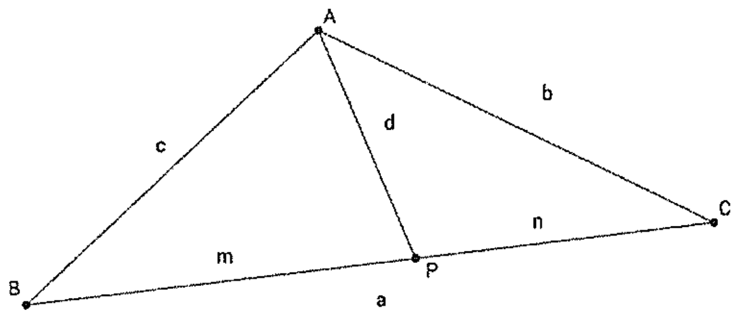
Announcements:

- A0 due tomorrow @ 8:25 AM.
- Office hours
  MTh: 1:00-2:00 , F 10:00-11:00
  NEW M 4:30-5:30.
- READ A1.
- Clicker Fridays.

Ls 92

Theorem 0.1. *Stewart's Theorem Let ABC be a triangle with $AB = c$, $AC = b$ and $BC = a$.*
If P is a point on BC with $BP = m$, $PC = n$ and $AP = d$, then $dad + man = bmb + cnc$.



Proof. **Proof A**

$$\begin{aligned}
c^2 &= m^2 + d^2 - 2md\cos\theta \\
b^2 &= n^2 + d^2 - 2nd\cos\theta' \\
b^2 &= n^2 + d^2 + 2nd\cos\theta \\
\frac{m^2 - c^2 + d^2}{-2md} &= \frac{b^2 - n^2 - d^2}{2nd} \\
nc^2 - nm^2 - nd^2 &= -mb^2 + mn^2 + md^2 \\
nc^2 - mb^2 &= mn^2 + md^2 + nm^2 + nd^2 \\
cnc + bmb &= nm(n+m) + d^2(m+n) \\
cnc + bmb &= man + dad
\end{aligned}$$

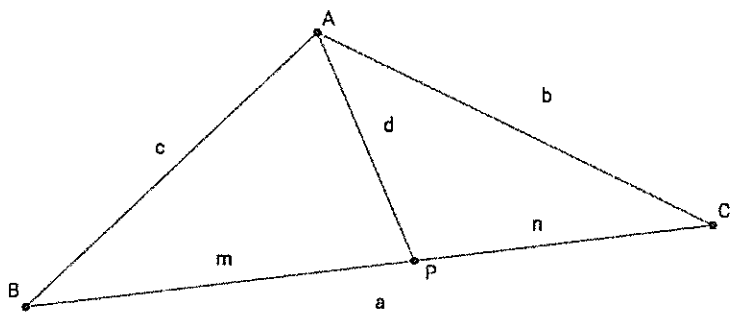
□

NO EXPLANATION

WHAT IS θ & θ'?

L3 P3

Theorem 0.2. *Stewart's Theorem Let ABC be a triangle with $AB = c$, $AC = b$ and $BC = a$.*
If P is a point on BC with $BP = m$, $PC = n$ and $AP = d$, then $dad + man = bmb + cnc$.



