Shortest Euclidean paths in the plane

Summary from last class

3 problem versions:

Given s, t

Given s, query t

Query s, t

shortest paths in a polygon

funnel algorithm for fixed s,t. $O(n)$

extension to query t

shortest path map + planar point location

Preprocessing = $O(n)$, Query time = $O(\log n + k)$, k = output size

shortest path in a polygonal domain

for fixed s,t compute visibility graph and use Dijkstra's algorithm. — make s, t point holes

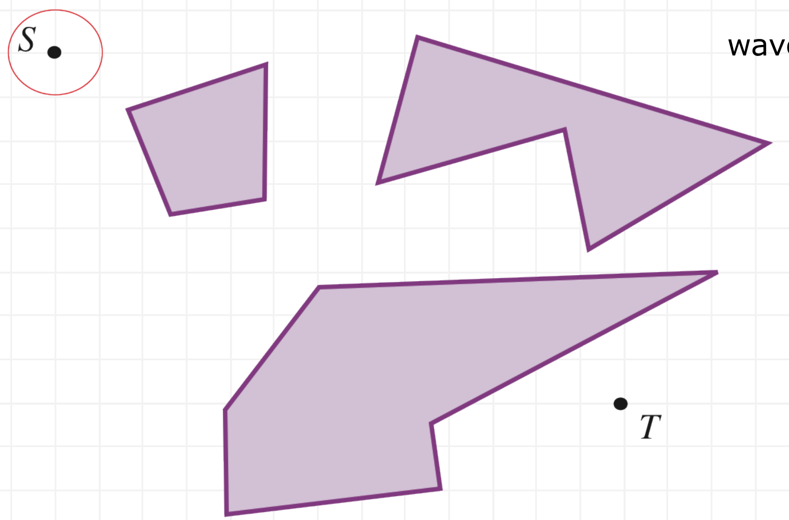
$O(n \log n + m)$, m = # edges in visibility graph, $O(n^2)$

