Show that forall $x, y \in \mathbb{R}$,

$$9x^4 + 4y^2 + x^2 \geqslant 12x^2y$$

Proof: Forall $x, y \in \mathbb{R}$, $9x^4 + 4y^2 + x^2 \geq 12x^2y$

$$9x^4 + 4y^2 - 12x^2y + x^2 \geq 0$$

$$9x^4 - 12x^2y + 4y^2 + x^2 \geq 0$$

$$(3x^2 - 2y)^2 + x^2 \geq 0$$

which is true.

Correct Solution: For all $x, y \in \mathbb{R}$

$$(3x^2 - 2y)^2 + x^2 \geq 0$$

Since the squares of real numbers is always non negative. Further, this implies that

$$9x^4 - 12x^2y + 4y^2 + x^2 \geq 0$$

and hence $$9x^4 + 4y^2 + x^2 \geq 12x^2y$$

Alternate:

For all $x, y \in \mathbb{R}$,

$$9x^4 + 4y^2 + x^2 \geq 12x^2y$$

$$\Leftrightarrow 9x^4 - 12x^2y + 4y^2 + x^2 \geq 0$$

$$\Leftrightarrow (3x^2 - 2y)^2 + x^2 \geq 0.$$

which is true.