

Does 13 divide $2^{70} + 3^{70}$?

Solution: By CISR, 13 divides this number if and only if

$$2^{70} + 3^{70} \equiv 0 \pmod{13}.$$

Examining the number modulo 13,

$$\begin{aligned} 2^{70} + 3^{70} &\equiv (2^2)^{35} + 3^{70} \pmod{13} \\ &\equiv (4)^{35} + 3^{70} \pmod{13} \\ &\equiv (4 - 13)^{35} + 3^{70} \pmod{13} \\ &\equiv (-9)^{35} + 3^{70} \pmod{13} \\ &\equiv -(9)^{35} + 3^{70} \pmod{13} \\ &\equiv -(3^2)^{35} + 3^{70} \pmod{13} \\ &\equiv -3^{70} + 3^{70} \pmod{13} \\ &\equiv 0 \pmod{13} \end{aligned}$$

Therefore, 13 divides $2^{70} + 3^{70}$.