

Week 10 List of Theorems

Properties of Conjugates (PCJ)

If z and w are complex numbers, then

1. $\overline{z + w} = \bar{z} + \bar{w}$.
2. $\overline{zw} = \bar{z} \bar{w}$.
3. $\overline{\bar{z}} = z$.
4. $z + \bar{z} = 2\text{Re}(z)$.
5. $z - \bar{z} = 2i\text{Im}(z)$.

Properties of Modulus (PM)

If z and w are complex numbers, then

1. $|z| = 0$ if and only if $z = 0$.
2. $|\bar{z}| = |z|$.
3. $|z|^2 = z\bar{z}$.
4. $|zw| = |z| |w|$.
5. $|z + w| \leq |z| + |w|$.

Polar Multiplication of Complex Numbers (PMCN)

If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ are complex numbers in polar form, then $z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$.

De Moivre's Theorem (DMT)

For any $\theta \in \mathbb{R}$ and $n \in \mathbb{Z}$, $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$.

Properties of Complex Exponentials (PCE)

If θ and ϕ are real numbers, then

$$\begin{aligned} e^{i\theta} \cdot e^{i\phi} &= e^{i(\theta+\phi)} \\ (e^{i\theta})^n &= e^{in\theta} \quad \forall n \in \mathbb{Z} \end{aligned}$$

Complex n -th Roots Theorem (CNRT)

If $a = r(\cos \theta + i \sin \theta)$, then the solutions to $z^n = a$ are $\sqrt[n]{r} \left[\cos \frac{\theta+2k\pi}{n} + i \sin \frac{\theta+2k\pi}{n} \right]$, $k = 0, 1, \dots, n-1$.

Division Algorithm for Polynomials (DAP)

If $f(x), g(x) \in \mathbb{F}[x]$ and $g(x)$ is not the zero polynomial, then there exist unique $q(x), r(x) \in \mathbb{F}[x]$ such that

$$f(x) = q(x)g(x) + r(x)$$

where $\deg(r(x)) < \deg(g(x))$ or $r(x)$ is the zero polynomial.