## The Relational Model

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### Introduction to Databases CS348

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# The Relational Model

### Idea

All information is organized in (a finite number of) relations.

Features:

- simple and clean data model
- powerful and declarative query/update languages
- semantic integrity constraints
- data independence

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# **Relational Structures/Databases**

Components:

Universe

• a set of values  ${\bf D}$  with equality (=)

Relation

- schema: name R, arity k (number of attributes)
- instance: a relation  $\mathbf{R} \subseteq \mathbf{D}^k$ .

### Database

- schema: finite set of relation schemes
- instance: a relation  $\mathbf{R}_{i}$  for each  $R_{i}$

NotationSignature: 
$$\rho = (R_1, \dots, R_k)$$
Instance:  $D = (\mathbf{D}, =, \mathbf{R}_1, \dots, \mathbf{R}_k)$ 

## **Examples of Relational Structures**

• the integer numbers with addition and multiplication:

 $(\mathbf{Z},=,\texttt{plus},\texttt{times})$ 

• a Bibliography Database:

(D,=,author,wrote,publication,...)

for D the set of strings and ints

• an (abstraction of) OS Kernel:

(D,=,process,file,waits-for,...)

for  ${\rm \mathbf{D}}$  the set of strings, ints, and addresses

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# Example: Bibliography

Relations (in the cs348 database) used in examples:

```
author(aid, name)
wrote(author, publication)
publication(pubid, title)
book(pubid, publisher, year)
journal(pubid, volume, no, year)
proceedings(pubid, year)
article(pubid, crossref, startpage, endpage)
```

 $\Rightarrow$  names of attributes will be important later (for SQL)

### Example (sample instance)

author	=	{	(1, John), (2, Sue)	
wrote	=	{	(1, 1), (1, 4), (2, 3)	}
$publication = \{$		{	<ul> <li>(1, Mathematical Logic),</li> <li>(3, Trans. Databases),</li> <li>(2, Principles of DB Syst.),</li> <li>(4, Query Languages)</li> </ul>	
book	=	{	(1, AMS, 1990)	}
journal	=	{	(3, 35, 1, 1990)	}
proceedings	=	{	(2, 1995)	}
article	=	{	(4, 2, 30, 41)	}

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# Example (tabular form)

author		wrote		
aid	name		author	publication
1	John		1	1
2	Sue		1	4
			2	3

publication					
pubid	title				
1	Mathematical Logic				
3	Trans. Databases				
2	Principles of DB Syst.				
4	Query Languages				

 $\Rightarrow$  that's why relations are often called "tables".

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## Example: OS Kernel

#### • schema:

process(pid, ppid, uid, stime, time, cmd)
file(fd, pid, name, dentry)
waits-for(pid, wpid)

instance—depends on what's going on in the kernel:

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#### schema:

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file(fd, pid, name, dentry)
waits-for(pid, wpid)

• instance—depends on what's going on in the kernel:

UID	PID	PPID	STIME	TIME	CMD
root	1	0	Sep16	00:00:00	init [2]
root	2	1	Sep16	00:00:00	[ksoftirqd/0]
root	3	1	Sep16	00:00:00	[watchdog/0]
root	4	1	Sep16	00:00:00	[events/0]
root	5	1	Sep16	00:00:00	[khelper]
root	6	1	Sep16	00:00:00	[kthread]
root	9	6	Sep16	00:00:00	[kblockd/0]
david	28382	5790	10:10	00:00:00	bash
david	29903	28382	10:10	00:00:00	ps -ef

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# Atomic Queries

### Idea

Relationships between objects (tuples) that are present in an instance are true, relationships absent are false.

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# **Atomic Queries**

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Relationships between objects (tuples) that are present in an instance are true, relationships absent are false.

In the sample Bibliography database instance

- "John" is an *author* with id "1":  $(1, John) \in author;$
- "Mathematical Logic" is a publication: (1, Mathematical Logic) ∈ publication; Moreover it is a book published by "AMS" in "1990":
  - $(1, \mathrm{AMS}, 1990) \in \mathtt{book};$

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- "John" wrote "Mathematical Logic":
- "John" has **NOT** written "Trans. Databases":
- $(1,1) \in \texttt{wrote};$
- $(1,3) \not\in \texttt{wrote};$

• etc.

# Relational Calculus: Syntax

### Idea

Complex statements about truth can be formulated using the language of first-order logic.

### Definition (Syntax)

Given a **database schema**  $\rho = (R_1, ..., R_k)$  and a set of *variable names*  $\{x_1, x_2, ...\}$ , formulas are defined by

$$arphi ::= \underbrace{ R_i(x_{i_1}, \ldots, x_{i_k}) \mid x_i = x_j \mid arphi \wedge arphi \mid \exists x_i. arphi}_{ ext{conjunctive formulas}} \mid arphi \lor arphi \mid \neg arphi}_{ ext{positive formulas}}$$

### First-order Variables and Valuations

How do we interpret variables?

