Find $\gcd(65, 40)$ and find integers x and y such that $65x + 40y = \gcd(65, 40)$

$$65 = 40(1) + 25 \tag{1}$$
$$40 = 25(1) + 15 \tag{2}$$
$$25 = 15(1) + 10 \tag{3}$$
$$15 = 10(1) + 5 \tag{4}$$
$$10 = 5(2) + 0 \tag{5}$$

Thus, the greatest common divisor of 65 and 40 is 5.

$$\begin{aligned}
5 &= 15 + 10(-1) && \text{By (4)} \\
&= 15 + (25 + 15(-1))(-1) && \text{By (3)} \\
&= 25(-1) + 15(2) \\
&= 25(-1) + (40 + 25(-1))(2) && \text{By (2)} \\
&= 40(2) + 25(-3) \\
&= 40(2) + (65 + 40(-1))(-3) && \text{By (1)} \\
&= 65(-3) + 40(5)
\end{aligned}$$

Therefore, integers satisfying the original equations are $x = -3$ and $y = 5$.