorithm parameters for a target instance set
- We used ParamILS [Hutter *et al.*, 2007, 2009]
- Offload tedious human tasks to machines

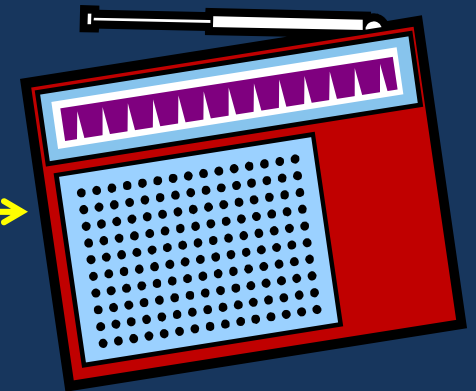
# Automated Configuration

Training Instance Set



Highly Parametric Algorithm

Automated  
Configurator  
(here: ParamILS)



Tuned  
(configured)  
Algorithm

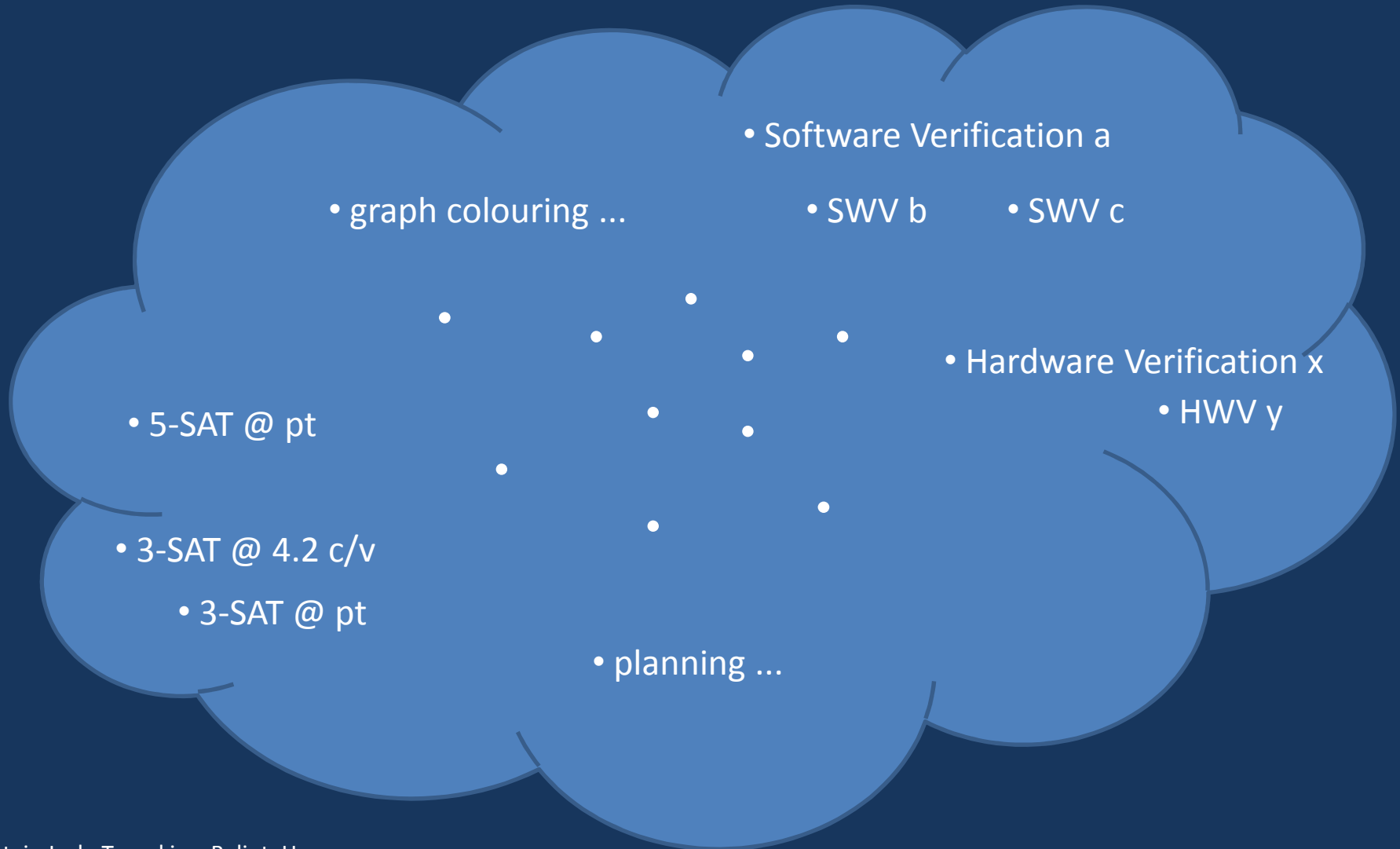
# Instance "Space"