Proof. **Proof B**

The Cosine Law on $\triangle APB$ tells us that

$$c^2 = m^2 + d^2 - 2md\cos(\angle APB).$$

Subtracting c^2 from both sides gives

$$0 = -c^2 + m^2 + d^2 - 2md\cos(\angle APB).$$

Adding $2md\cos\angle APB$ to both sides gives

$$2md\cos(\angle APB) = -c^2 + m^2 + d^2.$$

Dividing both sides by $2md$ gives

$$\cos(\angle APB) = \frac{-c^2 + m^2 + d^2}{2md}.$$

Now, the Cosine Law on $\triangle APC$ tells us that

$$b^2 = n^2 + d^2 - 2nd\cos\angle APC.$$

Since $\angle APC$ and $\angle APB$ are supplementary angles, then

$$\cos\angle APC = \cos(\pi - \angle APB) = -\cos(\angle APB).$$

Substituting into our previous equation, we see that

$$b^2 = n^2 + d^2 + 2nd\cos\angle APB.$$

Subtracting n^2 from both sides gives

$$b^2 - n^2 = d^2 + 2nd\cos(\angle APB).$$

L3 P4

Then subtracting d^2 from both sides gives

$$b^2 - n^2 - d^2 = 2nd\cos(\angle APB).$$

Dividing both sides by $2nd$ gives

$$\frac{b^2 - n^2 - d^2}{2nd} = \cos(\angle APB).$$

Now we have two expressions for $\cos(\angle APB)$ and equate them to yield

$$\frac{-c^2 + m^2 + d^2}{2md} = \frac{b^2 - n^2 - d^2}{2nd}.$$

WAY TOO MUCH WRITING.

Multiplying both sides by $2mnd$ shows us that

$$n(-c^2 + m^2 + d^2) = m(b^2 - n^2 - d^2).$$

Next we distribute to get

$$-nc^2 + nm^2 + nd^2 = mb^2 - mn^2 - md^2.$$

Adding $nc^2 + mn^2 + md^2$ to both sides gives

$$nm^2 + mn^2 + nd^2 + md^2 = mb^2 + nc^2.$$

Factoring twice gives:

$$nm(m + n) + d^2(m + n) = mb^2 + nc^2.$$

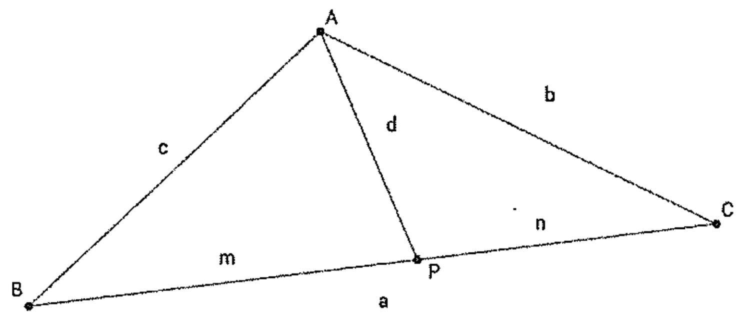
Since P lies on BC, then $a = m + n$ so we substitute to yield

$$nma + d^2a = mb^2 + nc^2.$$

Finally, we can rewrite this as $bmb + cnc = dad + man$.. $\square$

L3 P5

Theorem 0.3. *Stewart's Theorem Let ABC be a triangle with $AB = c$, $AC = b$ and $BC = a$.*
If P is a point on BC with $BP = m$, $PC = n$ and $AP = d$, then $dad + man = bmb + cnc$.



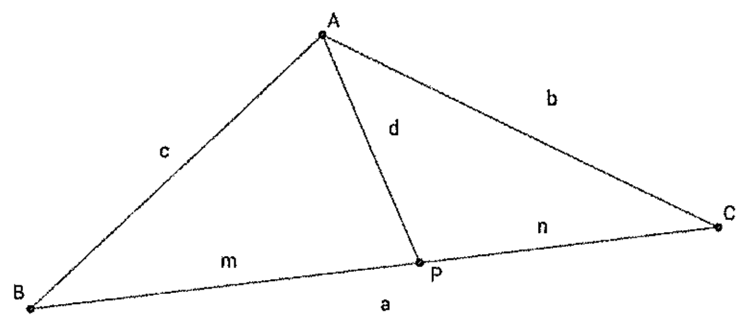
Proof. **Proof C**

Needs more steps.

Using the Cosine Law for supplementary angles $\angle APB$ and $\angle APC$, and then clearing denominators and simplifying gives $dad + man = bmb + cnc$ as required. $\square$

L3 P6

Theorem 0.4. *Stewart's Theorem Let ABC be a triangle with $AB = c$, $AC = b$ and $BC = a$.*
If P is a point on BC with $BP = m$, $PC = n$ and $AP = d$,
then $dad + man = bmb + cnc$.