today: for query t, continuous Dijkstra, P= $O(n \log n)$, Q= $O(\log n + k)$

Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

Given a polygonal domain, two points S, T, find the shortest path from S to T

Continuous Dijkstra approach

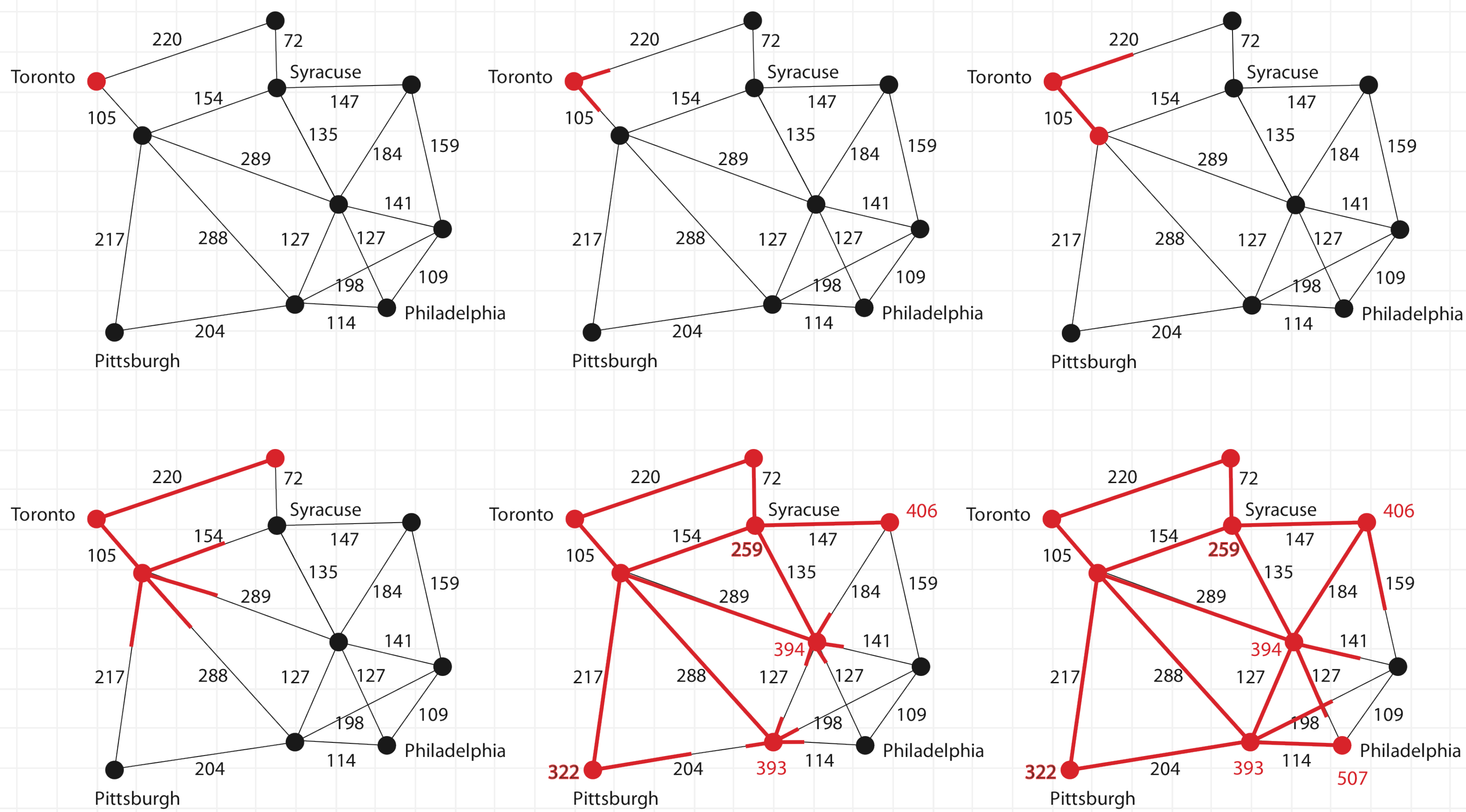


wavefront expands from S

Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

Given a polygonal domain, two points S, T, find the shortest path from S to T

geometric visualization of usual Dijkstra algorithm — imagine paint flowing along edges



Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

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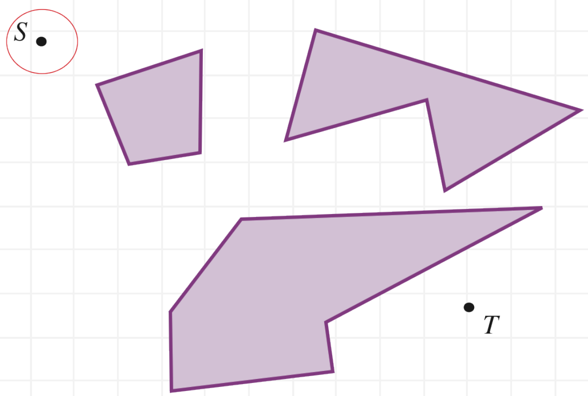

wavefront expands from S

Shortest paths among obstacles in the **plane**
JSB Mitchell - International Journal of Computational Geometry & …, 1996 - World Scientific
We give a subquadratic (O (n3/2+ϵ) time and O (n) space) algorithm for computing Euclidean **shortest paths** in the **plane** in the presence of polygonal **obstacles**; previous time bounds were at least quadratic in n, in the worst case. The method avoids use of visibility **...**
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$P = O(n^{3/2+\varepsilon})$

From: http://scholar.google.ca/scholar?hl=en&q=Shortest+Paths+Among+Obstacles+in+the+Plane&btnG=&as_sdt=1%2C5&as_sdtp=

