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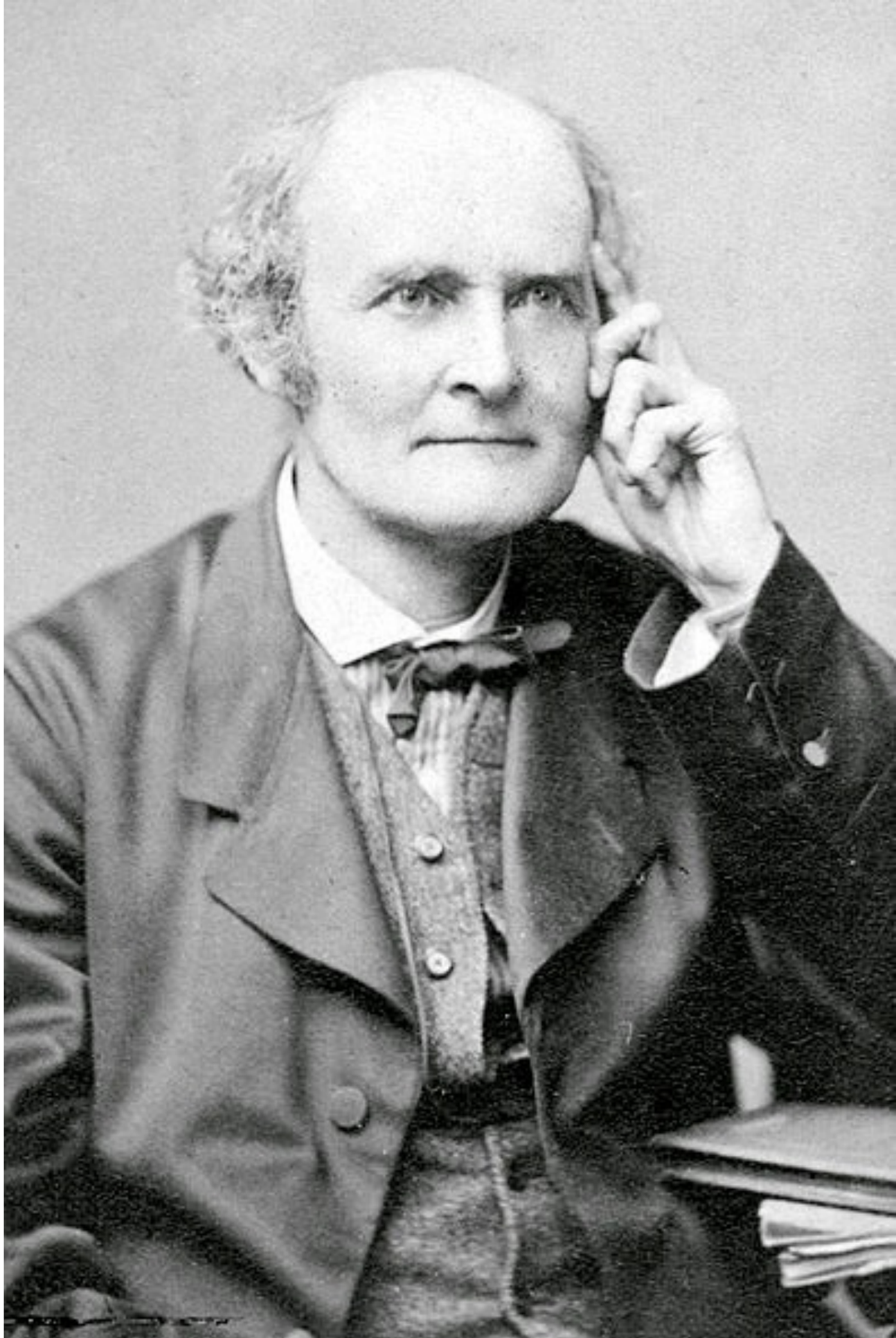
Sir Arthur Cayley (1821 – 1895)

- Collected Works published by Cambridge University Press contains 967 papers
- Cayley-Hamilton Theorem in linear algebra
- defined a group, advanced the study of abstract algebra
- contributed to non-Euclidean geometries
- contributed to study of invariants



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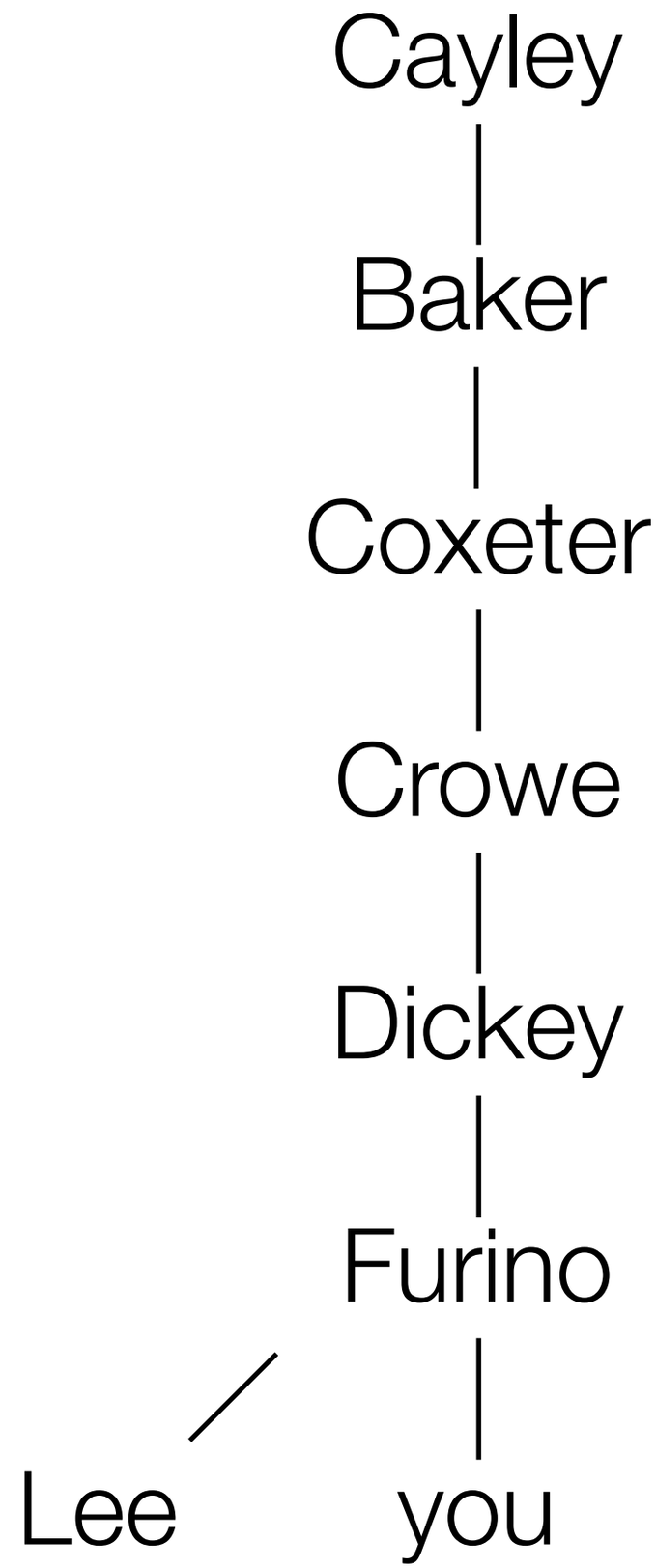
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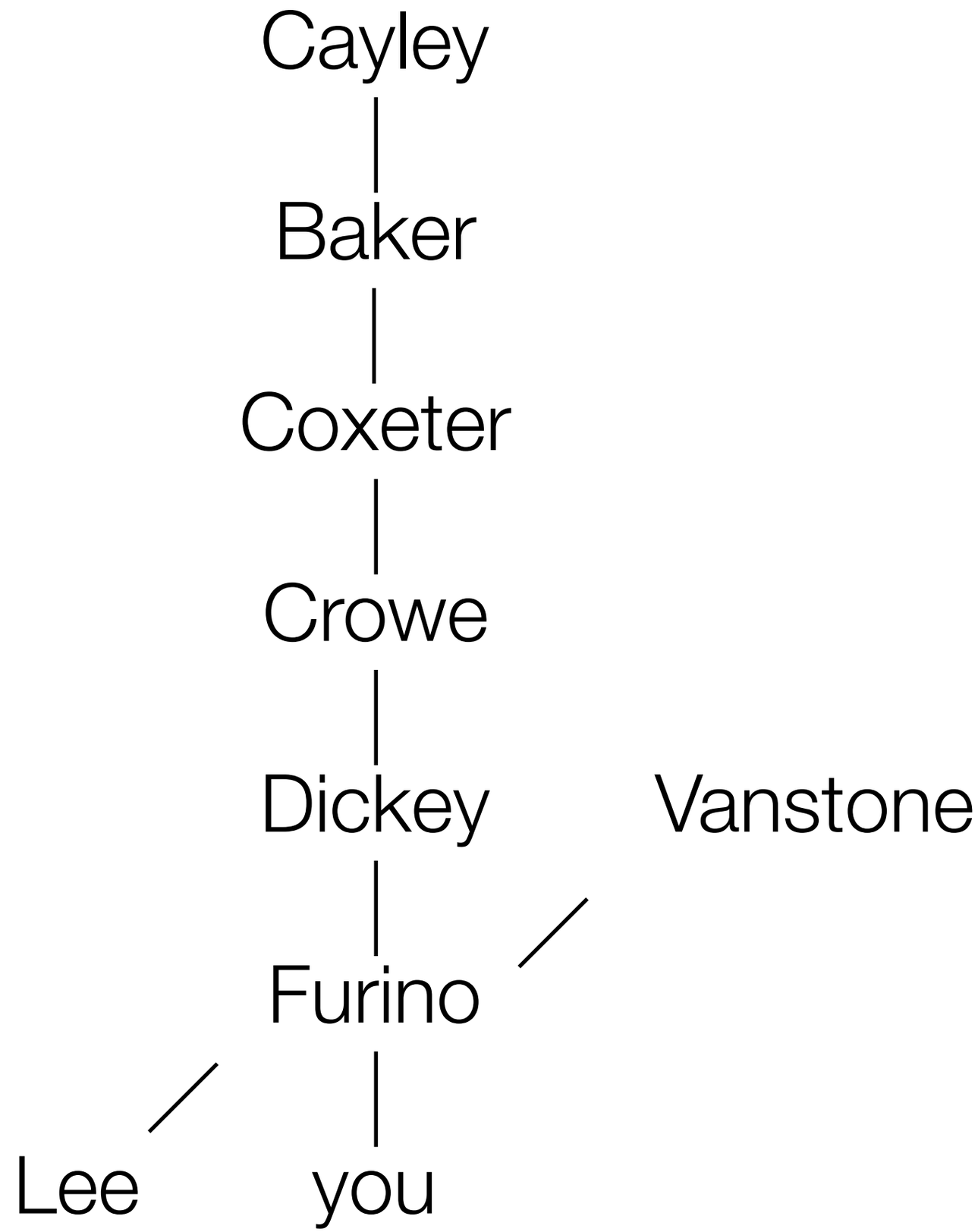
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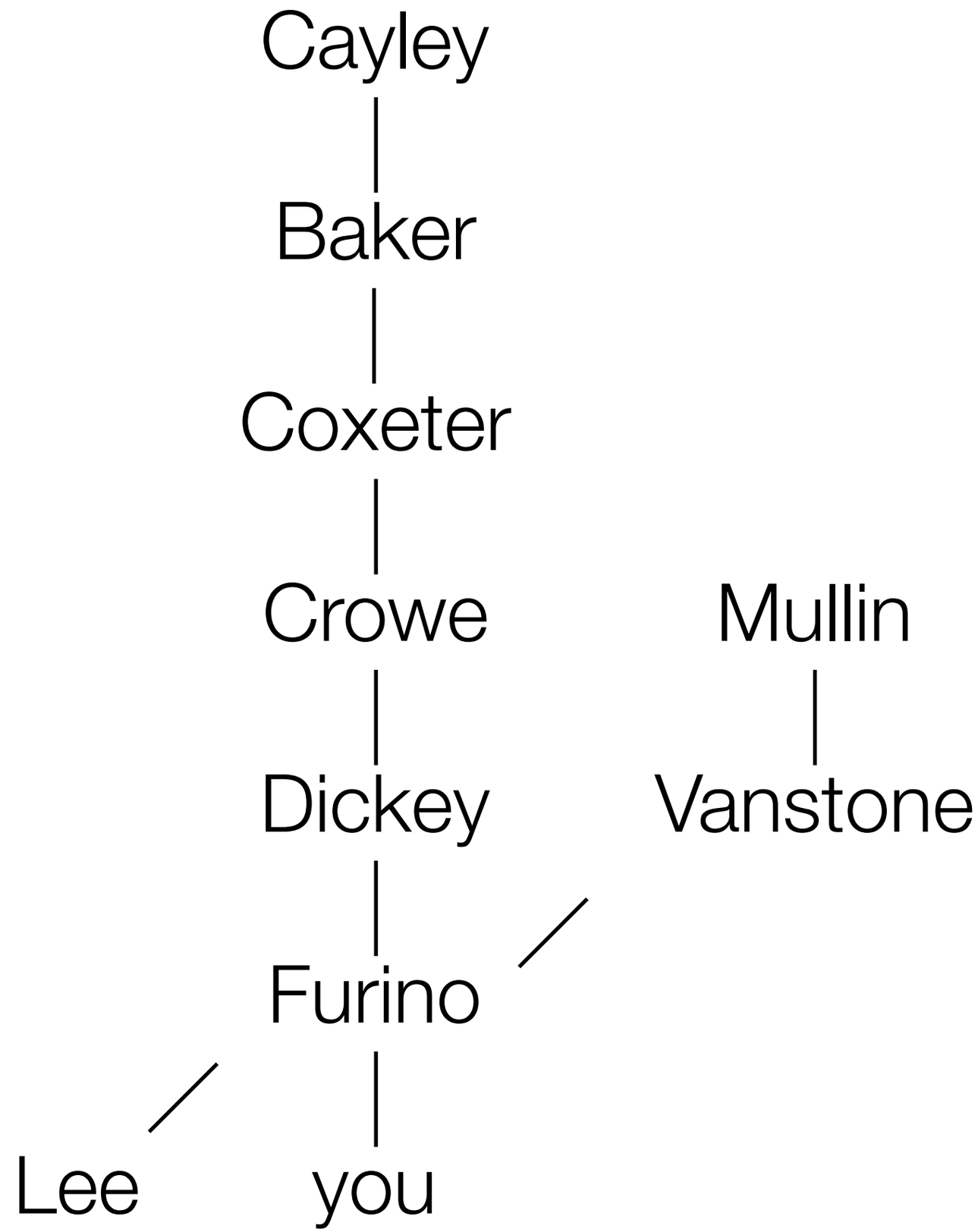
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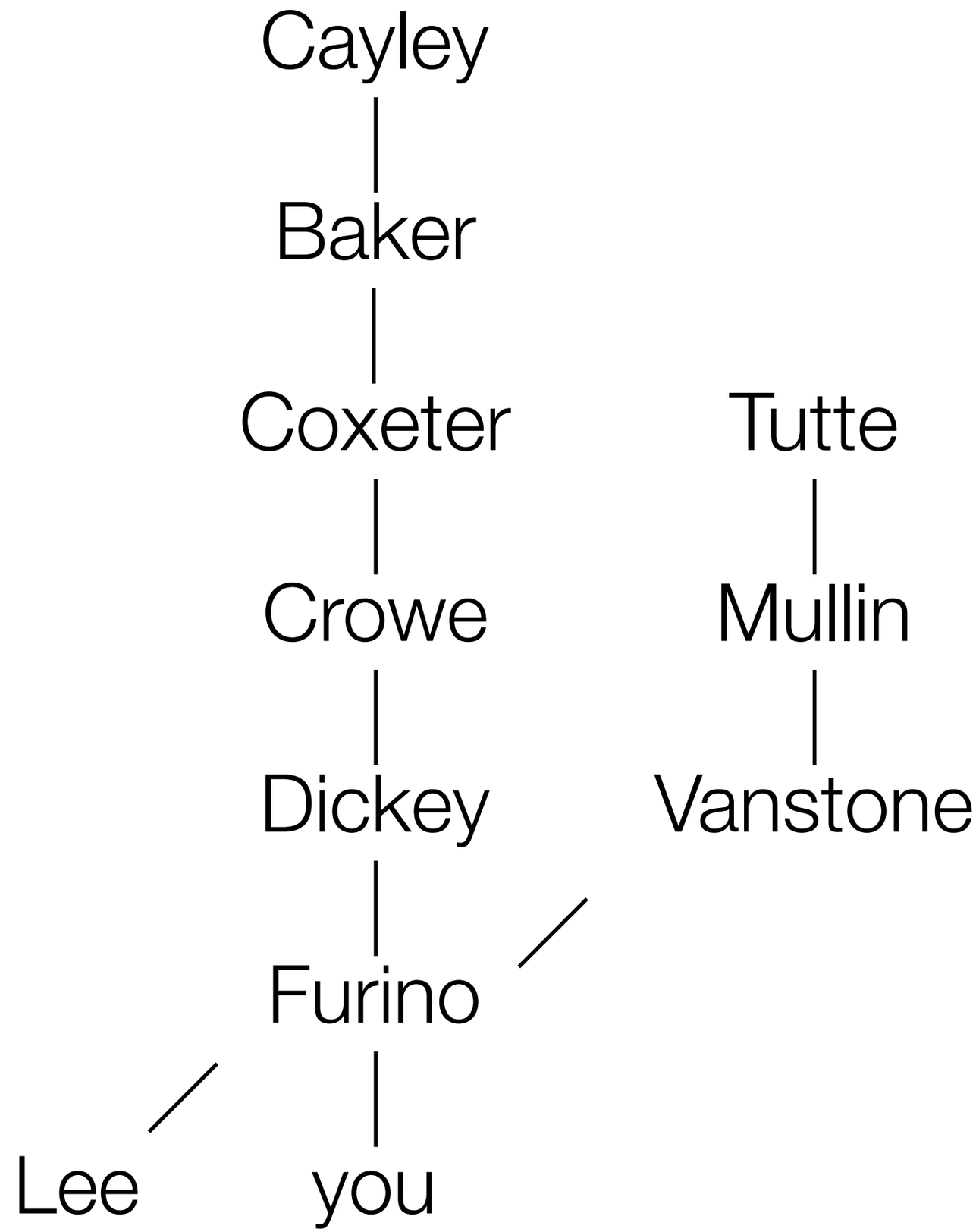
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William T Tutte
(1917 – 2002)