Proof. **Proof D**

The Cosine Law on $\triangle APB$ tells us that

$$c^2 = m^2 + d^2 - 2md \cos \angle APB.$$

Similarly, the Cosine Law on $\triangle APC$ tells us that

$$b^2 = n^2 + d^2 - 2nd \cos \angle APC.$$

Since $\angle APC$ and $\angle APB$ are supplementary angles, we have

$$b^2 = n^2 + d^2 + 2nd \cos \angle APB.$$

Equating expressions for $\cos \angle APB$ yields

$$\frac{-c^2 + m^2 + d^2}{2md} = \frac{b^2 - n^2 - d^2}{2nd}.$$

Clearing the denominator and rearranging gives

$$nm^2 + mn^2 + nd^2 + md^2 = mb^2 + nc^2.$$

Factoring yields

$$mn(m + n) + d^2(m + n) = mb^2 + nc^2.$$

Substituting $a = (m + n)$ gives $dad + man = bmb + cnc$ as required. $\square$

Throughout the lecture, let A, B, C be statements.

Def'n: $\neg A$ is NOT A.

A	$\neg A$
T	F
F	T

Def'n: $A \wedge B$ is A AND B.
$A \vee B$ is A OR B.

A	B	$A \wedge B$	$A \vee B$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Which of the following are true?

- π is irrational and $3 > 2$ TRUE
- 10 is even and $1 = 2$ FALSE
- 7 is larger than 6 or 15 is a multiple of 3 TRUE
- $5 \leq 6$ TRUE
- 24 is a perfect square or the vertex of parabola $x^2 + 2x + 3$ is $(1, 1)$ FALSE.
- 2.3 is not an integer TRUE.
- 20% of 50 is not 10 FALSE.
- $\underbrace{7 \text{ is odd or } \underbrace{1 \text{ is positive and } 2 \neq 2}}$

ORDER OF OPERATIONS.

$\neg$, $\wedge$, $\vee$.

LAST BULLET IS TRUE.

Def'n: The symbol $\equiv$ in logic means logically equivalent, that is, in a truth table, the LHS & RHS are equal.

Ex: Show $\neg(\neg A) \equiv A$.

A	$\neg A$	$\neg(\neg A)$
T	F	T
F	T	F

Since first & last columns are equal, $A \equiv \neg(\neg A)$.

Theorem: (De Morgan's Law).

$$\neg(A \vee B) \equiv \neg A \wedge \neg B$$

$$\neg(A \wedge B) \equiv \neg A \vee \neg B.$$

Pf:

A	B	$A \vee B$	$\neg(A \vee B)$	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Since $\neg(A \vee B)$ has the same truth as $\neg A \wedge \neg B$, we have $\neg(A \vee B) \equiv \neg A \wedge \neg B$.

Ex: $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

Implication $(A \Rightarrow B)$

Def'n:

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

In $A \Rightarrow B$, A is called the hypothesis
B is called the conclusion.

nota bene

$\boxed{NB}$ To <u>prove</u> $A \Rightarrow B$, we assume A is true and show B is true.

To <u>use</u> $A \Rightarrow B$, we <u>prove</u> A is true and use B as true.

In the following, identify the hypothesis, the conclusion and state whether the statement is true or false.

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

- If $\sqrt{2}$ is rational then $2 < 3$ TRUE
- If (1+1=2) then $5 \cdot 2 = 11$ FALSE.
- If C is a circle, then the area of C is πr^2 TRUE.
- If 5 is even then 5 is odd TRUE.
- If $4 - 3 = 2$ then $1 + 1 = 3$ TRUE.

Proposition: $A \Rightarrow B \equiv \neg A \vee B$.

Pf:

A	B	$A \Rightarrow B$	$\neg A$	$\neg A \vee B$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

equal

$\boxtimes$.

Divisibility.

Defn: Let $m, n \in \mathbb{Z}$ (integers) Zählen. We say that m divides n and write $m \mid n$ if (and only if) there exists a $k \in \mathbb{Z}$ such that

$$mk = n.$$

Ex: $3|6$, $2|2$, $7|49$, $5|0$, $0|0$.