3-SAT @ pt

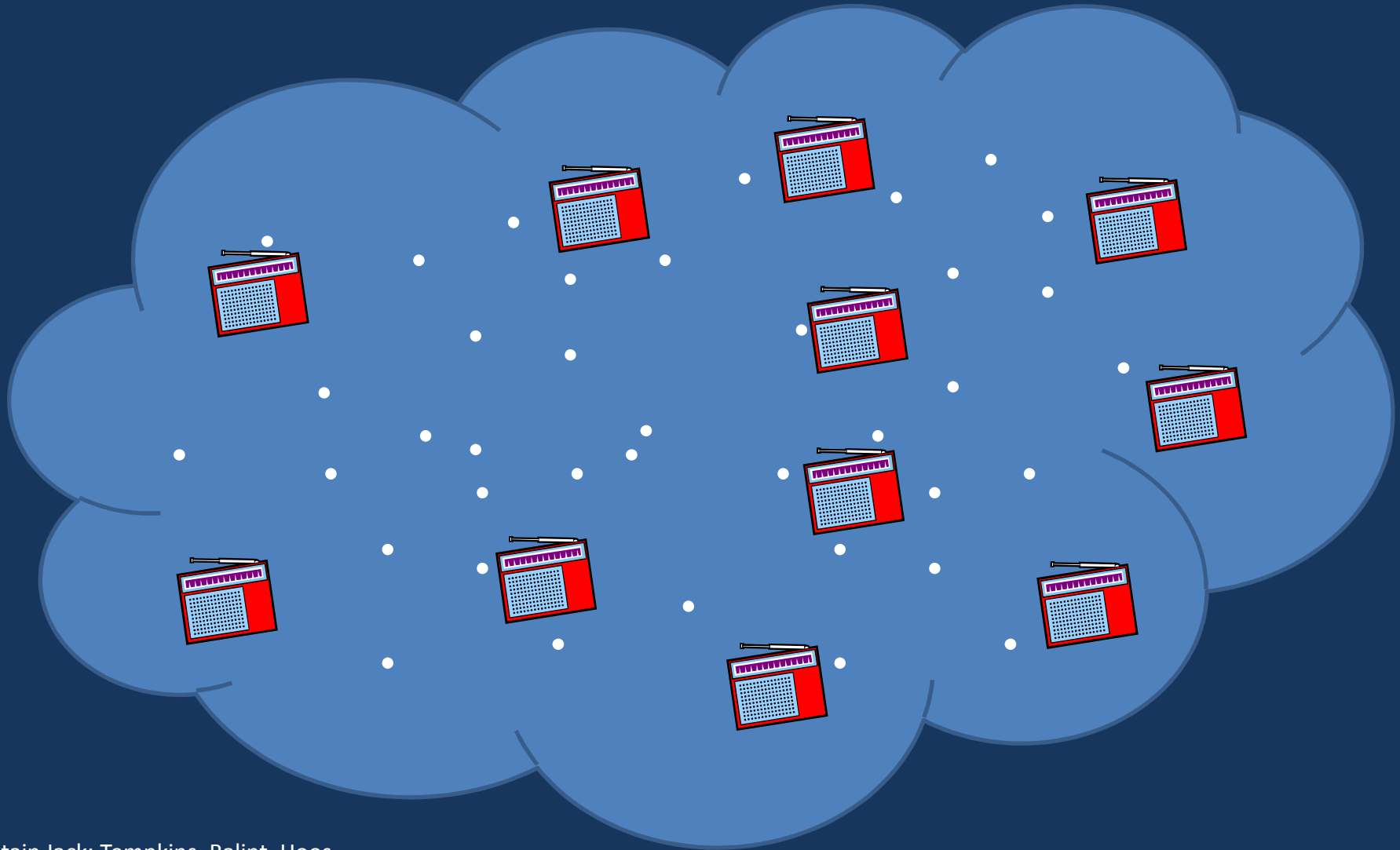




# Instance "Space"



# Algorithm "Space"



# Captain Jack



good performance

"interesting"  
configuration  
space

# Overview

- Motivation
- Captain Jack
  - Background
  - Design
  - New Contributions
- Results
- Future Work
- Conclusions

# Stochastic Local Search (SLS) for SAT

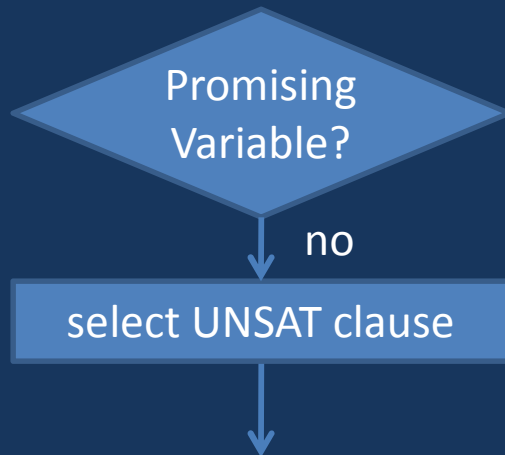