- “Foreign Office, Statistical” during WW II
- came to Canada in 1948
- came to Waterloo in 1962
- FRSC, FRS, OC 2001
- “the leading mathematician in combinatorics for three decades”

Your Lineage

- The Mathematics Genealogy Project
- http://genealogy.math.ndsu.nodak.edu

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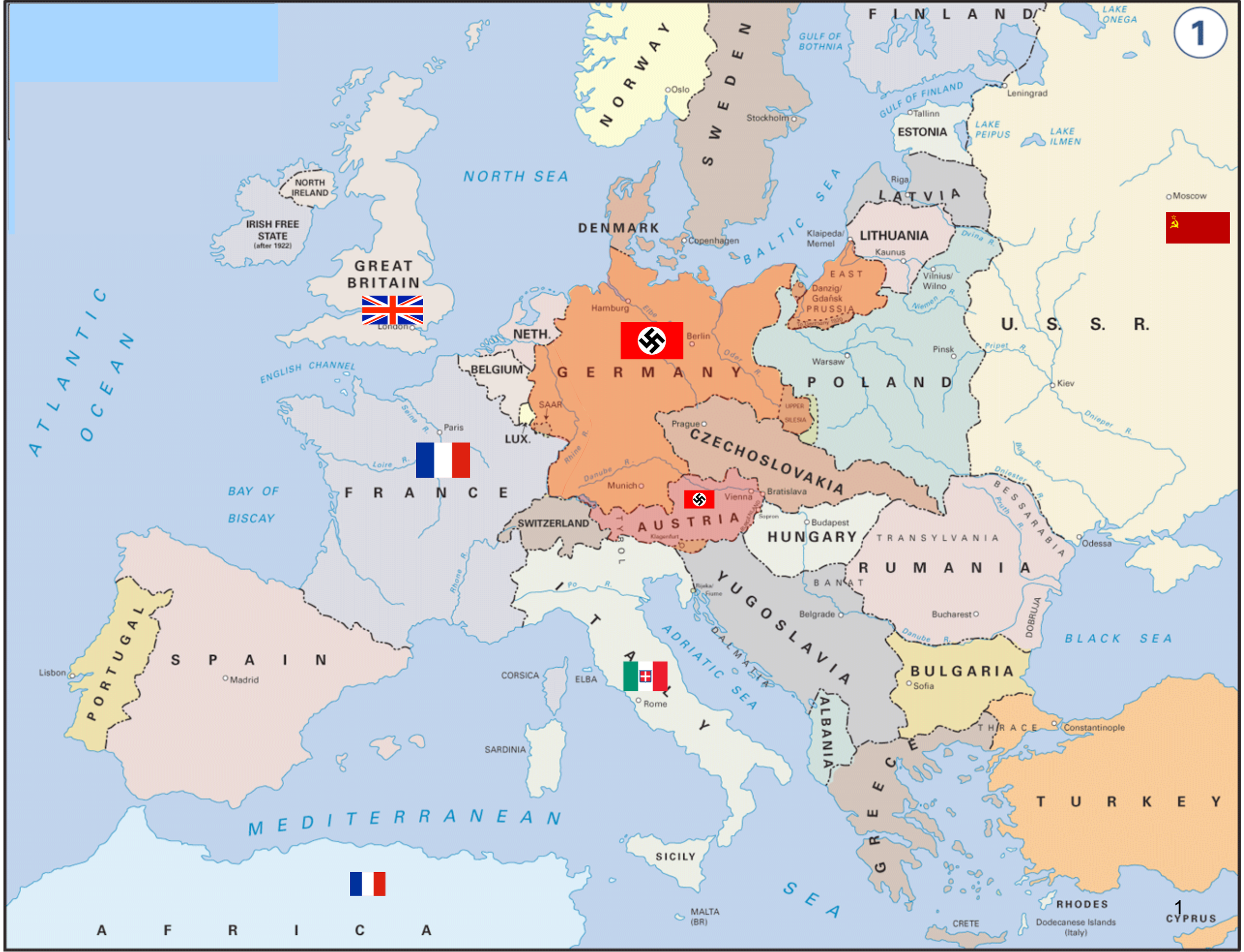
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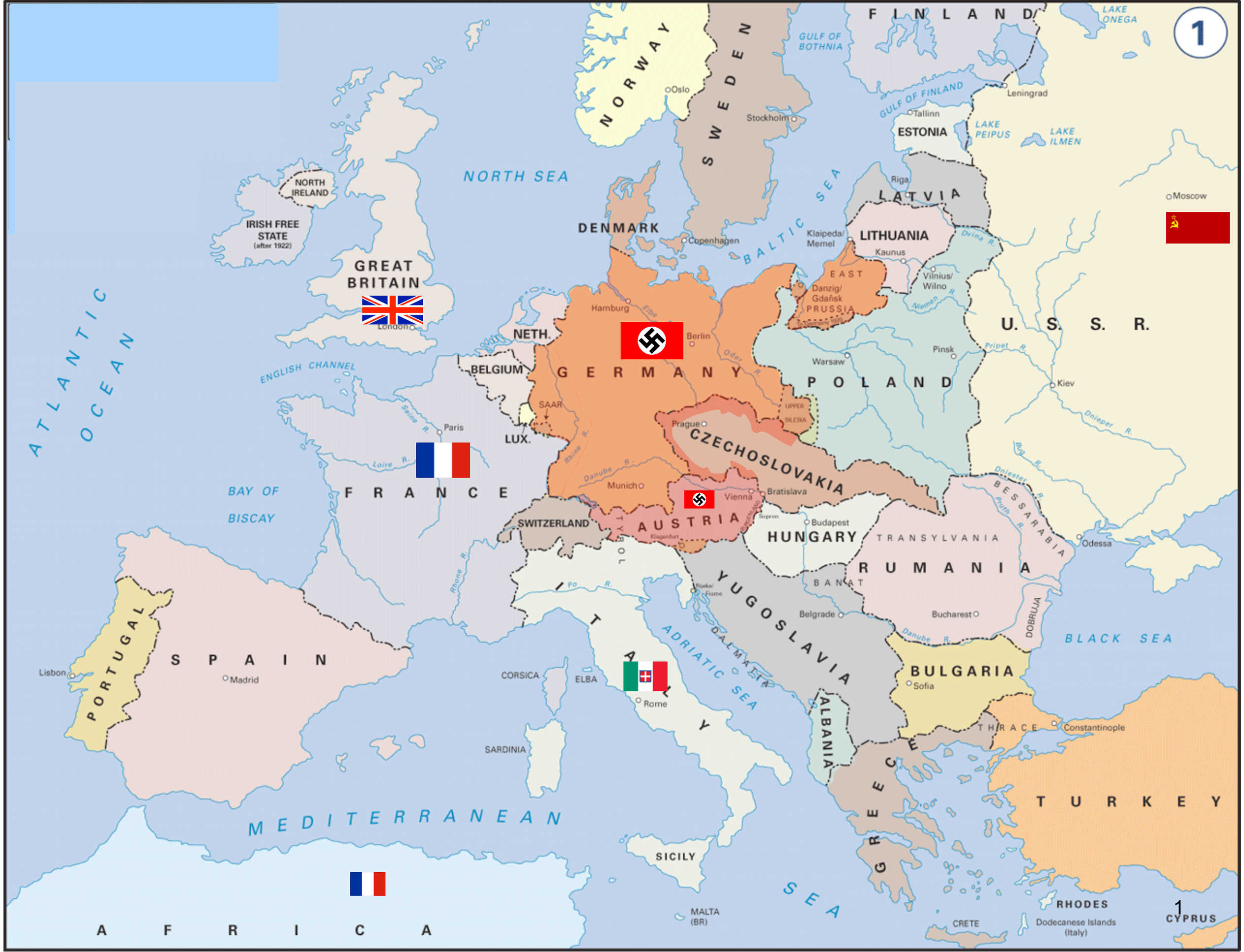
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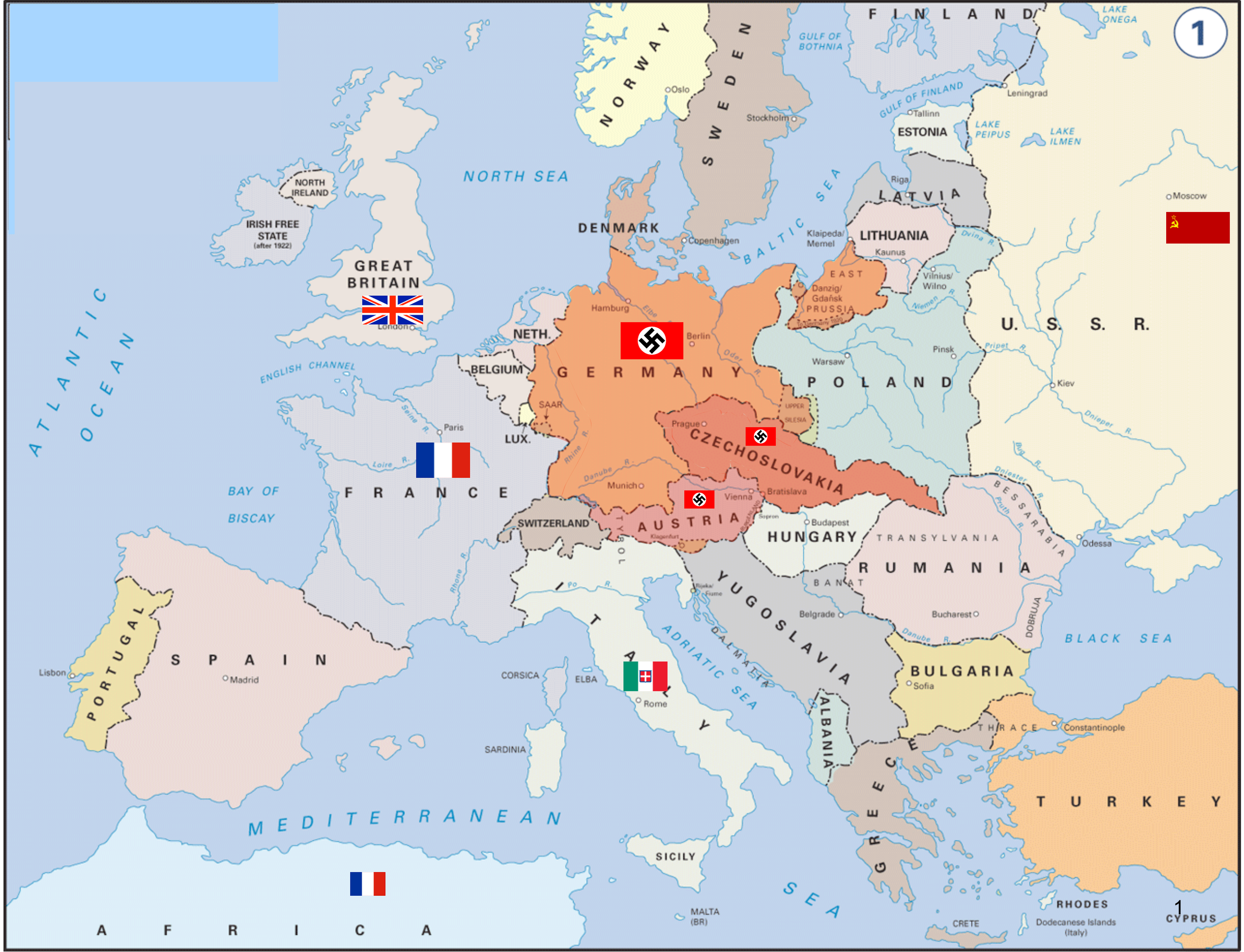
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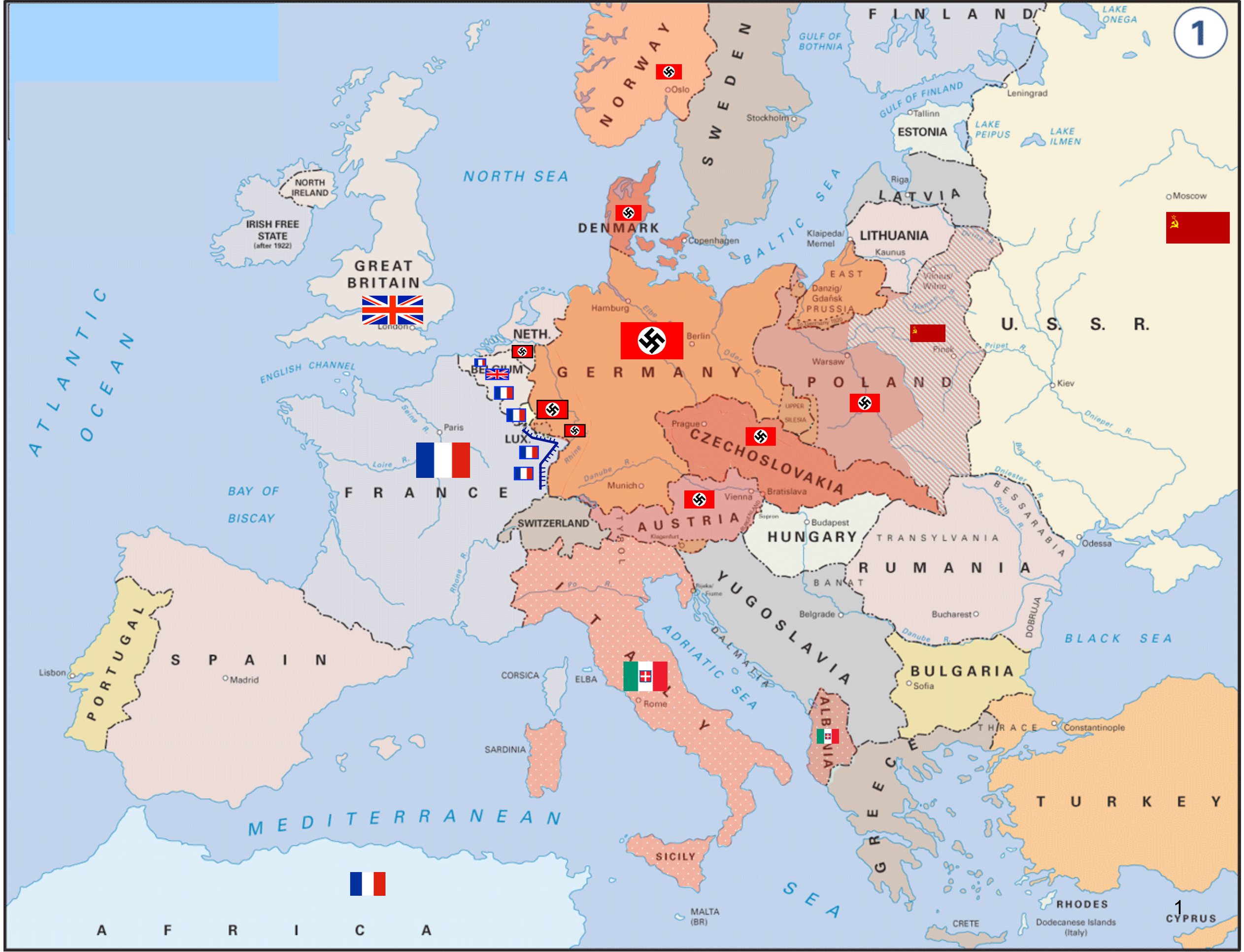
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Battle of the North Atlantic