**Definition** (Valuation)

A valuation is a function

$$heta:\{{\it x}_1,{\it x}_2,\ldots\}
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that maps variable names to values in the universe.

#### Idea

Answers to queries ⇔ valuations to free variables that make the formula true with respect to a database.

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# **Complete Semantics**

Definition

The truth of formulas is defined with respect to

1 a database instance D = (D, =, R, S, ...), and

**2** a valuation 
$$\theta$$
 : { $x_1, x_2, \ldots$ }  $\rightarrow$  D

as follows:

$$egin{aligned} D, & extsf{$\theta \models R(x_{i_1},\ldots,x_{i_k})$} extsf{ if } R \in 
ho, ( heta(x_{i_1}),\ldots, heta(x_{i_k})) \in \mathbf{R} \ D, & extsf{$\theta \models x_i = x_j$} & extsf{ if } heta(x_i) = heta(x_j) \ D, & extsf{$\theta \models \varphi \land \psi$} & extsf{ if } D, & extsf{$\theta \models \varphi$} extsf{ and } D, & extsf{$\theta \models \psi$} \ D, & extsf{$\theta \models \neg \varphi$} & extsf{ if not } D, & extsf{$\theta \models \varphi$} \ D, & extsf{$\theta \models \exists x_i. \varphi$} & extsf{ if } D, & extsf{$\theta \models \varphi$} extsf{ if } D, & extsf{$\theta \models \varphi$} \end{aligned}$$

Definition

An **answer** to a query  $\varphi$  over a database D is a **relation**:

$$arphi(D) = \{ heta_{\mid \mathrm{FV}(arphi)} : D, heta \models arphi \}$$

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# **Examples of Queries**

over integers:

- list all pairs of numbers that add to 10
- list all composite numbers
- list all prime numbers

over the bibliography database:

- list all publications
- list titles of all publications
- list titles of all books
- list all publications without authors
- list (pairs of) coauthor names
- list titles of publications written by a single author

w.r.t. the OS kernel:

- list pairs of PIDs of processes that share an open file
  - list PIDs of processes do not have any open file
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## Equivalences and Syntactic Sugar

#### **Boolean Equivalences**

• 
$$\neg(\neg \varphi) \equiv \varphi$$
  
•  $\varphi \lor \psi \equiv \neg(\neg \varphi \land \neg \psi)$   
•  $\varphi \to \psi \equiv \neg \varphi \lor \psi$   
•  $\varphi \leftrightarrow \psi \equiv (\varphi \to \psi) \land (\psi \to \varphi)$   
• ...

First-order Equivalences

• 
$$\forall x. \varphi \equiv \neg \exists x. \neg \varphi$$

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# **Integrity Constraints**

Relational *signature* captures only the structure of relations.

### Idea

Valid database instances satisfy additional integrity constraints.

- values of a particular attribute belong to a prescribed data type.
- values of attributes are unique among tuples in a relation (keys).
- values appearing in one relation must also appear in another relation (*referential integrity*).
- values cannot appear simultaneously in certain relations (*disjointness*).
- values in certain relation must appear in at least one of another set of relations (*coverage*).

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## Example Revisited (Integers)

What must be always true for the integers?

- plus is commutative
- plus is a (relation representing a) binary function
- plus and times obey the distributive law

#### Idea

Integrity constraints

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# Example Revisited (Bibliography)

Typing constraints

- Author id's are integers.
- Author names are strings.

Uniqueness of values/Keys

- Author id's are unique and determine author names.
- Publication id's are unique as well.
- Articles are identified by their id and the id of a collection they have appeared in.

Referential Integrity/Foreign Keys

- "books", "journals", "proceedings", and "articles" are "publications".
- The components of a "wrote" tuple must be an "author" and a "publication".

# Example Revisited (cont.)

### Disjointness

- "books" are different from "journals".
- "books" are different from "proceedings".

### Coverage

- Every "publication" is a "book" or a "journal" or a "proceedings" or an "article".
- Every "article" appears in a "book" or in a "journal" or in "proceedings".

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#### Idea

Answers to queries can be used to define derived relations (views)  $\Rightarrow$  extension of a DB schema

- subtraction, complement, ....
- collection-style publication, editor, ...

In general, a view is an integrity constraint of the form

$$orall x_1,\ldots,x_k.R(x_1,\ldots,x_k)\iff arphi$$

where *R* is a fresh relation name and  $x_1, \ldots, x_k$  are free variables of  $\varphi$ .

 $\Rightarrow$  why not simply an arbitrary formula containing R?

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## Story so far...

- **1** databases  $\Leftrightarrow$  relational structures
- 2 queries ⇔ formulas in First-Order logic
- ③ integrity constraints ⇔ closed formulas in FO logic

... so is there anything new here?

