Find the cube roots of $1-2i$. (You may leave part of your answer in terms of arctan).

Solution: We are trying to solve $z^3 = 1-2i$. Let $z = re^{i\theta}$. Then

$$r^3 e^{i3\theta} = 1-2i$$

Taking lengths gives (by PM)

$$|r^3 e^{i3\theta}| = |r^3||e^{i3\theta}| = |r|^3 = |1-2i| = \sqrt{(1)^2 + (-2)^2} = \sqrt{5}$$

Hence $r = \sqrt[6]{5}$. Next, we substitute this back into the first displayed equation to get that

$$e^{i3\theta} = r^{-1}(1-2i) = e^{i\alpha}$$

where $\alpha = \arctan((-2r^{-1})/(r^{-1})) = \arctan(-2)$. Hence

$$3\theta = \alpha + 2\pi k$$

for any integer k. Solving for θ gives

$$\theta = \alpha/3 + 2\pi k/3$$

As in class, $k \in \{0, 1, 2\}$. Hence,

$$z \in \{\sqrt[6]{5}e^{i(\arctan(-2)/3)}, \sqrt[6]{5}e^{i(\arctan(-2)/3+2\pi/3)}, \sqrt[6]{5}e^{i(\arctan(-2)/3)+4\pi/3)}\}$$