Winston Churchill

Winston Churchill, June 1940

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22.

~~The~~ battle of France is over

What General Weygand calls 'the battle of France' is over. The battle of Britain is about to begin. Upon this battle depends the survival of Christian civilisation. Upon it depends our own British life and the long continuity of our institutions, and our Empire. The whole fury and might of the enemy must very soon be turned on us. Hitler knows that ~~if~~ he will have to break us in this Island, or lose the war. If we can stand up to him, all Europe may be ~~liberated~~ freed, and the life of the world may move forward into broad and sunlit uplands. But if we fail, then the whole world, including the United States, and all that we have known and cared for, will sink into the abyss of a new Dark Age made more sinister & perhaps more protracted by the lights of perverted Science. Let us therefore brace ourselves to our duty, and so bear ourselves that if the British Empire and Commonwealth lasts for a thousand years, men will still say, 'This was their ~~my~~ finest hour.'

Winston Churchill, June 1940

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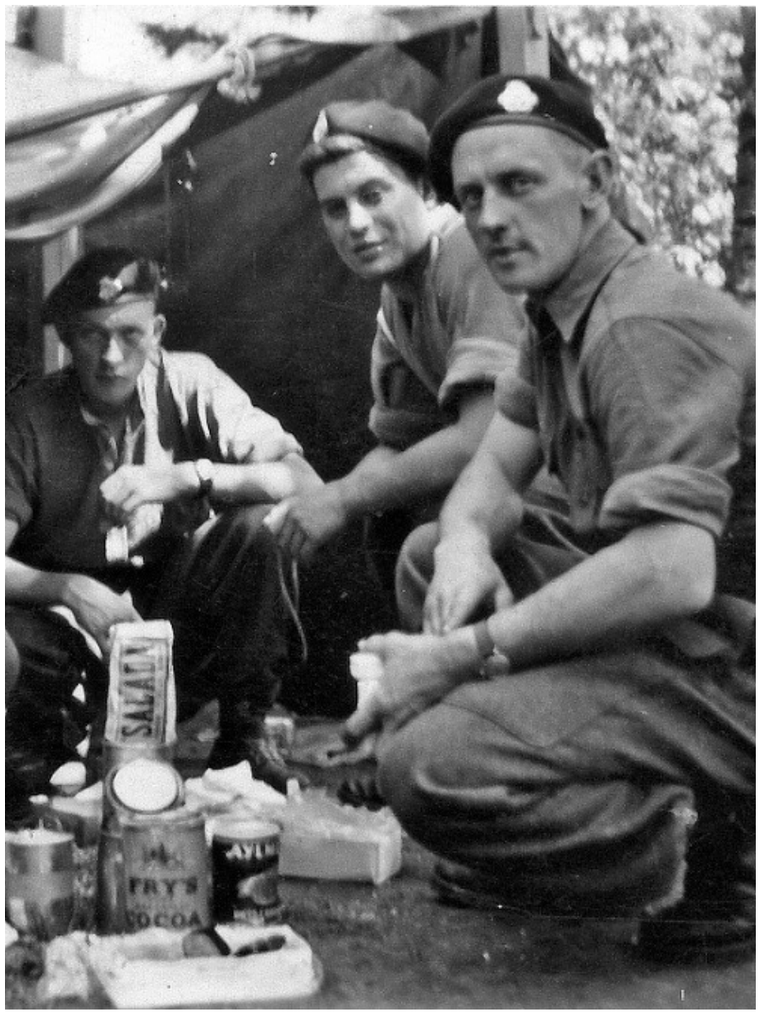

Evo Furino (8th Canadian (NB) Hussars), Gerald Merchant (HMCS St Croix), Frank Merchant (pilot, RCAF)

Ottawa Lists 146 as Lost In Sinking of Destroyer

Ottawa, Oct. 1.—(CP)—Following is the Royal Canadian Navy's official casualty list in the sinking of the destroyer St. Croix, containing the names of 146 of those missing:

MISSING

Missing on war service, due to enemy action, while serving on convoy duty in the Atlantic:

Officers

DeFreitas, Percival Francis Mayow, Lt.-Cmdr., Oak Bay, Victoria.

Dobson, Andrew Hedley, D.S.C., Lt.-Cmdr., Halifax.

Gallagher, John Fraser, Lieut., St. John's, Nfld.

King, William Lyon Mackenzie, Surgeon Lt., Bedford, N.S.

Major Paul Simon, Lt.-Cmdr., Mount Royal, Que.

Page, William Leonard, Sub-Lt., Regina.

Porter, Robert Noel Timothy, Lieut., Vancouver.

Ridge, Derrick, Lieut. (engineer), Hampstead, Montreal.

Ross, Charles Alexander, Lieut., Halifax.

Wright, George Bucknam, Lieut., Winnipeg.

Ratings

Adamson, Selwyn Arthur, Leading Coder, Port Credit, Ont.

Armstrong, Hugh, Leading Stoker, Glengrove P.O., Ont.

Armstrong, William Morrison, Mechanician, Kamloops, B.C.

Bailey, Morris Benjamin, O.S., Claresholm, Alta.

Badour, William Rufus, A.B., Kingston, Ont.

Barnhart, Gordon Franklin, Stoker, Fort Erie, Ont.

Barriault, Joseph Ernest, A.B., Wellington, P.E.I.

Barwis, William Donald, A.B., Nelson, B.C.

Bass, George Othoe, Leading Stoker, Cardinal, Ont.

Bedford, Andrew Lawrence, A.B., South Porcupine, Ont.

Bell, Joseph Griffiths, Coder, Sarnia, Ont.

Berrisford, Gordon Harold, O.S., Selkirk, Man.

Booth, Robert Gordon, O.S., Saskatoon.

Botham, Robert John, A.B., Plato, Sask.

Bottomley, Stanley Rhodes, A.B., Edmonton.

Bowser, Fred Douglas, Supply P.O., Saint John, N.B.

Boyle, Thomas, Sigmn., Toronto.

Brennan, James Edward Stewart, Engine Room Artificer, Kingston, N.S.

Brett, Walter Bretwalda, Engine Room Artificer, Regina.

Brockman, Stanley Bertram, O.S., Burnaby, B.C.

Bruce, William, L.S., Dundee, Scotland.

Burgess, Ira Urban, A.B., Southwest Port Mouton, N.S.

Butterfield, Thomas William James, E.R.A., Vancouver.

Bydwell, Leslie Wellington, E.R.A., Saint John, N.B.

Carignan, Joseph Jean Baptiste, Stoker, St. Angele De Laval, Que.

Craig, Frederick Thomas, Leading Stoker, Halifax.

Dean, Alfred Costello, Leading Stoker, Morton, Ont.

Deeks, William Richard, O.S., Vernon, B.C.

DeGeer, Carl Gerhard Ludvig, A.B., Saint John, N.B.

Demers, Honore Wilfrid Joseph, Electrical Artificer, Melrose Highlands, Mass.

Denneny, James Gordon, Telegraphist, Montreal.

Des Brisay, Gordon Montgomery, Ordinary Signalman, Vancouver.

Deschamp, Winslow Alfonzo, A.B., Shelburne, N.S.

Dinner, Lorne, Supply Assistant, Montreal.

Dowell, Charles Brannagan, O.S., Windsor, Ont.

Edmonds, John Celestine, Stoker P.O., St. John's, Nfld.

Evans, Llewellyn, A.B., Peterboro', Ont.

Evans, William Leo, Stoker, Point St. Charles, Que.

Forbes, Garry Ferguson, A.B., Port Arthur.

Fulton, Henry, Stoker, St. John's, Nfld.

Good, James Hamilton, Ordinary Telegraphist, Raith, Ont.

Gordon, Cyril James Albert, leading sick berth attendant, Regina.

Goreham, Leslie Mackenzie, A.B., Ottawa.

Goulet, Joseph Philippe Lionel, Steward, St. Evariste, Que.

Grandy, Charles Reginald, L.S., Grand Bank, Nfld.

Grant, Ronald Earl, A.B., Toronto.

Greggio, William Alexander, Ordinary Telegraphist, Humberstone, Ont.

Grenon, Albert Joseph, A.B., Somerset, Man.

Guay, Vincent Elwood, A.B., Corbyville, Ont.

Hann, Peter Martin Joseph, P.O., Steward, Halifax.

Hillier, George Abraham, A.B., Lamalin, Nfld.

Hodgson, Albert John, Stoker P.O., Blue Ridge, Alta.

Hutton, Jack, Leading Sigmn., Vancouver.

Kidson, Weldon Alexander, Leading Stoker, Victoria.

Lanrigan, Frederick, Stoker (1st class), Toronto.

Lane, Arthur Douglas, A.B., Windsor, Ont.

Lanktree, William, Leading Stoker, Port Arthur.

Lillyman, Raymond Frederick, Ordinary Telegraphist, Winnipeg.

Locke, Oscar James, Stoker, Ansonville, Ont.

Long, Ronald James, A.B., Windsor, Ont.

Lopuck, Antoni, Leading Stoker, East Transcona, Man.

Luchan, Edmund Victor, Stoker, Schumacher, Ont.

Marmon, Thomas, L.S., Verdun, Que.

Melnitsky, Theodore, Cook (seamen's), St. Lambert, Que.

Meloche, Joseph Orphee, Telegraphist, Montreal.

Mercham, Joseph Gerald, Electrical Artificer, Sturgeon Falls, Ont.

McKeown, John Edward, Leading Stoker, Toronto.

McKinney, Lawrence Albert, A.B., Montreal.

McKinnon, Alexander, Chief Stoker, Rosemount, Montreal.

McMaster, Alexander Peter, L.S., Toronto.

McNeney, James Robert, Yeoman of Signals, New Westminster, B.C.

McPherson, Clifford Albert, A.B., Tilbury, Ont.

Newhouse, John Stanley, Leading Telegraphist, Calgary.

Newstead, Kenneth Edwin, Cook (seamen's), Toronto.

O'Connor, Thomas Patrick, Stoker, P.O., Montreal.

Osborne, Robert James, Leading Stoker, Montreal.

Parker, Robert Borden, Stoker, P.O., Halifax.

Parnell, George Frederick, E.R.A. (4th class), Hoey, Sask.

Pasquantonio, Ralph Joseph, Leading Stoker, A1228, R.C.N.R., Mrs. Elisa Currcio (sister), Port Colborne, Ont.

Pastorek, John O.S., Hammond, B.C.

Payne, Sydney Rudolph, Stoker (1st class), Fort William.

Pidlaski, Nicholas, O.S., Winnipeg.

