Q1. I enjoy trying to discover and write MATH 135 proofs.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

CODE

BC

Q2.When I have difficulties with MATH 135 proofs, I know I can handle them.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

Q3. Suppose A, B and C are all true statements.

The compound statement $(\neg A) \vee (B \wedge \neg C)$ is

A) True

B) False ←

$F \vee (T \wedge F)$

$F \vee F$

Q4. In class, I would prefer the use of

A) Document Camera

B) Blackboard

Divisibility: Let $m, n \in \mathbb{Z}$. Then $m \mid n$ if (and only if) there exists a $k \in \mathbb{Z}$ such that

$$mk = n.$$

Ex. $5 \mid 15$ $\because$ $5 \cdot 3 = 15$.

$-7 \mid 0$ $\because$ $(-7) \cdot 0 = 0$.

$0 \mid 0$ $\because$ $0 \cdot 0 = 0$.

$3 \mid -27$ $\because$ $3 \cdot (-9) = -27$

If m does not divide n, we write $m \nmid n$

Ex: $5 \nmid 7$ since there no integer satisfying $5k = 7$.

NB $\pi \mid 3\pi$ doesn't make sense since in the def'n of "$\mid$", $m, n \in \mathbb{Z}$.

Q: (Direct Proof) $n \in \mathbb{Z} \wedge 14 \mid n \Rightarrow 7 \mid n$.

"there exists" $\exists$

Soln: Let $n \in \mathbb{Z}$ and suppose $14 \mid n$. Then $\exists$ $k \in \mathbb{Z}$ s.t. $14k = n$. Then $(7 \cdot 2)k = n$. By

s.t. = such that.

associativity, $7(2k) = n$. Since $2k \in \mathbb{Z}$, $7 \mid n$

$\blacksquare$

Q: Let $x \in \mathbb{Z}$. Suppose 2^{2x} is an odd integer. Show that 2^{-2x} is odd.

Pf: Recall: An integer n is...

(i) Even if $2 \mid n$.

(ii) Odd if $2 \mid (n-1)$

First, note $x \geq 0$ for 2^{2x} to be an integer.

If $x \geq 1$ then $2^{2x} = 2 \cdot (\underbrace{2^{2x-1}}_{\in \mathbb{Z}})$ so $2 \mid 2^{2x}$.

and thus, 2^{2x} is not odd.

$\therefore x = 0$. Hence $2^{2x} = 1$ and $2^{-2x} = 1$ is odd!

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Defn. An integer p is said to be prime if (and only if) $p > 1$ and its only positive divisors are 1 and p.

Ex: Show p & $p+1$ are prime only when $p = 2$.

Prop: Bounds by Divisibility (BBD).

$$a|b \wedge b \neq 0 \Rightarrow |a| \leq |b|.$$

Pf: Let $a, b \in \mathbb{Z}$ s.t. $a|b$ and $b \neq 0$. Then $\exists\, k \in \mathbb{Z}$ s.t. $ak = b$. Since $b \neq 0$, we know that $k \neq 0$. Thus $a = \frac{b}{k}$. Also $|a| = \left|\frac{b}{k}\right| \leq |b|$ ∎

(Alt: $|a| \leq |a||k| = |b|$) (since $|k| \geq 1$, since $k \neq 0$)

Prop: Transitivity of Divisibility

If $a|b \wedge b|c \Rightarrow a|c$.

Pf: $\exists k \in \mathbb{Z}$ s.t. $ak = b$; $\exists \ell \in \mathbb{Z}$ s.t. $b\ell = c$

$\Rightarrow (ak)\ell = c \Rightarrow a(\underbrace{k\ell}_{\in \mathbb{Z}}) = c$ so $a|c$ ■

Prop: Divisibility of Integer Combinations. (DIC)

Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $a|c$ then for any $x, y \in \mathbb{Z}$ we have $a|(bx + cy)$