2) Let P and Q be statements. Define the statement $P \circ Q$ by the following truth table:

P	Q	$P \circ Q$
T	T	F
T	F	T
F	T	T
F	F	T

Show that $P \circ Q$ is equivalent to the statement $\neg(P \wedge Q)$

Solution: Creating a truth table yields:

P	Q	$P \circ Q$	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

Since the third and the fifth columns are the same, the two headings are logically equivalent

(b) Is $P \wedge Q$ equivalent to $(P \circ Q) \circ (Q \circ P)$?

Solution: For part (b), We use a sequence of logical equivalences to see that:

$$
\begin{aligned}
(P \circ Q) \circ (Q \circ P) &\equiv (\neg(P \wedge Q)) \circ (\neg(Q \wedge P)) && \text{By part (a)} \\
&\equiv (\neg P \vee \neg Q) \circ (\neg Q \vee \neg P) && \text{By DeMorgan's Laws} \\
&\equiv \neg((\neg P \vee \neg Q) \wedge (\neg Q \vee \neg P)) && \text{By part (a)} \\
&\equiv \neg(\neg P \vee \neg Q) \vee \neg(\neg Q \vee \neg P) && \text{By DeMorgan's Laws} \\
&\equiv (P \wedge Q) \vee (Q \wedge P) && \text{By DeMorgan's Laws} \\
&\equiv (P \wedge Q) \vee (P \wedge Q) \\
&\equiv P \wedge Q
\end{aligned}
$$