Pook, William Frederick, Assistant Cook (seamen's), Seattle, Wash.

Priske, Sydney Robert, P.O., Port Arthur.

Prosch, Joseph, Leading Stoker, West Hartlepool, Eng.

Pudney, William Sydney David, Sigmn., Toronto.

Reid, Edward Roddick, A.B., Brandon.

Richardson, Jasper James, Ordinary Coder, Brandon, Man.

Riecke, Harry Carl, P.O., Montreal.

Rigby, Robert Charles, Ordinary Telegrapher, Sarnia, Ont.

Rising, Jack Humphrey, Ordnance Artificer, Moncton, N.B.

Robertson, Theodore Maxwell, Stoker P.O., Halifax.

Roder, William Norman, Stoker (2nd class), Arkona, Ont.

Rowell, David Bayford, A.B., V8359, R.C.N.V.A., Miss Sulta Rowell (sister), 208 Maple avenue, Hamilton.

Russ, Frank Abseth, P.O., Telegraphist, Toronto.

Rosso, Anthony Francis, L.S., Montreal.

Savage, Francis Herbert, A.B., Ashton, Ont.

Scudamore, Frederick Herbert, Leading Stoker, Mimico, Ont.

Simard, Joseph Hidola Michel, A.B., Quebec.

Sims, Hartman William, L.S., London, Ont.

Smith, Joseph Walter, Leading Cook (seamen's), Emerald Junction, P.E.I.

Stephenson, Francis Woodrow, P.O., Montreal.

Stephenson, Carmen Ernest, L.S., Montreal.

Stevenson, William Lyle, O.S., Truro, N.S.

Strange, William Robert, Stoker (1st class), Edmonton.

Tagg, James, E.R.A., Scotland.

Terry, John Malcolm, A.B., Toronto.

Thicke, Charles Frederick, A.B., Wellington, B.C.

Tipe, Carl, Stoker (1st class), Toronto.

Turner, Alexander, A.B., Toronto.

Urquhart, Donald LeRoy, A.B., Taymouth, N.B.

VanSickle, Harold Allen, O.S., Swan River, Man.

Vey, John Wellington, A.B., Sainte John, N.B.

Walker, George Edward, A.B., Montreal.

Walsh, Patrick Douglas, Stoker, Jasmin, Sask.

Warner, Gordineer Wendell, Stoker (1st class), Phillipsville, Ont.

Watson, Horace William, C.P.O., Halifax.

Whitaker, Ronald, Steward, Attercliff, Sheffield, Yorkshire, Eng.

White, Robert Edwin, A.B., Calgary.

Winch, Wilmer James, Leading Stoker, Wiarton, Ont.

Wyckoff, Gordon Colquhoun, L.S., Calgary.

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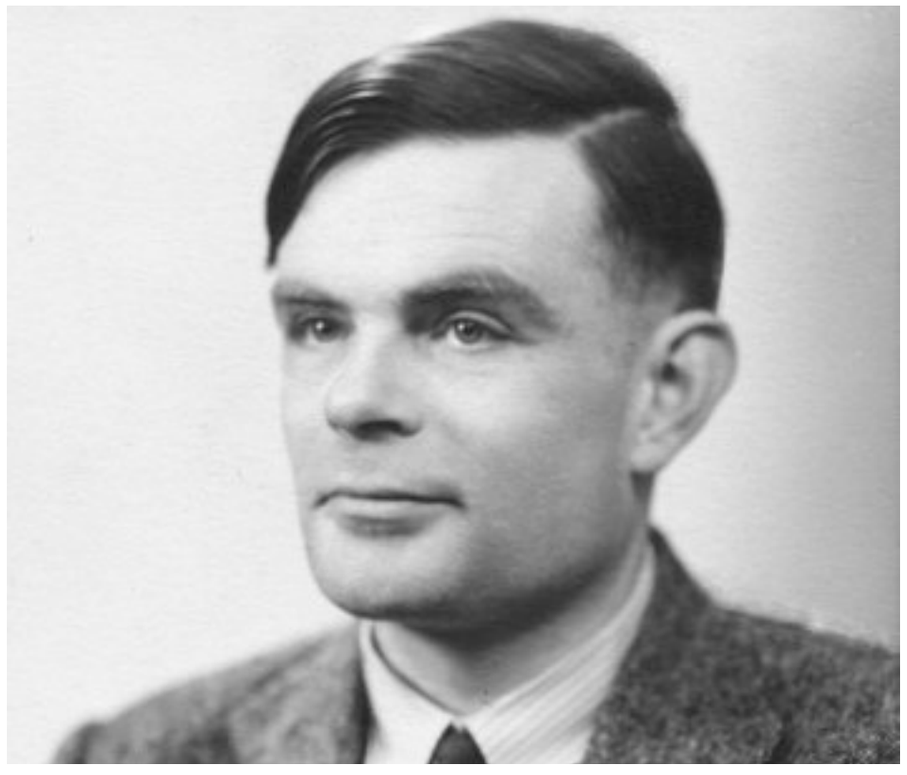
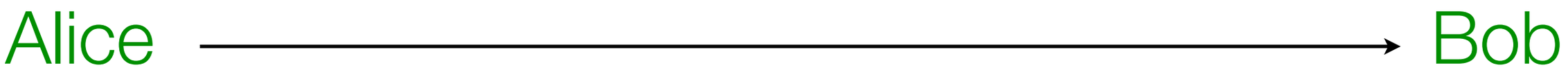
Mercham, Joseph Gerald, Electrical Artificer, Sturgeon Falls, Ont.

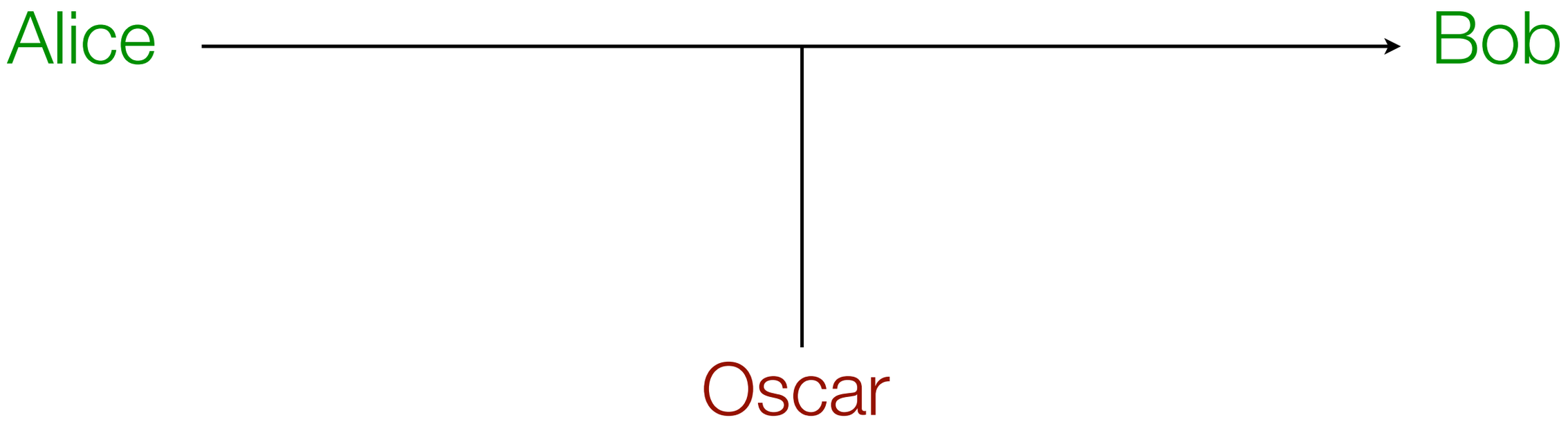


Enigma machine, Bletchley Park (centre for British decryption efforts in WW II), Alan Turing

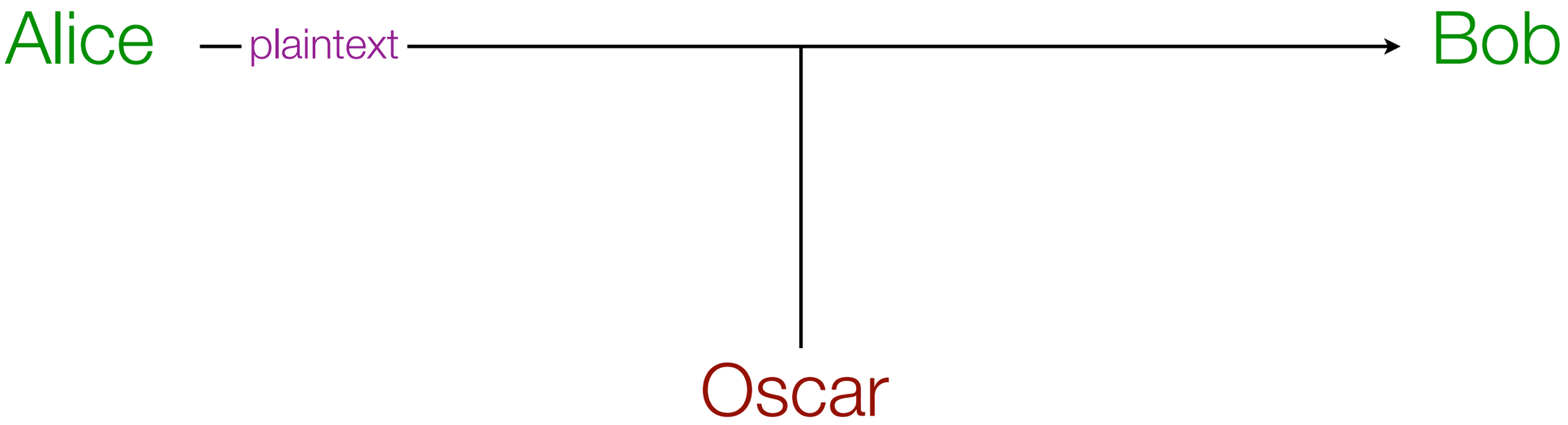
Cryptography



Cryptography



Cryptography



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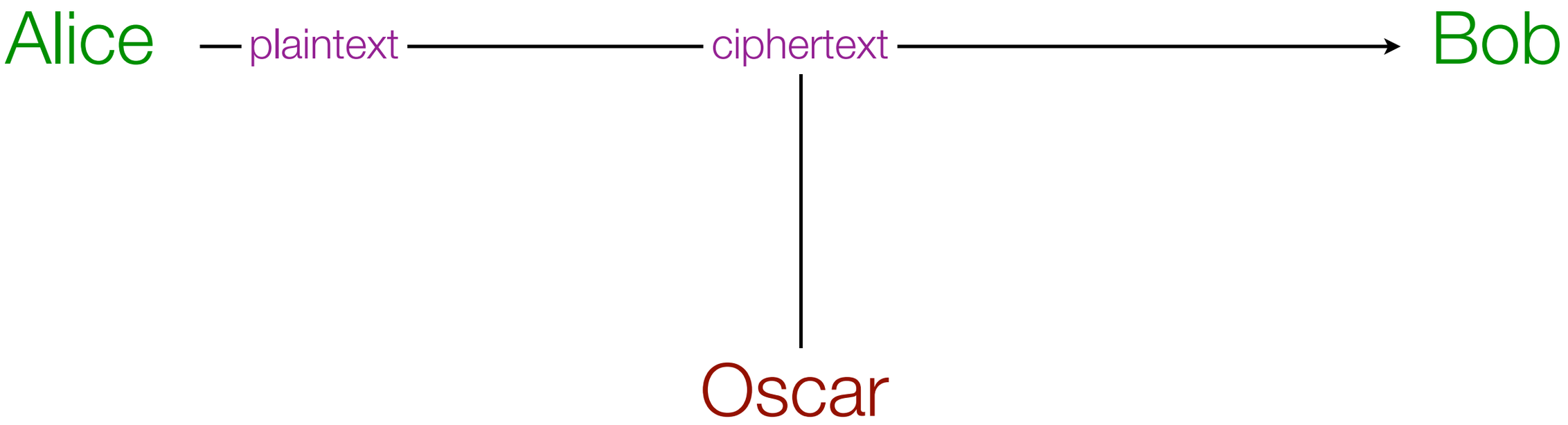
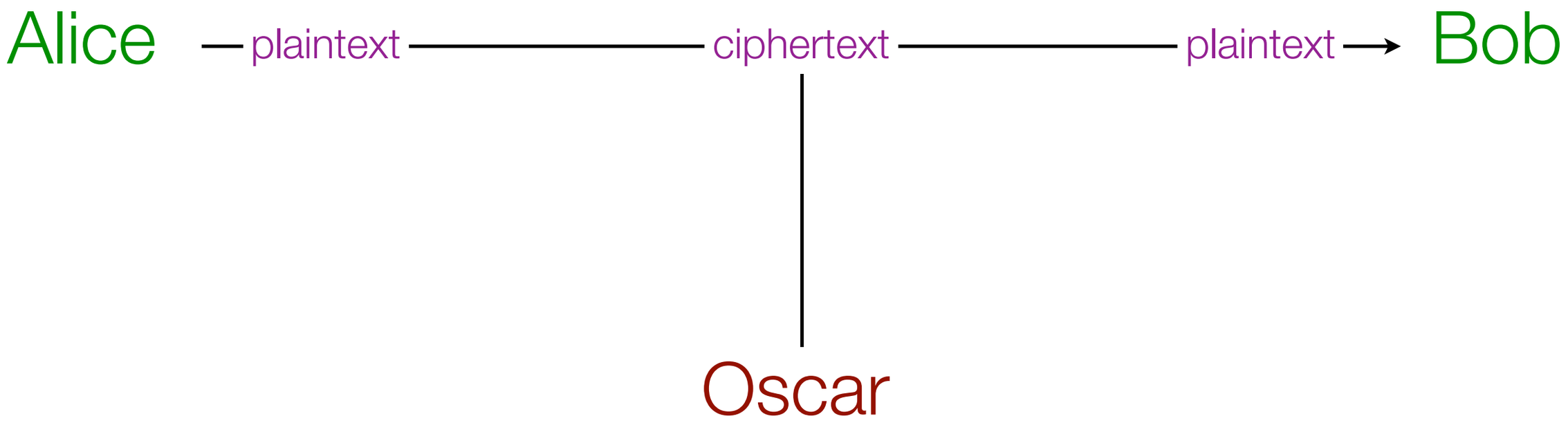
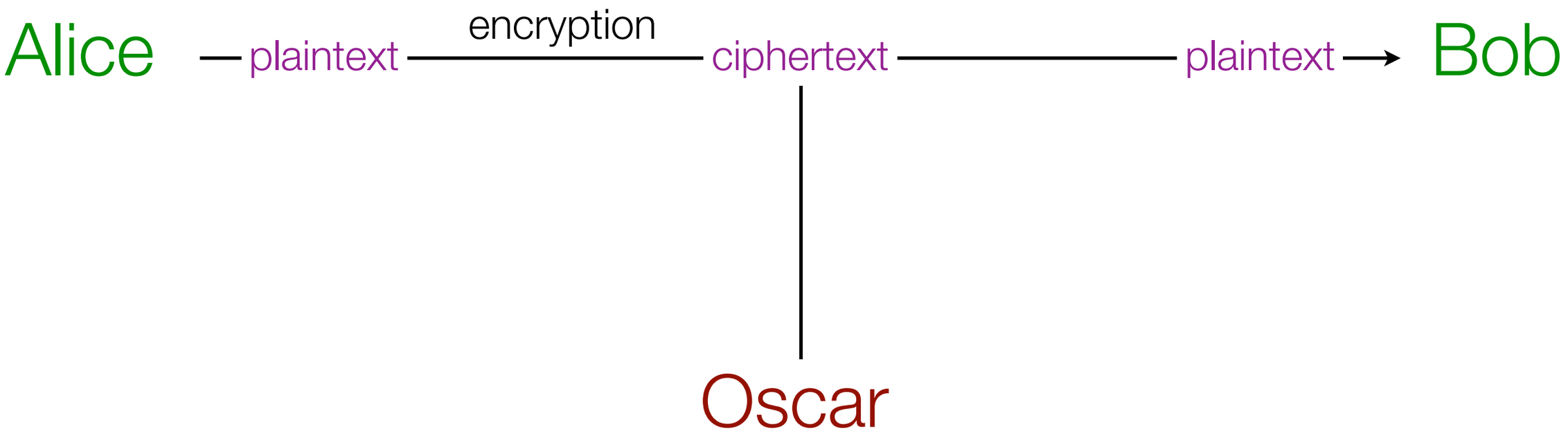