randomly initialize all variables  
while (formula not satisfied)  
    **select a variable** and “flip” it

$$(\neg x_1 \vee x_2 \vee \neg x_5)$$

Evaluate each variable  
(Variable Expression)

Variable Selection  
Mechanism (VSM)

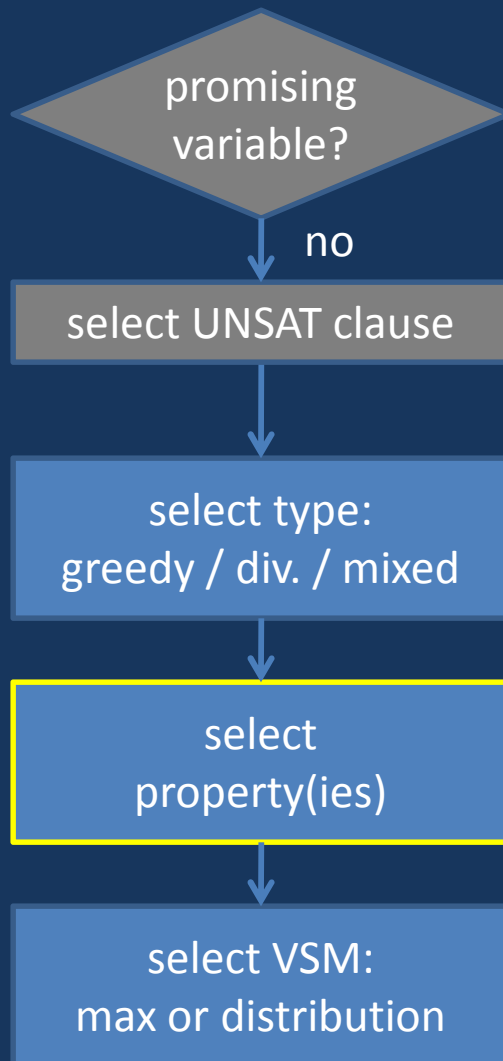
# Captain Jack Controller



Captain Jack can use promising variables (if they exist)  
[G<sup>2</sup>WSAT: Li, Huang, 2005]