→YES: database instances must be finite

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... so is there anything new here?

 $\Rightarrow$  YES: database instances must be finite

- $\neg \exists x. author(x, y)$
- book(x, y, z)  $\lor$  proceedings(x, y)
- x = y

 $\Rightarrow$  we want only queries with finite answers (over finite databases).

### Definition (Domain-independent Query)

A query is *domain-independent* if its answer depends only in the database instance (but not on the domain D).

### Theorem

Answers to domain-independent queries contain only values that exist in the database instance (or as a constant in the query).

### Domain-independent + finite database $\Rightarrow$ "safe"

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# Safety and Query Satisfiability

### Theorem

Satisfiability<sup>a</sup> of first-order formulas is undecidable;

- co-r.e. in general
- r.e for finite databases

<sup>a</sup>Is there a database for which the answer is non-empty?

### Proof.

Reduction from PCP (see Abiteboul et. al. book, p.122-126).

### Theorem

Domain-independence of first-order queries is undecidable.

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arphi is satisfiable iff  $x=y\wedge arphi$  is not domain-independent.

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### Proof.

 $\varphi$  is satisfiable iff  $x = y \land \varphi$  is not domain-independent.

### **Range-restricted Queries**

$$\begin{array}{c} \text{Definition (Range restricted queries)} \\ Q & ::= & R(x_{i_1},\ldots,x_{i_k}) \\ & & Q \wedge Q \\ & & Q \wedge x_i = x_j \\ & & \exists x_i.Q \\ & & Q_1 \vee Q_2 \\ & & Q_1 \wedge \neg Q_2 \end{array} \right\} \quad x_i,x_j \in FV(Q) \\ & & FV(Q_1) = FV(Q_2) \end{array}$$

. . .

#### Theorem

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Range-restricted  $\Rightarrow$  Domain-independent.

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### Theorem

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Range-restricted  $\Rightarrow$  Domain-independent.

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## Safe v.s. Range-restricted

Do we lose expressiveness by using to Range-restricted queries?

#### Theorem

*Every* domain-independent query *can be written equivalently as a* Range restricted *query.* 

### Proof.

- **1** restrict every variable in  $\varphi$  to *active domain* (constants present in the database instance),
- express the active domain using a *unary query* over the database instance.

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# **Computational Properties**

- Evaluation of every query terminates
   ⇒ relational calculus is not *Turing complete*
- Data Complexity in the size of the database, for a *fixed* query. ⇒ in PTIME
  - $\Rightarrow \text{ in LOGSPACE}$
  - $\Rightarrow$  AC<sub>0</sub> (constant time on polynomially many CPUs in parallel)
- Combined complexity
  - $\Rightarrow$  in PSPACE
  - $\Rightarrow$  can express NP-hard problems (encode SAT)

Query Evaluation vs. Theorem Proving

**Query Evaluation** 

Given a query  $\varphi$  and a finite database instance *D* find substitutions  $\theta$  such that  $D, \theta \models \varphi$ .

**Query Satisfiability** 

Given a query  $\varphi$  find whether there is a (finite) database instance *D* and a substitution  $\theta$  such that  $D, \theta \models \varphi$ .

- much harder (undecidable) problem
- can be solved for fragments

(conjunctive/positive) queries

# Query Equivalence and Schema

### Do we ever need the power of theorem proving?

Definition (Query Subsumption)

 $\varphi$  subsumes  $\psi$  (with respect to a schema  $\Sigma$ ) if

 $\varphi(D)\subseteq\psi(D)$ 

for every database D such that  $D \models \Sigma$ .

- necessary for query simplification
- equivalent to proving

$$\bigwedge_{\phi_i\in\Sigma}\phi_i
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undecidable in general; decidable for fragments of RC

# Query Equivalence and Schema

Do we ever need the power of theorem proving?

Definition (Query Subsumption)

 $\varphi$  subsumes  $\psi$  (with respect to a schema  $\Sigma$ ) if

 $arphi(D)\subseteq \psi(D)$ 

for every database *D* such that  $D \models \Sigma$ .

- necessary for query simplification
- equivalent to proving

$$igwedge_{\phi_i\in\Sigma}\phi_i o (orall x_1,\dots x_k.arphi o\psi)$$

• undecidable in general; decidable for fragments of RC

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### Note

RC is not Turing-complete

 $\Rightarrow$  there must be computable queries that cannot be written in RC.

### **Built-in Operations**

• ordering, arithmetic, string operations, etc.

### **Counting/Aggregation**

• cardinality of sets (parity)

Reachability/Connectivity/Transitive closure

paths in a graph (binary relation)?

Model extensions: Incompleteness/Inconsistency

- tuples with unknown (but existing) values
- incomplete relations and open world assumption
- conflicting information (e.g., from different sources)

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