Cryptography



Cryptography



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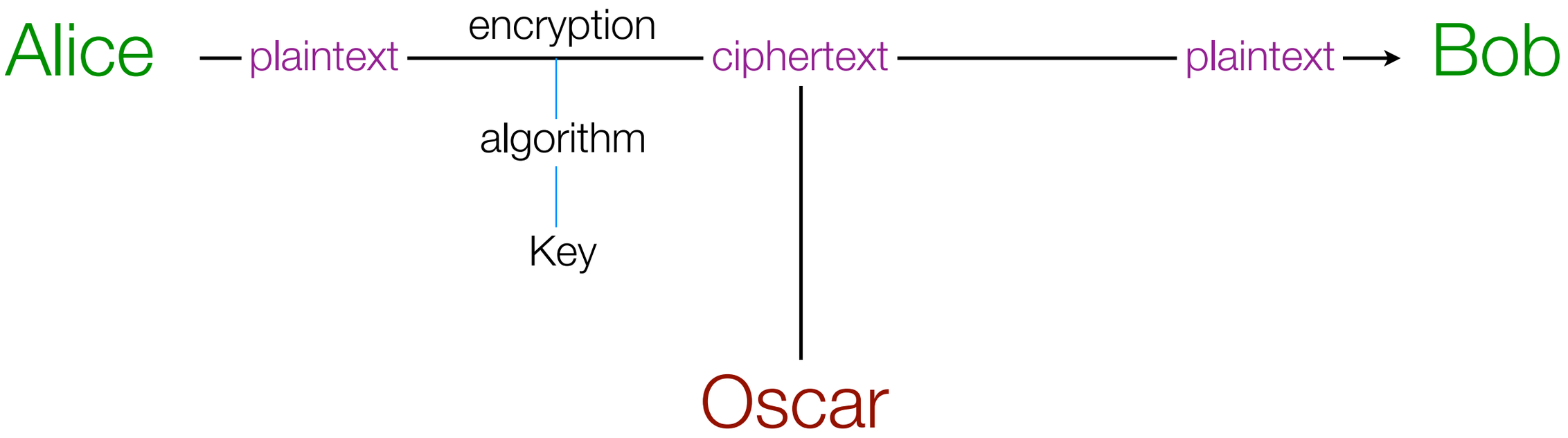
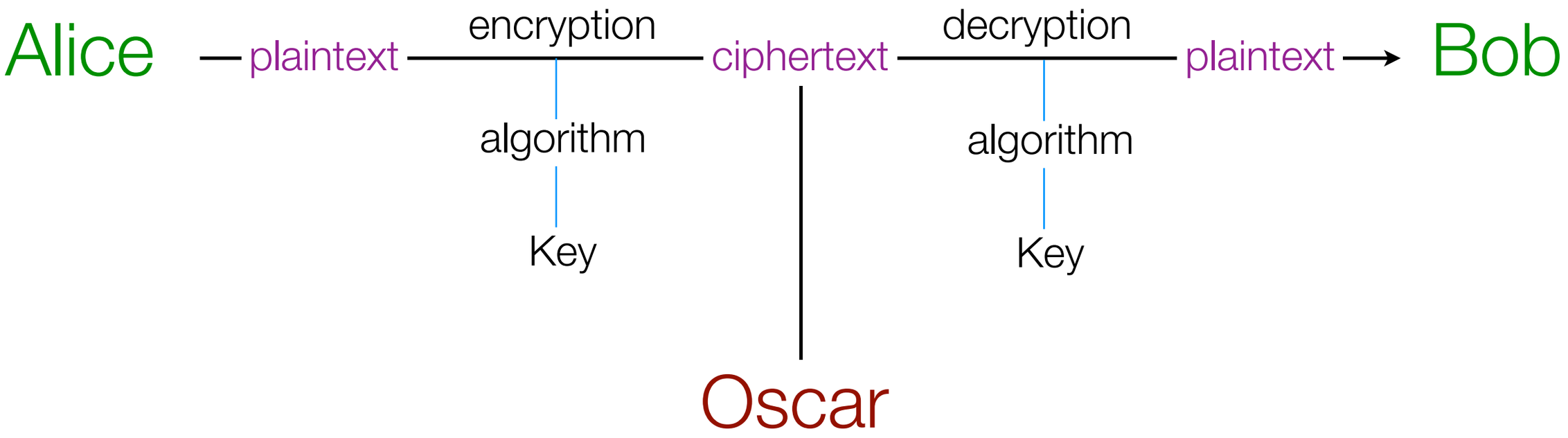


Cryptography



Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Q	M	W	N	B	E	R	V	T	C	Y	X	U	Z	I	L	O	K	P	H	A	G	S	F	J	D

Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Q	M	W	N	B	E	R	V	T	C	Y	X	U	Z	I	L	O	K	P	H	A	G	S	F	J	D

Plaintext

```
We shall fight on the beaches,
We shall fight on the landing grounds,
We shall fight in the fields and in the streets,
We shall fight in the hills;
We shall never surrender
```

Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
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Ciphertext

Substitution Cipher

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Q	M	W	N	B	E	R	V	T	C	Y	X	U	Z	I	L	O	K	P	H	A	G	S	F	J	D