Select UNSAT clause uniformly at random  
[Papadimitriou, 1991]  
[WalkSAT: Selman, Kautz, Cohen, 1994]

# Captain Jack Controller



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[G<sup>2</sup>WSAT: Li, Huang, 2005]

Select UNSAT clause uniformly at random  
[Papadimitriou, 1991]  
[WalkSAT: Selman, Kautz, Cohen, 1994]

# Variable Properties

- Greedy Properties

**make** = # of clauses that become **satisfied** if we flip  $x$

**break** = ... **unsatisfied** ...

**score** = (**make** - **break**) [GSAT: Selman, Levesque, Mitchell, 1992]



# Variable Properties

- Greedy Properties

make = # of clauses that become satisfied if we flip  $x$

break = ... unsatisfied ...

score = (make – break) [GSAT: Selman, Levesque, Mitchell, 1992]

- Diversification Properties

age = # of steps since  $x$  was flipped [TABU: Glover, 1986]

flips = # of times  $x$  has been flipped [HSAT: Gent, Walsh, 1993]

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- Variable Expressions

$$\text{sparrowAge} \quad 1 + \left( \frac{\text{age}}{c_d} \right)^{c_e}$$

# Captain Jack Variable Properties

- Greedy

- make
- break
- score
- sparrowScore<sub>2</sub>
- score
- scoreRatio
- rel\*

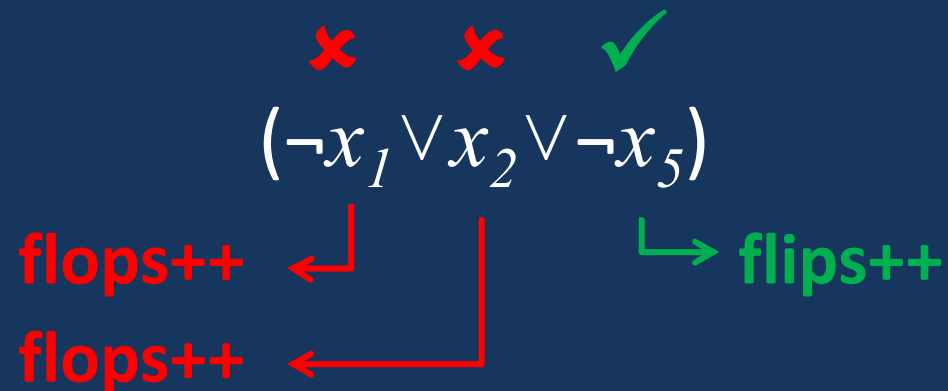
- Diversification

- rand
- flat
- fair
- last
- age
- age<sub>1</sub>
- age<sub>5</sub>
- ageRange
- sparrowAge
- tabu
- flips
- flops
- normFlops
- resetFlops
- rel\*

