Using the fact that $\tan(\pi/12) = 2 - \sqrt{3}$, find the polar coordinates of

$$-\sqrt{3} - (2\sqrt{3} - 3)i.$$

Solution: First we compute the length of the given complex number:

$$\begin{aligned} r &= |-\sqrt{3} - (2\sqrt{3} - 3)i| \\ &= \sqrt{(-\sqrt{3})^2 + (-(2\sqrt{3} - 3))^2} \\ &= \sqrt{3 + 4(3) - 12\sqrt{3} + 9} \\ &= \sqrt{24 - 12\sqrt{3}} \\ &= 2\sqrt{6 - 3\sqrt{3}} \end{aligned}$$

Now, to compute θ, the angle of rotation with respect to the x-axis, we perform:

$$\begin{aligned} \arctan\left(\frac{y}{x}\right) &= \arctan\left(\frac{-(2\sqrt{3} - 3)}{-\sqrt{3}}\right) \\ &= \arctan\left(\frac{\sqrt{3}(2 - \sqrt{3})}{\sqrt{3}}\right) \\ &= \arctan(2 - \sqrt{3}) \\ &= \pi/12 \end{aligned}$$

Thus, since our original complex point is in the third quadrant, we have that $\theta = \pi + \pi/12 = 13\pi/12$. Hence, in polar coordinates,

$$\begin{aligned} -\sqrt{3} - (2\sqrt{3} - 3)i &= re^{i\theta} \\ &= r(\cos(\theta) + i\sin(\theta)) \\ &= (2\sqrt{6 - 3\sqrt{3}})(\cos(13\pi/12) + i\sin(13\pi/12)) \end{aligned}$$