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SB PVQXX ETRVH IZ HVB XQZNTZR RKIAZNP
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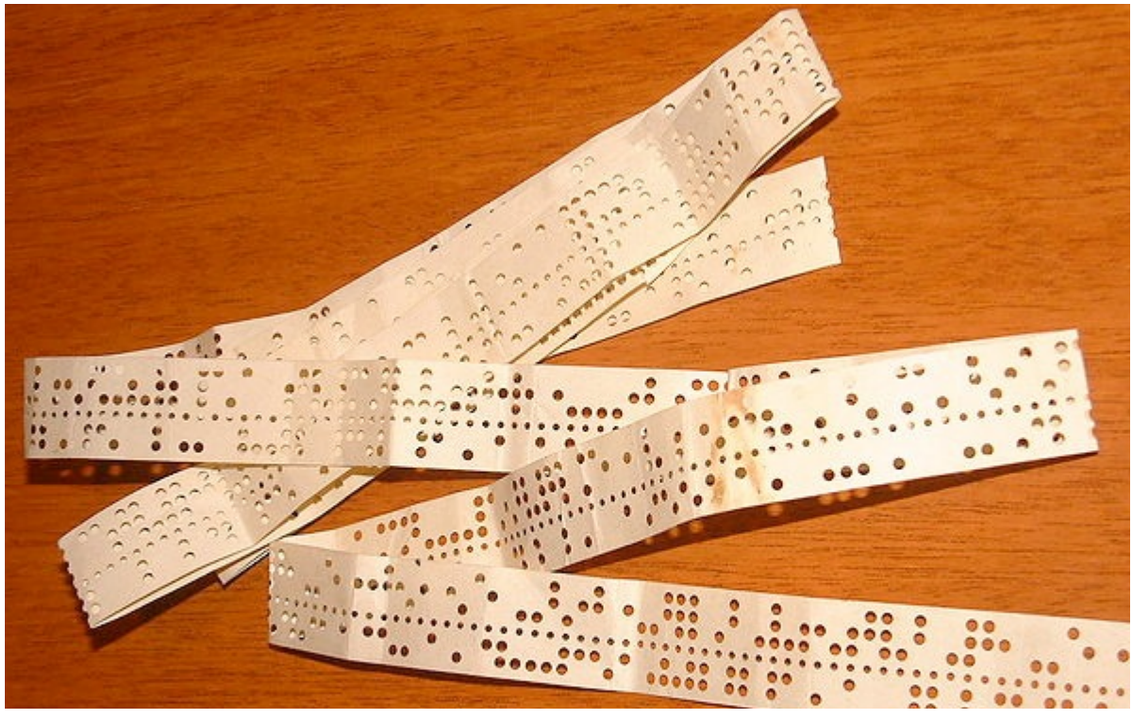
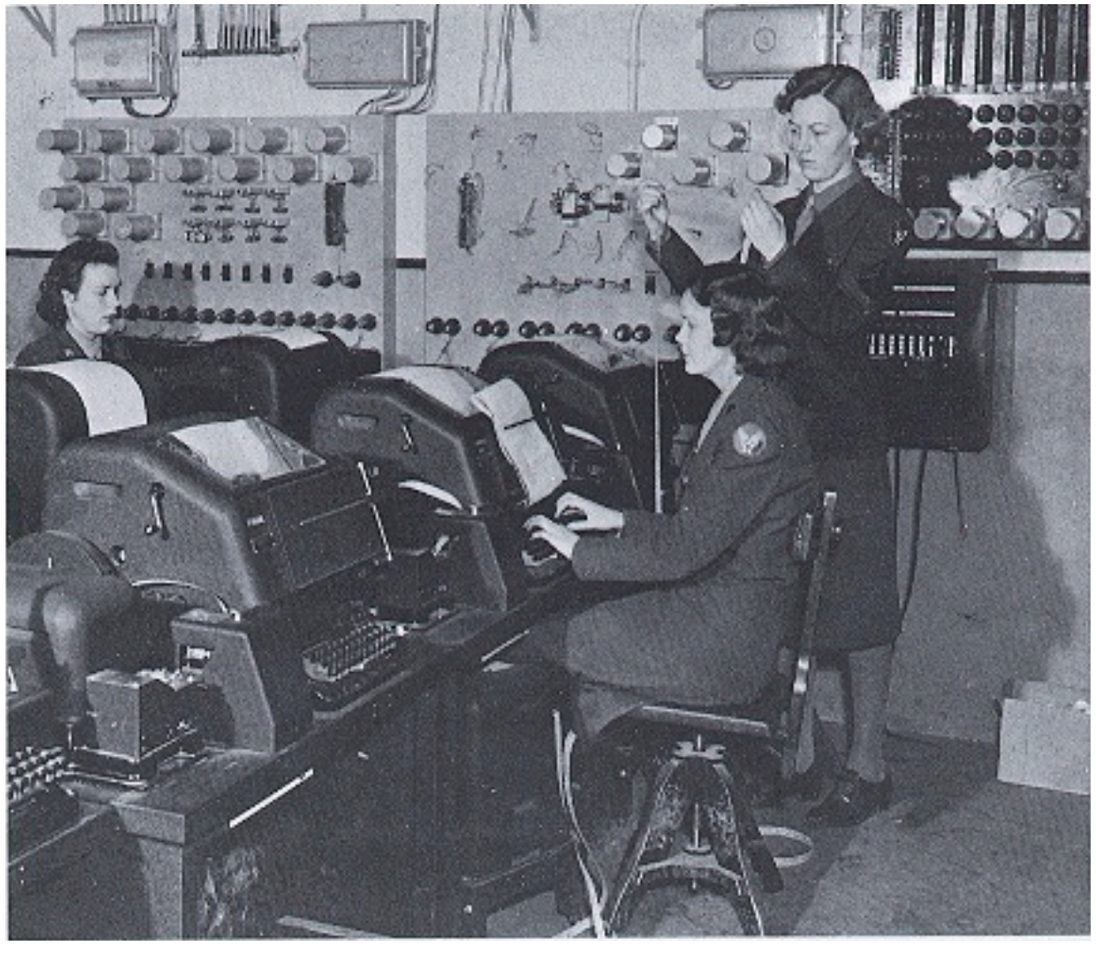
SB PVQXX ETRVH TZ HVB VTXXP
SB PVQXX ZBGBK PAKKBZNBK
```

Fish

- a new cryptographic system, more sophisticated than Enigma
- teletype





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- carried highest level of intelligence, encrypted communication between German High Command and Army Group headquarters in the field
- Tutte was assigned to Fish in October 1941

Tutte has problems

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- Based on radio intercepts that may introduce errors
- No computing capabilities other than pencil and paper

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- There is no wikipedia or electronic library
- He cannot talk about the problem with others outside his group

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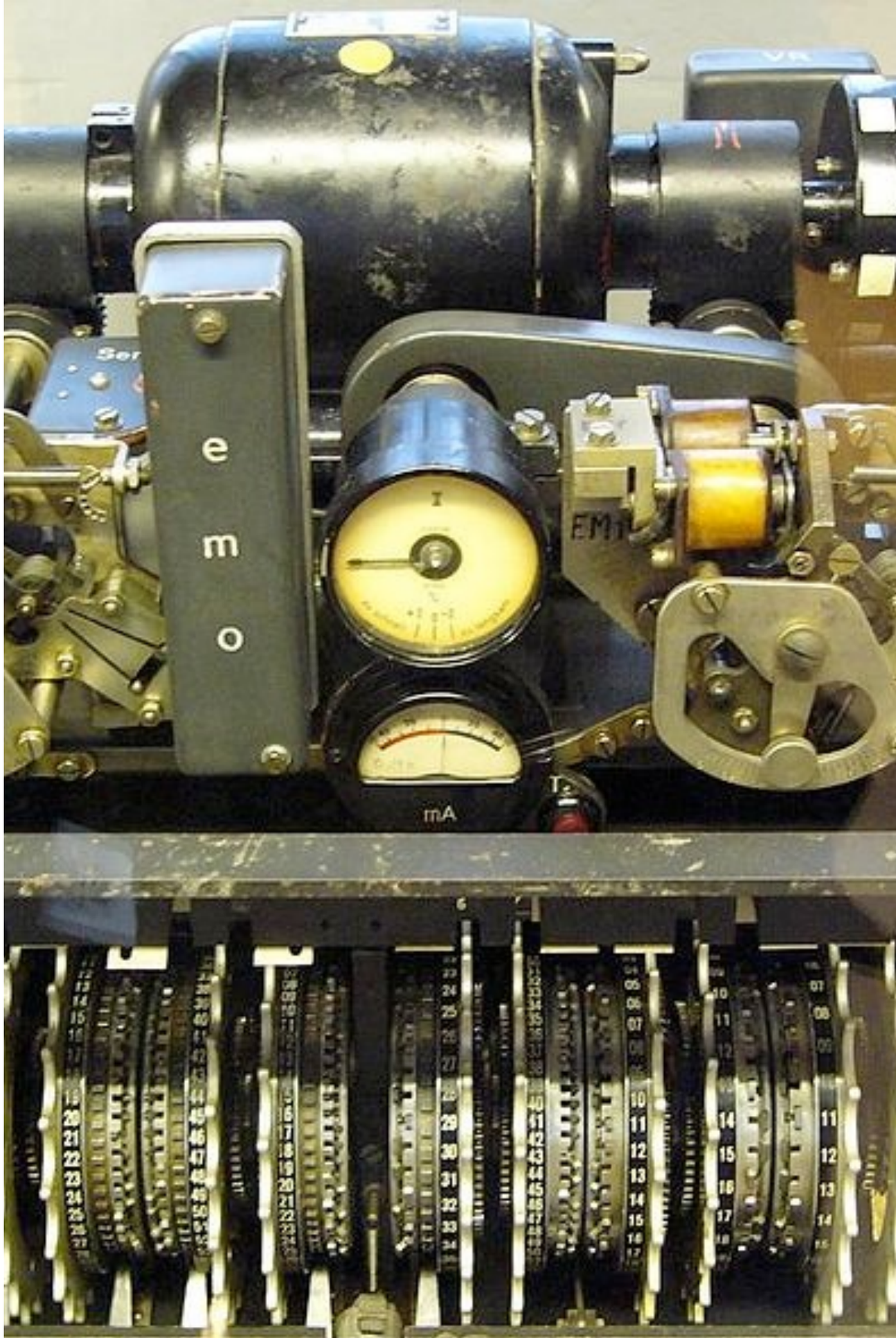
- Head of the Research Section, Major Gerry Morgan, says to Tutte "See what you can do with this."
- Using only this message and working by hand with the teletype codes (not the letters of the alphabet) Tutte was able to identify the internal structure and behaviour of the encryption machine

Lorenz Schlusselzusatz 40

- 12 wheels all with a different number of teeth
- some with irregular movements
- arranged in three groups
- using 5 bitstreams



How dramatic was this?

- “the greatest intellectual feat of the whole war”, Tony Sales quoted in “Colossal Adventures”, New Scientist, 10 May 1997
- “only six people outside the Lorenz codebreaking team ever understood the significance”, Tony Sales

Attacks on Current Traffic

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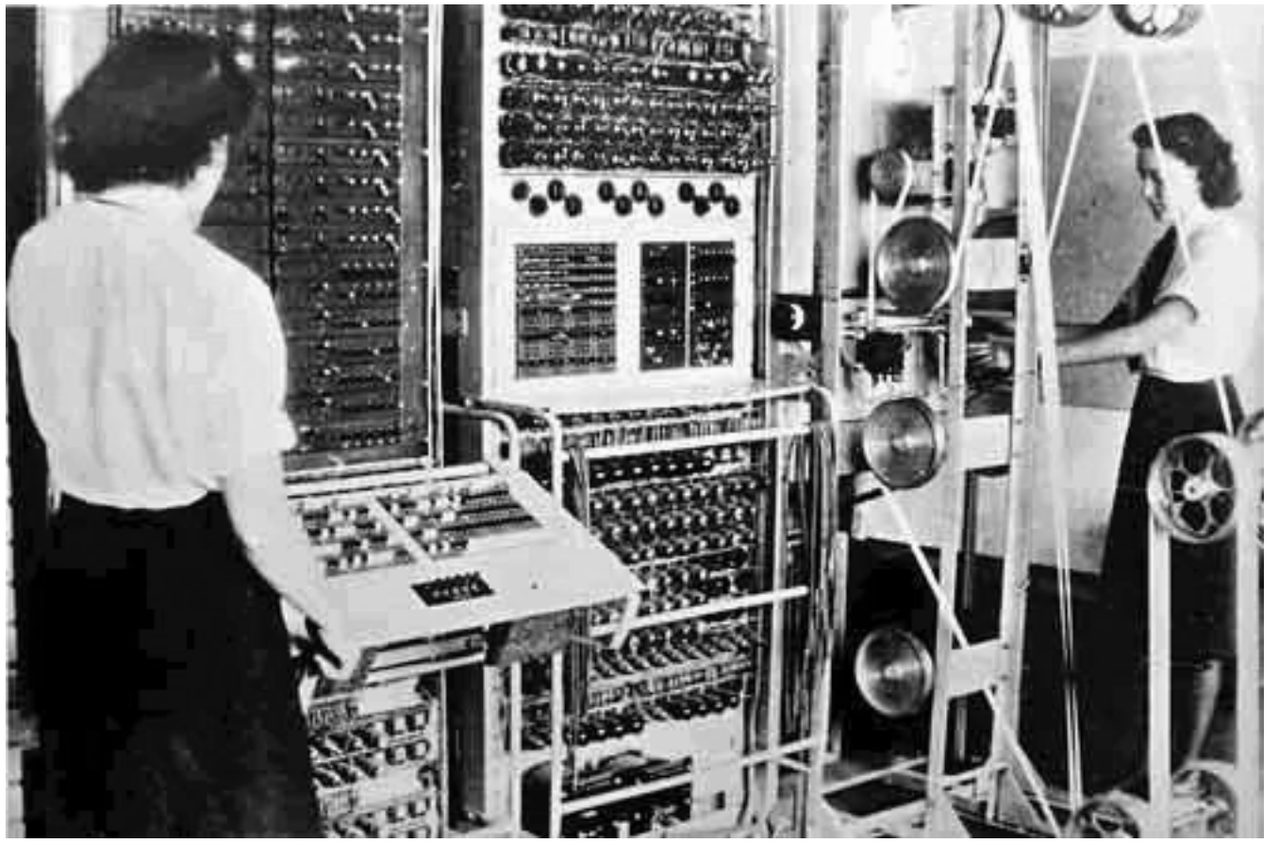
- Tutte devised the “statistical method” that would work against Fish but...
- What was needed was an analysis of each message that was thousands of times faster than computations by hand
- Could a machine be invented to do this?
- Tutte explained his ideas to Gerry Morgan and Max Newman

Colossus

- Prototype finished in Dec 1943
- Designed by Tommy Flowers of the Post Office
- Used an unprecedented number of vacuum tubes
- Its existence was not publicly acknowledged until decades after the war



Battle of Kursk, USSR, Jul and Aug 1943

The Armies

	Germany	USSR
Soldiers	781,000	1,910,000
Tanks	3,000	5,000
Artillery	10,000	25,000
Aircraft	2,100	2,800
Casualties	170,000 (54,000 d)	863,000 (180,000 d)
Tanks Lost	720	2,600
Aircraft Lost	680	2,000

Fish Decrypt

To OKH/OP. ABT. and to OKH/Foreign Armies East, from Army Group South IA/01, No. 411/43, signed von Weichs, General Feldmarschall, dated 25/4:-

Comprehensive appreciation of the enemy for "Zitadelle"

In the event of "Zitadelle", there are at present approximately 90 enemy formations west of the line Belgorod--Kursk--Maloarkhangelsk. The attack of the Army Group will encounter stubborn enemy resistance in a deeply echeloned and well developed main defence zone, (with numerous dug in tanks, strong artillery and local reserves) the main effort of the defence being in the key sector Belgorod--Tamarovka.

If the enemy throws in all strategic reserves on the Army Group front into the Kursk battle, the following may appear on the battle field:- On day 1 and day 2, 2 armoured divisions and 1 cavalry corps. On day 3, 2 mech and 4 armoured corps. On day 4, 1 armoured and 1 cavalry corps. On day 5, 3 mech corps. On day 6, 3 cavalry corps. On day 6 and/or day 7, 2 cavalry corps.

About the Importance of Codebreaking

- “[the war] was shortened by not less that two years and probably by four years”, Sir Harry Hinsley, official historian, British intelligence efforts in WW II
- An exhibit in 2003 on “Secret War” at the Imperial War Museum, in London, quoted British Prime Minister Winston Churchill telling King George VI, “It was thanks to [codebreaking] that we won the war.”

Attacking a Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Ciphertext (26! = 403,291,461,126,605,635,584,000,000 possible keys)

```
SB PVQXX ETRVH IZ HVB MBQWVBP
SB PVQXX ETRVH IZ HVB XQZNTZR RKIAZNP
SB PVQXX ETRVH TZ HVB ETBXNP QZN TZ HVB PHKBBHP
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Possible Plaintext

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SB PVQXX ETRVH IZ HVB MBQWVBP
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```

Attacking a Substitution Cipher

	E						T													H				
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Ciphertext

```
SB PVQXX ETRVH IZ HVB MBQWVBP
SB PVQXX ETRVH IZ HVB XQZNIZG GKIAZNP
SB PVQXX ETRVH TZ HVB ETBXNP QZN TZ HVB PHKBBHP
SB PVQXX ETRVH TZ HVB VTXXP
SB PVQXX ZBHBK PAKKBZNBK
```

Possible Plaintext

```
SE PHQXX ETRHT IZ THE MEQWHEP
SE PHQXX ETRHT IZ THE XQZNTZR RKIAZNP
SE PHQXX ETRHT TZ THE ETEXNP QZN TZ THE PTKEETP
SE PHQXX ETRHT TZ THE HTXXP
SE PHQXX ZEGEK PAKKEZNEK
```

Attacking a Substitution Cipher

	E						T						D			A					H				N
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Ciphertext

```
SB PVQXX ETRVH IZ HVB MBQWVBP
SB PVQXX ETRVH IZ HVB XQZNIZG GKIAZNP
SB PVQXX ETRVH TZ HVB ETBXNP QZN TZ HVB PHKBBHP
SB PVQXX ETRVH TZ HVB VTXXP
SB PVQXX ZBHBK PAKKBZNBK
```

Possible Plaintext

```
SE PHAXX ETRHT IN THE MEAWHEP
SE PHAXX ETRHT IN THE XANDTNR RKIANDP
SE PHAXX ETRHT TN THE ETEXDP AND TN THE PTKEETP
SE PHAXX ETRHT TN THE HTXXP
SE PHAXX NEGEK PAKKENDEK
```

A Better Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Q	M	W	N	B	E	R	V	T	C	Y	X	U	Z	I	L	O	K	P	H	A	G	S	F	J	D

Plaintext

```
We shall fight on the beaches,
We shall fight on the landing grounds,
We shall fight in the fields and in the streets,
We shall fight in the hills;
We shall never surrender
```

Ciphertext

A Better Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Q	M	W	N	B	E	R	V	T	C	Y	X	U	Z	I	L	O	K	P	H	A	G	S	F	J	D