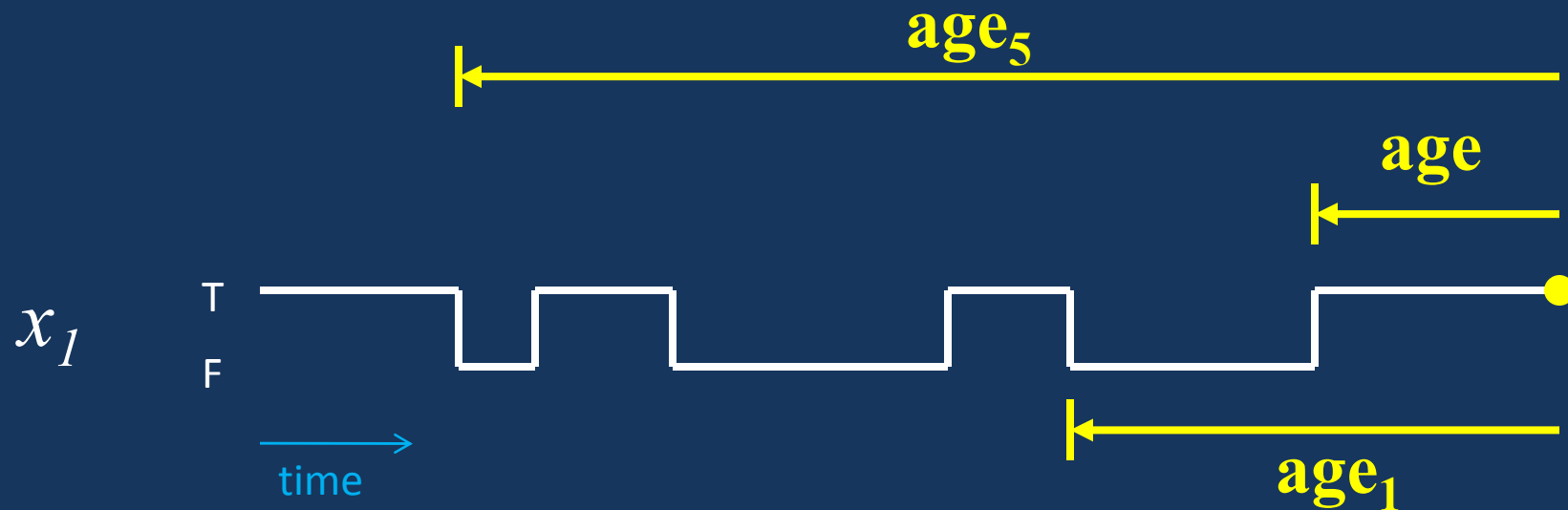
# New Variable Properties

- scoreRatio: (make)/(make + break)

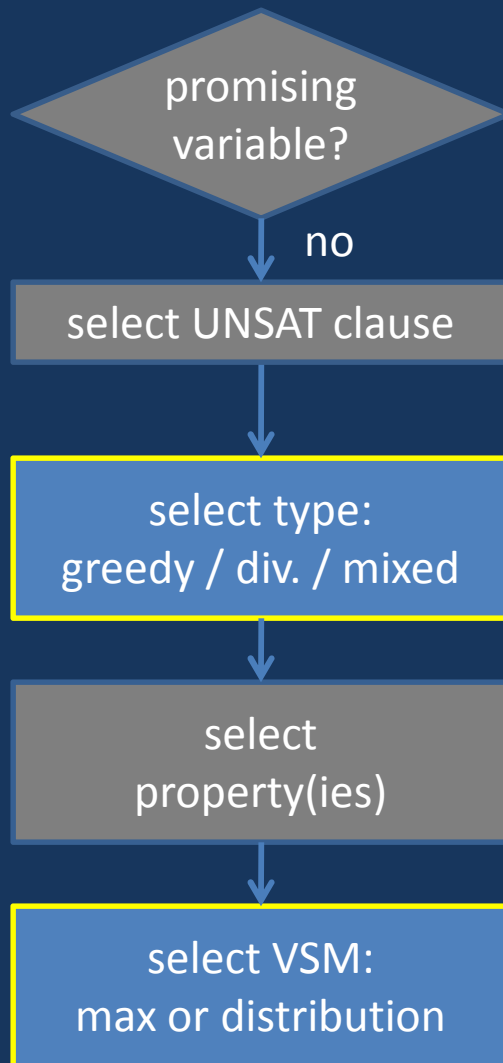
- flops:



# age<sub>k</sub> property



# Captain Jack Controller



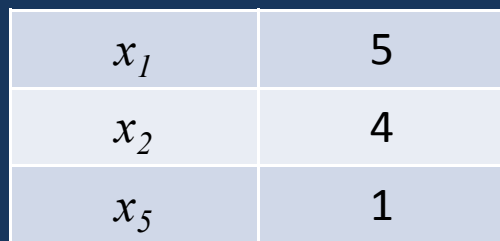
# Mixed Variable Expressions

- VE-Sampler  
 $f(\text{greedy}) + f(\text{diversification})$
- Sparrow  
 $f(\text{greedy}) \cdot f(\text{diversification})$
- both use a **mixed** VE
  - First introduced with VW2 [Prestwich, 2005]
- Captain Jack
  - 3 Options: greedy, diversification, mixed
  - $\text{greedy} \cdot \text{diversification}$

# Variable Selection Mechanism

$$(\neg x_1 \vee x_2 \vee \neg x_5)$$

Variable Expression

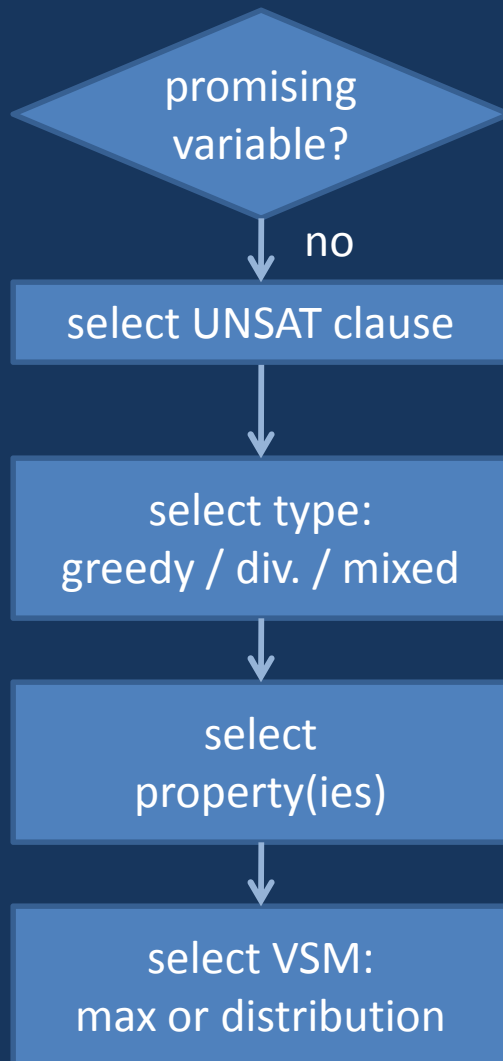


|       |   |
|-------|---|
| $x_1$ | 5 |
| $x_2$ | 4 |
| $x_5$ | 1 |

- Maximum ("best")
  - select  $x_1$
- Probability Distribution
  - select  $x_1$  50%
  - select  $x_2$  40%
  - select  $x_5$  10%



