Prove that $A \subseteq B$ if and only if $A \cap B = A$.

Proof; For the forward direction, assume that $A \subseteq B$. To show $A \cap B \subseteq A$, we note that $A \cap B$ is the set of all elements contained in both A and B and so all elements are contained in A. To show $A \subseteq A \cap B$, let $x \in A$. Then since $A \subseteq B$, we have that $x \in B$. As $x \in A$ and $x \in B$, we have that $x \in A \cap B$ by definition. Since $A \cap B \subseteq A$ and $A \subseteq A \cap B$, we have that $A = A \cap B$.

For the reverse direction, assume that $A \cap B = A$. As before, we have that $A \cap B \subseteq B$ since $A \cap B$ is all elements contained in both A & B. As $A = A \cap B$, for any $x \in A$, $x \in A \cap B$ and since $A \cap B \subseteq B$, we have that $x \in B$. Therefore $A \subseteq B$.