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Ciphertext

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IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
ETRVH TZHVB VTXXP SBPVQ XXZBL BKPAK KBZNB
```
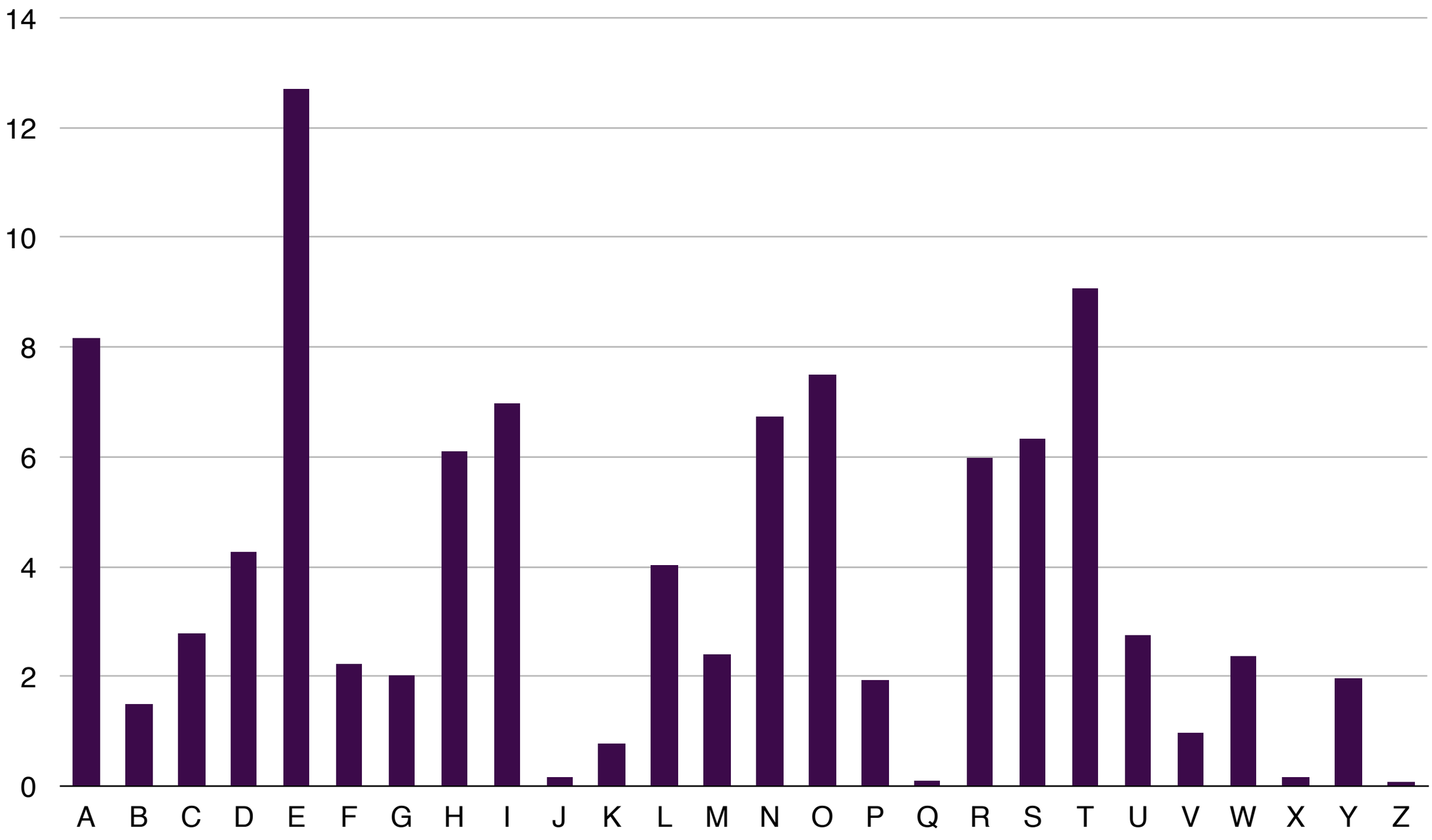

Relatively Frequency of the English Alphabet



Letters Grouped by Relative Frequency

- E
- T, A, O, I, N, S, H, R
- D, L
- C, U, M, W, F, G, Y, P, B
- V, K, J, X, Q, Z
- Might also consider relative frequencies of letters that start or end words

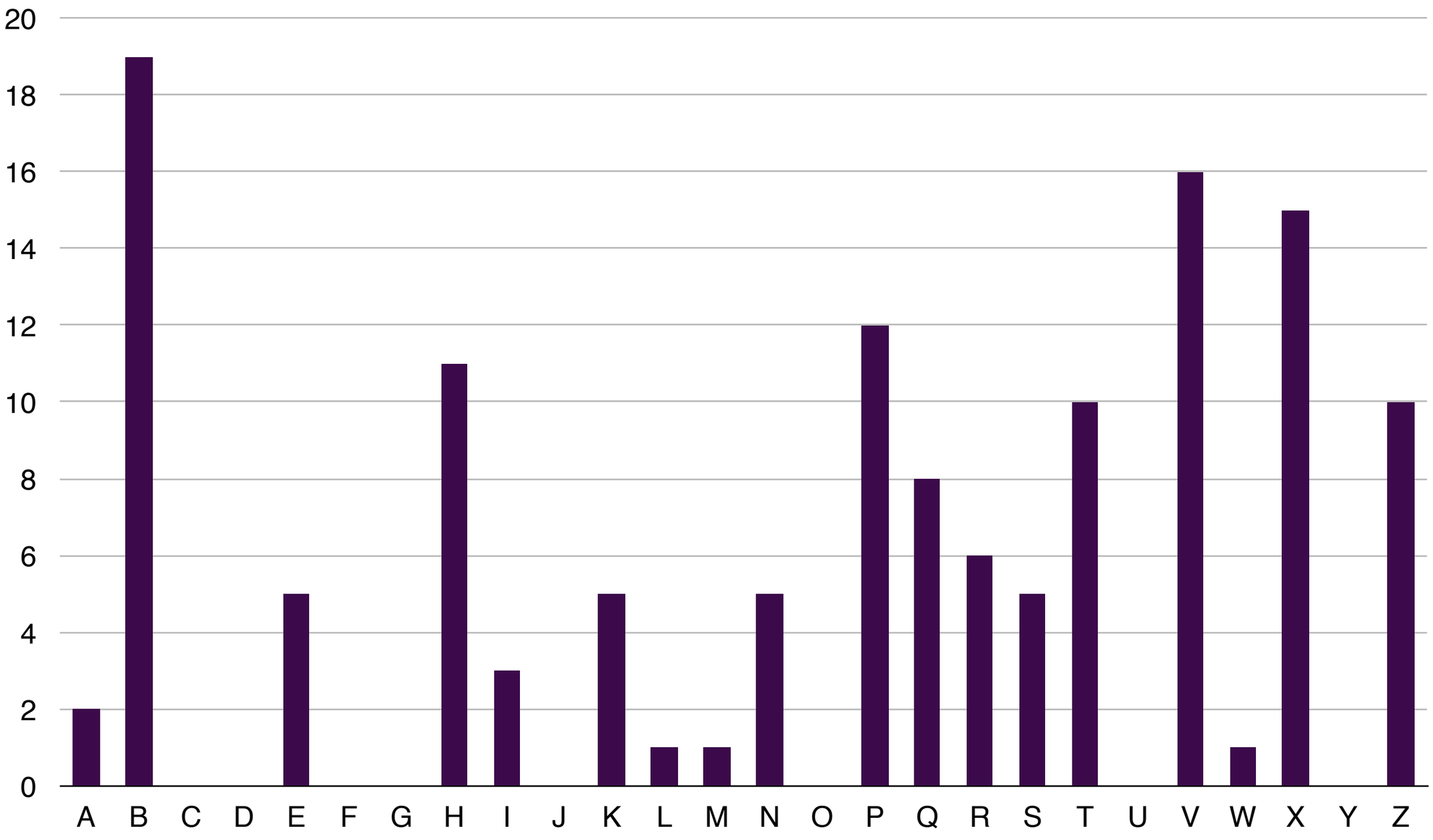
Common Pairs of Letters

- TH (3.015% of all pairs of letters)
- HE (3.004%)
- IN (1.872%)
- ER (1.860%)
- AN (1.419%)
- Might also consider pairs of identical letters: TT, LL, RR

Common Triples of Letters

- THE (2.032% of all triplets of letters)
- ING (0.747%)
- AND (0.667%)
- HER (0.547%)
- ERE (0.448%)

Relatively Frequency of the Ciphertext



Decrypting A Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10

Ciphertext

```
SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
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Plaintext

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
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	E																								

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```
SEPVQ XXETR VHIZH VEMEQ WVEPS EPVQX XETRV HIZHV EXQZN TXRRK
IAZNP SEPVQ XXETR VHTZH VEETB XNPQZ NTZHV EPHKE EHPSE PVQXX
ETRVH TZHVE VTXXP SEPVQ XXZEL EKPAK KEZNE
```

Decrypting A Substitution Cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10
	E																				T				

Ciphertext

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SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
ETRVH TZHVB VTXXP SBPVQ XXZBL BKPAK KBZNB
```

Plaintext

```
SEPTQ XXETR THIZH TEMEQ WTEPS EPTQX XETRT HIZHT EXQZN TXRRK
IAZNP SEPTQ XXETR THTZH TEETB XNPQZ NTZHT EPHKE EHPSE PTQXX
ETRTH TZHTE TTXXP SEPTQ XXZEL EKPAK KEZNE
```

“T” Didn’t Work, Look for “THE”

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10
	E																								

Ciphertext

```
SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
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```

Plaintext

```
SEPVQ XXETR VHIZH VEMEQ WVEPS EPVQX XETRV HIZHV EXQZN TXRRK
IAZNP SEPVQ XXETR VHTZH VEETB XNPQZ NTZHV EPHKE EHPSE PVQXX
ETRVH TZHVE VTXXP SEPVQ XXZEL EKPAK KEZNE
```

“T” Didn’t Work, Look for “THE”

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10
	E						T														H				

Ciphertext

```
SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
ETRVH TZHVB VTXXP SBPVQ XXZBL BKPAK KBZNB
```

Plaintext

```
SEPHQ XXETR HTIZT HEMEQ WHEPS EPHQX XETRH TIZTH EXQZN TXRRK
IAZNP SEPHQ XXETR HTTZT HEETB XNPQZ NTZTH EPTKE ETPSE PHQXX
ETRHT TZTHE HTXXP SEPHQ XXZEL EKPAK KEZNE
```

Realigning The Text

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10
	E						T														H				

Plaintext

```
SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
ETRVH TZHVB VTXXP SBPVQ XXZBL BKPAK KBZNB
```

Plaintext (Rearranged)

```
SEPHQXXETRHTIZ THE MEQWHEPSEPHQXXETRHTIZ THE XQZNTXRRKIAZNPSEPHQ
XXETRHTTZ THE ETBXNPQZNTZ THE PTKEETPSEPHQXXETRHTTZ THE
HTXXPSEPHQ XXZELEKPAKKEZNE
```

What About “XX”?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10
	E						T														H				

Plaintext

```
SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
ETRVH TZHVB VTXXP SBPVQ XXZBL BKPAK KBZNB
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SEPHQXXETRHTIZ THE MEQWHEPSEPHQXXETRHTIZ THE XQZNTXRRKIAZNPSEPHQ
XXETRHTTZ THE ETBXNPQZNTZ THE PTKEETPSEPHQXXETRHTTZ THE
HTXXPSEPHQ XXZELEKPAKKEZNE
```

What About “XX”?

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	19	0	0	5	0	0	11	3	0	5	1	1	5	0	12	8	6	5	10	0	16	1	15	0	10
	E						T														H		L		

Plaintext

```
SBPVQ XXETR VHIZH VBMBQ WVBPS BPVQX XETRV HIZHV BXQZN TXRRK
IAZNP SBPVQ XXETR VHTZH VBETB XNPQZ NTZHV BPHKB BHPSB PVQXX
ETRVH TZHVB VTXXP SBPVQ XXZBL BKPAK KBZNB
```

Plaintext (Rearranged)

```
SEPHQLLETRHTIZ THE MEQWHEPSEPHQLLETRHTIZ THE LQZNTLRRKIAZNPSEPHQ
LLETRHTTZ THE ETBLNPQZNTZ THE PTKEETPSEPHQLLETRHTTZ THE
HTLLPSEPHQ LLZELEKPAKKEZNE
```

Vigenere Ciphers

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

As Letters

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

As Letters

Plaintext	I	T		W	A	S		A		D	A	R	K		A	N	D		S	T	O
Key																					
Ciphertext																					

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

As Letters

Plaintext	I	T		W	A	S		A		D	A	R	K		A	N	D		S	T	O
Key																					
Ciphertext																					

As Numbers

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

As Letters

Plaintext	I	T		W	A	S		A		D	A	R	K		A	N	D		S	T	O
Key																					
Ciphertext																					

As Numbers

Plaintext	8	19		22	0	18		0		3	0	17	10		0	13	3		18	19	14
Key																					
Ciphertext																					

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

As Letters

Plaintext	I	T		W	A	S		A		D	A	R	K		A	N	D		S	T	O
Key	S	N		O	O	P		Y		S	N	O	O		P	Y	S		N	O	O
Ciphertext																					

As Numbers

Plaintext	8	19		22	0	18		0		3	0	17	10		0	13	3		18	19	14
Key	18	13		14	14	15		24		18	13	14	14		15	24	18		13	14	14
Ciphertext																					

Vigenere Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

As Letters

Plaintext	I	T		W	A	S		A		D	A	R	K		A	N	D		S	T	O
Key	S	N		O	O	P		Y		S	N	O	O		P	Y	S		N	O	O
Ciphertext	A	G		K	O	H		Y		V	N	F	Y		P	L	V		F	H	C

As Numbers

Plaintext	8	19		22	0	18		0		3	0	17	10		0	13	3		18	19	14
Key	18	13		14	14	15		24		18	13	14	14		15	24	18		13	14	14
Ciphertext	0	6		10	14	7		24		21	13	5	24		15	11	21		5	7	2

Vigenere Cipher

Plaintext

Key

Ciphertext

Vigenere Cipher

Plaintext

```
IT WAS A DARK AND STORMY NIGHT; THE RAIN FELL IN TORRENTS —
EXCEPT AT OCCASIONAL INTERVALS, WHEN IT WAS CHECKED BY A
VIOLENT GUST OF WIND WHICH SWEPT UP THE STREETS (FOR IT IS IN
LONDON THAT OUR SCENE LIES), RATTLING ALONG THE HOUSETOPS, AND
FIERCELY AGITATING THE SCANTY FLAME OF THE LAMPS THAT STRUGGLED
AGAINST THE DARKNESS.
```

Key

Ciphertext

Vigenere Cipher

Plaintext