# Captain Jack Controller

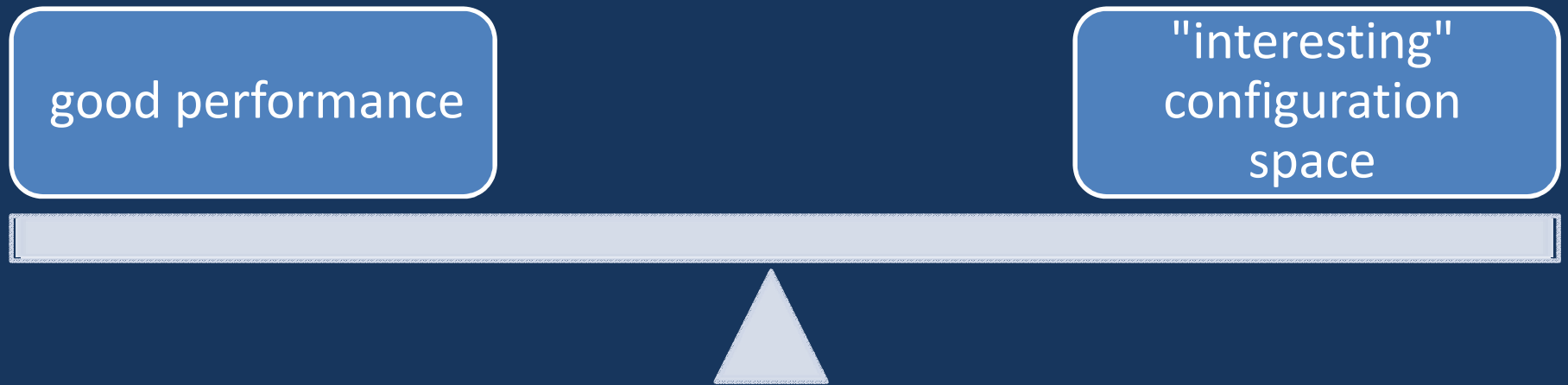


Each type of step assigned a weight (%)

Each property is assigned a weight (%)  
(both a greedy & div. property selected for mixed)

Prob. of selecting the VSM is based on the type

# Captain Jack



# Instance "Space"

Industrial-Like Random  
(Ansótegui et. al., 2009)

Software Verification  
CBMC (binary search)  
SWV (static checking)

Random k-SAT:  
3-SAT (@ pt, large 4.2)  
5-SAT (@ pt, large)  
7-SAT (@ pt, large)

# Performance Results

|              | 3-SAT       |             | 5-SAT       |             | 7-SAT       |             | IL50k       | CBMC        | SWV         |
|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|              | 1k          | 10k         | 100         | 500         | 60          | 90          |             |             |             |
| Captian Jack | 2.06        | <b>1.00</b> | 1.72        | 3.66        | 1.83        | 4.43        | <b>1.00</b> | 4.38        | 1.59        |
| Sparrow*     | 1.88        | 5.07        | <b>1.00</b> | 3.87        | <b>1.00</b> | 2.08        | 1.65        | 867         | 1.54        |
| VE-Sampler*  | 2.46        |             |             |             |             |             |             | <b>1.00</b> | <b>1.00</b> |
| SATenstein*  | <b>1.00</b> | 1.67        | 1.17        | <b>1.00</b> | 1.14        | 2.19        | 1.42        | 7.75        | 1.62        |
| TNM          | 2.25        | 15.96       | 1.22        | 8.20        | 2.37        | 1.96        | 422         | 6,563       | 1.62        |
| gNovelty+2   | 2.35        | 60.15       | 1.50        | 3.38        | 1.36        | 1.42        | 2,291       |             |             |
| AG2009++     | 2.28        | 54.81       | 1.17        | 7.35        | 1.90        | <b>1.00</b> | 8.27        | 40,837      | 1.48        |

time relative to fastest solver [PAR10]



# Greedy Properties (%)

|                           | 3-SAT |     | 5-SAT |     | 7-SAT |    | IL50k | CBMC | SWV |
|---------------------------|-------|-----|-------|-----|-------|----|-------|------|-----|
|                           | 1k    | 10k | 100   | 500 | 60    | 90 |       |      |     |
| make*                     |       | 40  | 3     |     |       |    | 19    | 15   |     |
| break*                    |       | 10  | 3     |     | 50    | 47 | 9     | 6    | 9   |
| score*                    | 10    | 10  | 3     | 20  |       | 3  | 71    | 27   | 12  |
| sparrowScore <sub>2</sub> | 79    | 40  | 90    | 78  | 50    | 47 |       |      | 3   |
| scoreRatio*               | 10    |     |       | 2   |       |    |       | 52   | 75  |