```
IT WAS A DARK AND STORMY NIGHT; THE RAIN FELL IN TORRENTS —
EXCEPT AT OCCASIONAL INTERVALS, WHEN IT WAS CHECKED BY A
VIOLENT GUST OF WIND WHICH SWEPT UP THE STREETS (FOR IT IS IN
LONDON THAT OUR SCENE LIES), RATTLING ALONG THE HOUSETOPS, AND
FIERCELY AGITATING THE SCANTY FLAME OF THE LAMPS THAT STRUGGLED
AGAINST THE DARKNESS.
```

Key `SNOOPY`

Ciphertext

Vigenere Cipher

Plaintext

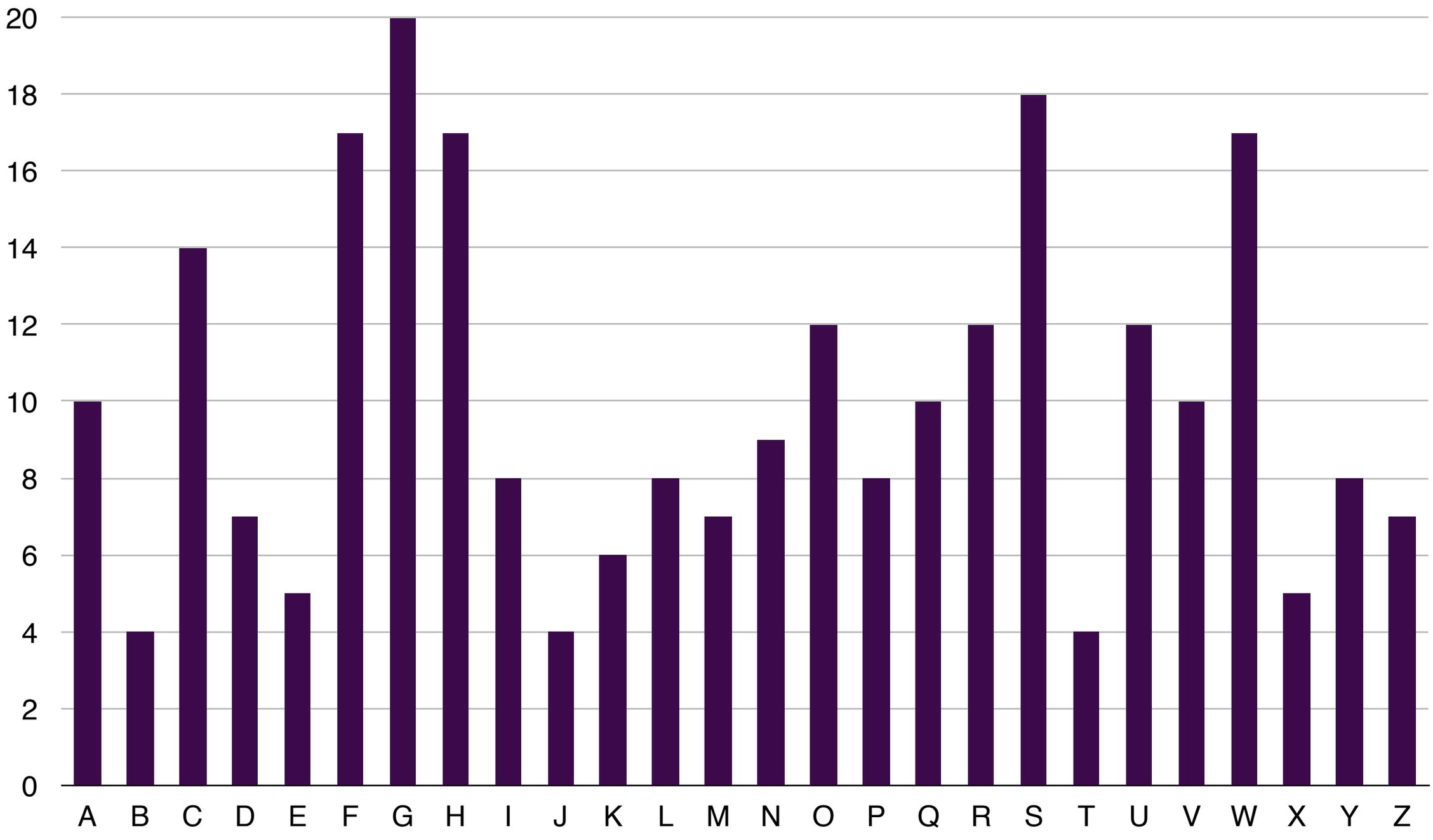
```
IT WAS A DARK AND STORMY NIGHT; THE RAIN FELL IN TORRENTS —
EXCEPT AT OCCASIONAL INTERVALS, WHEN IT WAS CHECKED BY A
VIOLENT GUST OF WIND WHICH SWEPT UP THE STREETS (FOR IT IS IN
LONDON THAT OUR SCENE LIES), RATTLING ALONG THE HOUSETOPS, AND
FIERCELY AGITATING THE SCANTY FLAME OF THE LAMPS THAT STRUGGLED
AGAINST THE DARKNESS.
```

Key `SNOOPY`

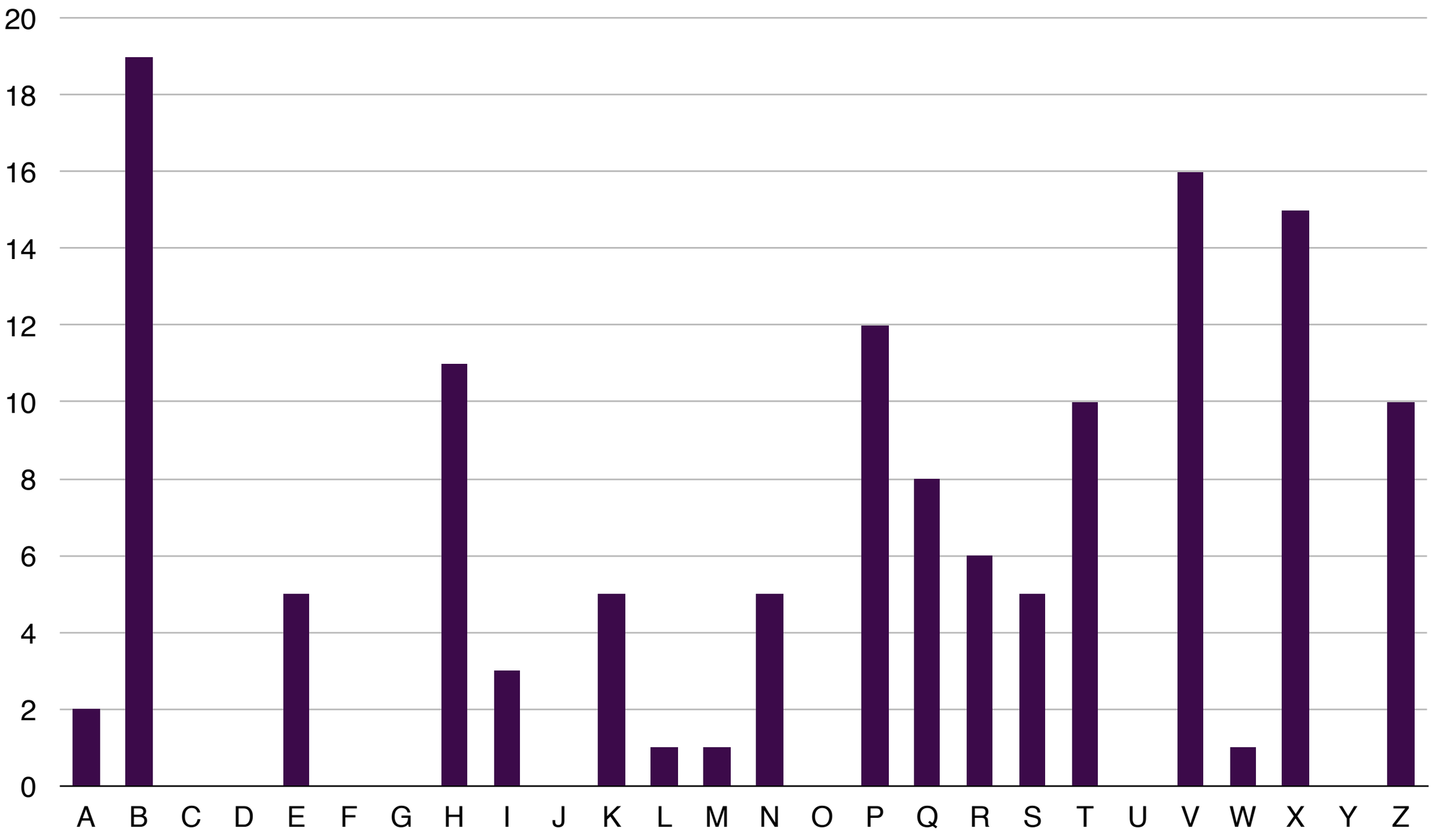
Ciphertext

```
AGKOH YVNFY PLVFH CGKQA WUWRL USFPG FSSZA GFGCF GCFGG SMAWC
HOIMU POGXM FNZWC RWEJO AQOUS BXRON GQWCU XSRQW SIWCA CFGUI
HRGSK WCBOU WQWQO RDHJN LUSGI PWRHG UMJVH WHGFY CBSMF GVOIM
MEGQT LWYWS HPSGH ZXLYN ZCCEL USVDS KRHCE QSART XCJPS ZNYYV
HOIGF THVTQ UNBHN DDNAS DDLUS ZPKHF HVPRK GFIVE DRROV YAAGH
IFWQO FZLWF G
```

Relatively Frequency of the Vigenere Ciphertext



Relatively Frequency of the Substitution Ciphertext



Need A Plan To Attack The Vigenere Cipher

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- If the key used in the Vigenere cipher is of length k, then the Vigenere cipher is simply k substitution (additive) ciphers.

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Need A Plan To Attack The Vigenere Cipher

- If the key used in the Vigenere cipher is of length k, then the Vigenere cipher is simply k substitution (additive) ciphers.
- We know how to attack substitution ciphers. In fact, each of these k ciphers is the simplest kind of substitution cipher, just translate all of the letters the same amount.
- If we could determine the length of the key, k, then we could divide the ciphertext into k streams each of which could be examined. The letters in stream 1 would be all the letters whose position is 1 mod k, that is, the letters in position 1, k+1, 2k+1, and so on. The letters in stream i would be all the letters whose position is i mod k.

Dividing the Cipher Text

Key Length 6

Ciphertext

```
AGKOH YVNFY PLVFH CGKQA WUWRL USFPG FSSZA GFGCF GCFGG SMAWC
HOIMU POGXM FNZWC RWEJO AQOUS BXRON GQWCU XSRQW SIWCA CFGUI
HRGSK WCBOU WQWQO RDHJN LUSGI PWRHG UMJVH WHGFY CBSMF GVOIM
MEGQT LWYWS HPSGH ZXLYN ZCCEL USVDS KRHCE QSART XCJPS ZNYYV
HOIGF THVTQ UNBHN DDNAS DDLUS ZPKHF HVPRK GFIVE DRROV YAAGH
IFWQO FZLWF G
```

```
AVVQLF... translated by S
GNFAUS... translated by N
KFHWSS... translated by O
OYCUFZ... translated by O
HPGWPA... translated by P
YLKRGG... translated by Y
```

How Do We Find The Key Length k?

- Consider pairs of letters.
- Suppose we have a random sequence of letters from the English alphabet and we choose the i-th letter. Since there are 26 possibilities and they are all equally likely, the probability of the choice being any particular letter is 1/26.
- Now suppose that there are two random sequences of letters and we examine the ordered pair of characters from the i-th position in the sequence. Since the two sequences are independent, the probability of choosing any particular ordered pair is (1/26)^2. The probability of choosing an *identical* pair of letters is

$$26 \times \left(\frac{1}{26}\right)^2 = \frac{1}{26} \approx 0.038$$

Random Passages Instead of Random Sequences

- If we replace the two random sequence of letters by two randomly chosen passages of English text, the probabilities change. Instead of being uniformly 1/26 they are the probabilities that appear in the table of relative frequencies. If p_λ is the probability that the letter λ appears in a given position in a string, then the probability that λ appears in the same position in two strings is

$$\sum_{\lambda=A}^{Z} (p_\lambda)^2 \approx 0.065$$

Measuring “Flatness”

- If the histogram of relative frequencies for letters in a ciphertext was absolutely flat, then each letter would occur with probability 1/26 and there would be relatively little information statistical information available for the cryptanalyst. If the histogram is not flat, we need some way to measure how much it varies from the flat histogram to help us determine the relative frequency of the letters. As a first guess we could try

$$\sum_{\lambda=A}^{Z}\left(p_\lambda-\frac{1}{26}\right)$$

- But that fails since

$$\sum_{\lambda=A}^{Z}\left(p_\lambda-\frac{1}{26}\right)=\sum_{\lambda=A}^{Z}p_\lambda-\sum_{\lambda=A}^{Z}\frac{1}{26}=1-26\times\frac{1}{26}=0$$

Measuring “Flatness” - Usefully

- The problem with the previous guess is that positive differences cancel negative differences. We can accentuate the differences by squaring the terms.

$$\sum_{\lambda=A}^{Z} \left(p_\lambda - \frac{1}{26} \right)^2$$

$$
\begin{aligned}
\sum_{\lambda=A}^{Z}\left(p_\lambda-\frac{1}{26}\right)^2 &= \sum_{\lambda=A}^{Z}\left(p_\lambda\right)^2-2\sum_{\lambda=A}^{Z}\frac{1}{26}p_\lambda+\sum_{\lambda=A}^{Z}\left(\frac{1}{26}\right)^2 \\
&= \sum_{\lambda=A}^{Z}\left(p_\lambda\right)^2-\frac{2}{26}\sum_{\lambda=A}^{Z}p_\lambda+\left(\frac{1}{26}\right)^2\sum_{\lambda=A}^{Z}1 \\
&= \sum_{\lambda=A}^{Z}\left(p_\lambda\right)^2-\frac{2}{26}\times 1+26\times\left(\frac{1}{26}\right)^2 \\
&= \sum_{\lambda=A}^{Z}\left(p_\lambda\right)^2-\frac{1}{26} \\
&= \sum_{\lambda=A}^{Z}\left(p_\lambda\right)^2-0.038 \\
&\approx 0.065-0.038 \\
&= 0.027
\end{aligned}
$$

Another Way To Count

- Suppose we have just one ciphertext. As before, let p_λ be the probability that the letter λ appears in a given position in the ciphertext. If we choose a second position in the ciphertext independent of the first choice, the probability of λ appearing again is p^2_λ.

- Thus the probability that two positions, randomly selected, will contain the same letter λ is

$$\sum_{\lambda=A}^{Z} (p_\lambda)^2$$

Estimating $\sum_{\lambda=A}^{Z} (p_\lambda)^2$

- Let f_λ be the frequency, not the probability, that the letter λ occurs in a particular cipher text. The number of unordered pairs containing two λs is $\binom{f_\lambda}{2}$
- Thus, the total number of unordered pairs containing two λs is

$$\sum_{\lambda=A}^{Z} \binom{f_\lambda}{2}$$

Index of Coincidence

- If the ciphertext has n characters then the the total number of unordered pairs is n choose 2 and so the likelihood of two randomly chosen letters being the same is

$$I_C = \frac{\sum_{\lambda=A}^{Z} \binom{f_\lambda}{2}}{\binom{n}{2}} = \frac{\sum_{\lambda=A}^{Z} f_\lambda \left(f_\lambda - 1\right)}{n\left(n-1\right)}$$

- I_C is the **index of coincidence** and is an approximation of

$$\sum_{\lambda=A}^{Z} \left(p_\lambda\right)^2$$

Estimating Key Length

- Suppose we possess a ciphertext of length n created by a Vigenere cipher with a key of length k all of whose characters are different. Then we can write our ciphertext in k rows such that letters in the same row have been enciphered using the same additive cipher but any two letters from different rows come from different additive ciphers. We then have k rows with approximately n/k letters in each row.

- If we pick two positions in any one row then the probability of getting identical letters is 0.065 since they originate from the same substitution cipher.

- If we pick two positions in different rows, then the probability of getting identical letters is 0.038 since they are randomly correlated.

Count The Number of Unordered Pairs of Positions

- Let us count the number of unordered pairs of positions in the ciphertext in two ways.
- Since there are n positions in the ciphertext, there are n choose 2 unordered pairs of positions.
- Alternatively, choose any position. We can do this in n ways. How do we choose the second position? If the second position is in the same row as the first position, it can be chosen in any of the remaining n/k - 1 positions. Thus the number of unordered pairs from one row is

$$\frac{1}{2}n\left(\frac{n}{k}-1\right)$$

- If the second position is in a different row from the first position, it can be chosen in any of the remaining n - n/k positions. Thus the number of unordered pairs from other rows is

$$\frac{1}{2}n\left(n-\frac{n}{k}\right)$$

Count The Number of Identical Pairs

- Now we count the number of unordered pairs of identical letters in two ways.
- First, we take the product of all unordered positions and the probability that a given unordered position contains identical letters.

$$\frac{1}{2}n(n-1)I_C$$

- Second, we add the probability that pairs of positions in the same rows generate identical letters and pairs in different positions generate identical letters.

$$\frac{1}{2}n\left(\frac{n}{k}-1\right)\times 0.065+\frac{1}{2}n\left(n-\frac{n}{k}\right)\times 0.038$$

- The two are approximately, not exactly, equal so

$$\frac{1}{2}n(n-1)I_C \approx \frac{1}{2}n\left(\frac{n}{k}-1\right)\times 0.065+\frac{1}{2}n\left(n-\frac{n}{k}\right)\times 0.038$$

Estimating Key Length

- Rearranging gives

$$k \approx \frac{0.027n}{I_C(n-1) - 0.038n + 0.065}$$

Attacking a Vigenere Cipher

- Approximate the key length k.
- Divide the ciphertext into k separate additive ciphers where the character in position i of the cipher text belongs to the (i mod k)-th additive cipher.
- Use the previously discussed relative frequency method to attack each additive cipher.
- Repeat as necessary by modifying your estimate of k.

More about William T Tutte

- http://www.math.uwaterloo.ca/CandO_Dept/William_Tutte/Canmath.shtml, by Prof. Dan Younger
- http://de.uwaterloo.ca/demo/math680/modules/Module_01/ history, interviews and accessible examples of the types of problems that Tutte worked on

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