property weights as percentages, values  $\leq 1$  not shown

# Diversification Properties

|                  | 3-SAT |     | 5-SAT |     | 7-SAT |    | IL50k | CBMC | SWV |
|------------------|-------|-----|-------|-----|-------|----|-------|------|-----|
|                  | 1k    | 10k | 100   | 500 | 60    | 90 |       |      |     |
| random/flat/fair | 15    | 11  | 2     | 46  |       | 52 | 2     | 6    | 2   |
| last             | 15    | 5   | 4     |     | 10    | 13 |       |      |     |
| age*             | 59    | 75  | 87    | 51  | 84    | 18 | 47    | 46   | 47  |
| flips*           | 4     |     | 2     |     |       |    | 48    | 42   | 45  |
| flops*           | 8     | 10  | 5     | 3   | 3     | 18 | 3     | 7    | 4   |

property weights as percentages, values  $\leq 1$  not shown

# Age-based Properties

|                  | 3-SAT |     | 5-SAT |     | 7-SAT |    | IL50k | CBMC | SWV |
|------------------|-------|-----|-------|-----|-------|----|-------|------|-----|
|                  | 1k    | 10k | 100   | 500 | 60    | 90 |       |      |     |
| age              | 4     | 10  | 33    |     | 3     | 13 |       |      | 3   |
| age <sub>1</sub> |       | 20  | 16    |     |       |    | 43    |      |     |
| age <sub>5</sub> | 15    | 40  | 4     |     | 20    |    |       | 42   | 44  |
| ageRange         | 7     |     |       | 3   |       |    |       |      |     |
| sparrowAge       | 29    |     |       |     | 20    | 2  | 3     |      |     |
| tabu             | 4     | 5   | 33    | 44  | 41    | 3  |       |      |     |

property weights as percentages, values  $\leq 1$  not shown

# Cross-Testing

| Configuration | 3-SAT |      | 5-SAT       |      | 7-SAT |             | IL50k | CBMC   | SWV         |
|---------------|-------|------|-------------|------|-------|-------------|-------|--------|-------------|
|               | 1k    | 10k  | 100         | 500  | 60    | 90          |       |        |             |
| CJ [3sat1k]   | 1     | 61.5 | 1.38        | 95.7 | 1.08  | 1.03        | 157   | 5 876  | 1.02        |
| CJ [3sat10k]  | 2.65  | 1    | 1.41        | 545  | 1.99  | 3.99        | 167   | 1 890  | 1.02        |
| CJ [5sat100]  | 2.56  | 135  | 1           | 93.2 | 1.18  | <b>0.72</b> | 170   | 7 108  | 1.03        |
| CJ [5sat500]  | 24.3  | 200  | 1.35        | 1    | 1.00  | <b>0.97</b> | 1 271 | 10 014 | 1.00        |
| CJ [7sat60]   | 99.1  | 200  | <b>0.82</b> | 539  | 1     | 2.33        | 786   | 9 989  | 1.02        |
| CJ [7sat90]   | 105   | 200  | 1.82        | 12.1 | 1.44  | 1           | 1 929 | 3 088  | <b>0.98</b> |
| CJ [IL50k]    | 16.6  | 200  | 4.50        | 567  | 2.20  | 15.8        | 1     | 1 106  | <b>0.83</b> |
| CJ [CBMC]     | 19.9  | 200  | 6.71        | 483  | 2.97  | 7.70        | 1 236 | 1      | 1.02        |
| CJ [SWV]      | 148   | 200  | 17.6        | 567  | 9.47  | 79.2        | 2.29  | 2.43   | 1           |

time relative to target configuration [PAR10]



# Additional Observations

- No promising steps for CMBC & SWV
- Mixed Steps were preferred
- Variable Selection:
  - No clear winner between:  
Max/Probability Distribution
- No clear results on clause-length settings

# Key Contributions

- CJ is a new *highly parametric* SLS algorithm
- Introduced several new variable properties
- Performs well on both random, industrial\* and industrial-like
- Insights into configurations and properties used on different instances

# Future Work

- $\text{age}_k$  properties
  - framework to introduce new properties
  - "knock-out" algorithm properties
  - adaptive strategies
  - lead to specialized light-weight algorithms
- 
- thanks...



arrrr.....  
there any  
questions?