An **optimal algorithm** for **Euclidean shortest paths** in the **plane**
J Hershberger, S Suri - SIAM Journal on Computing, 1999 - SIAM
We propose an **optimal**-time **algorithm** for a classical problem in **plane** computational geometry: computing a **shortest path** between two points in the presence of polygonal obstacles. Our **algorithm** runs in worst-case time O (n log n) and requires O (n log n) space **...**
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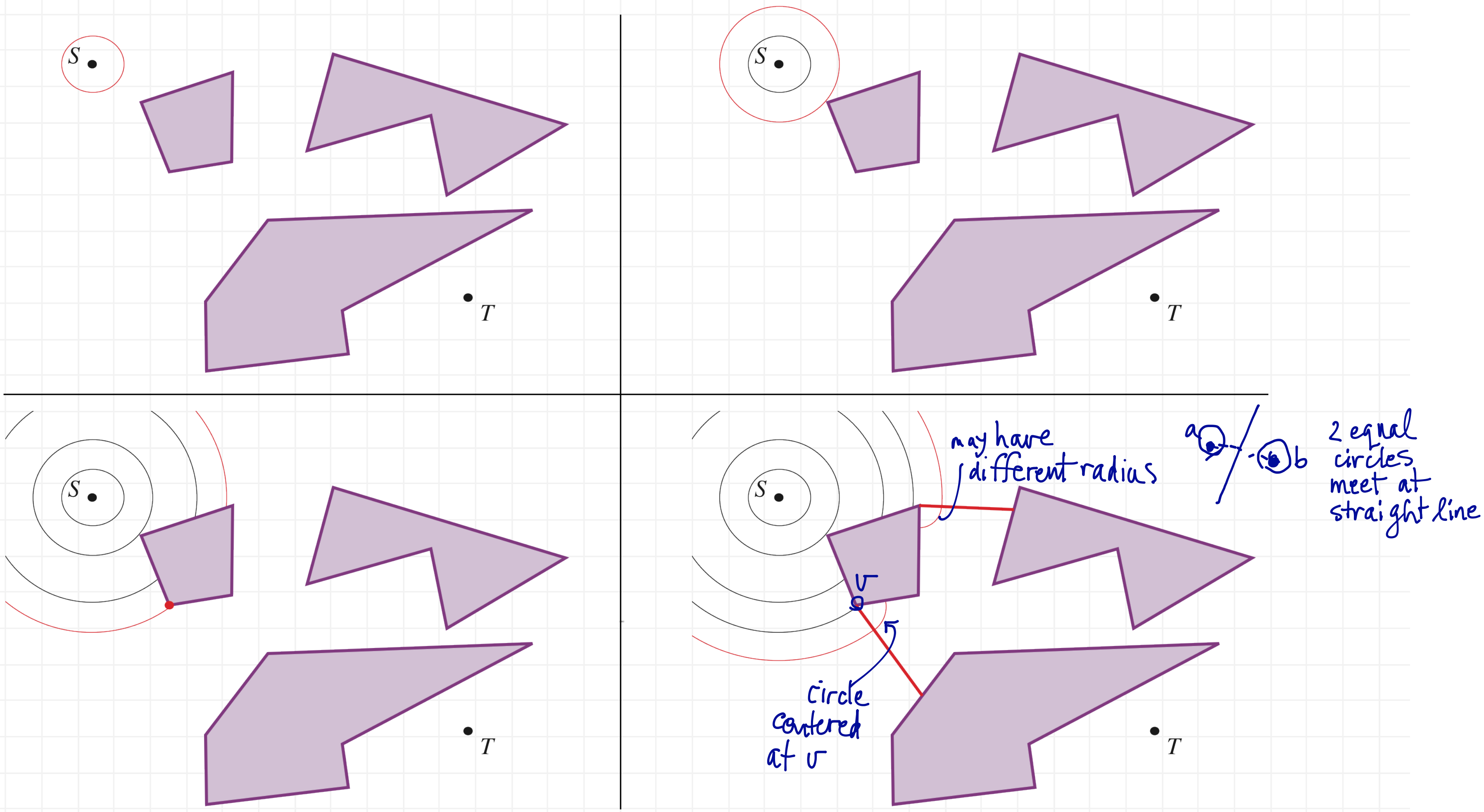
$P = O(n \log n)$

From: http://scholar.google.ca/scholar?q=AN+OPTIMAL+ALGORITHM+FOR+EUCLIDEAN+SHORTEST+PATHS+IN+THE+PLANE&btnG=&hl=en&as_sdt=0%2C5

Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

Given a polygonal domain, two points S, T, find the shortest path from S to T

Continuous Dijkstra approach wavefront expands from S



Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

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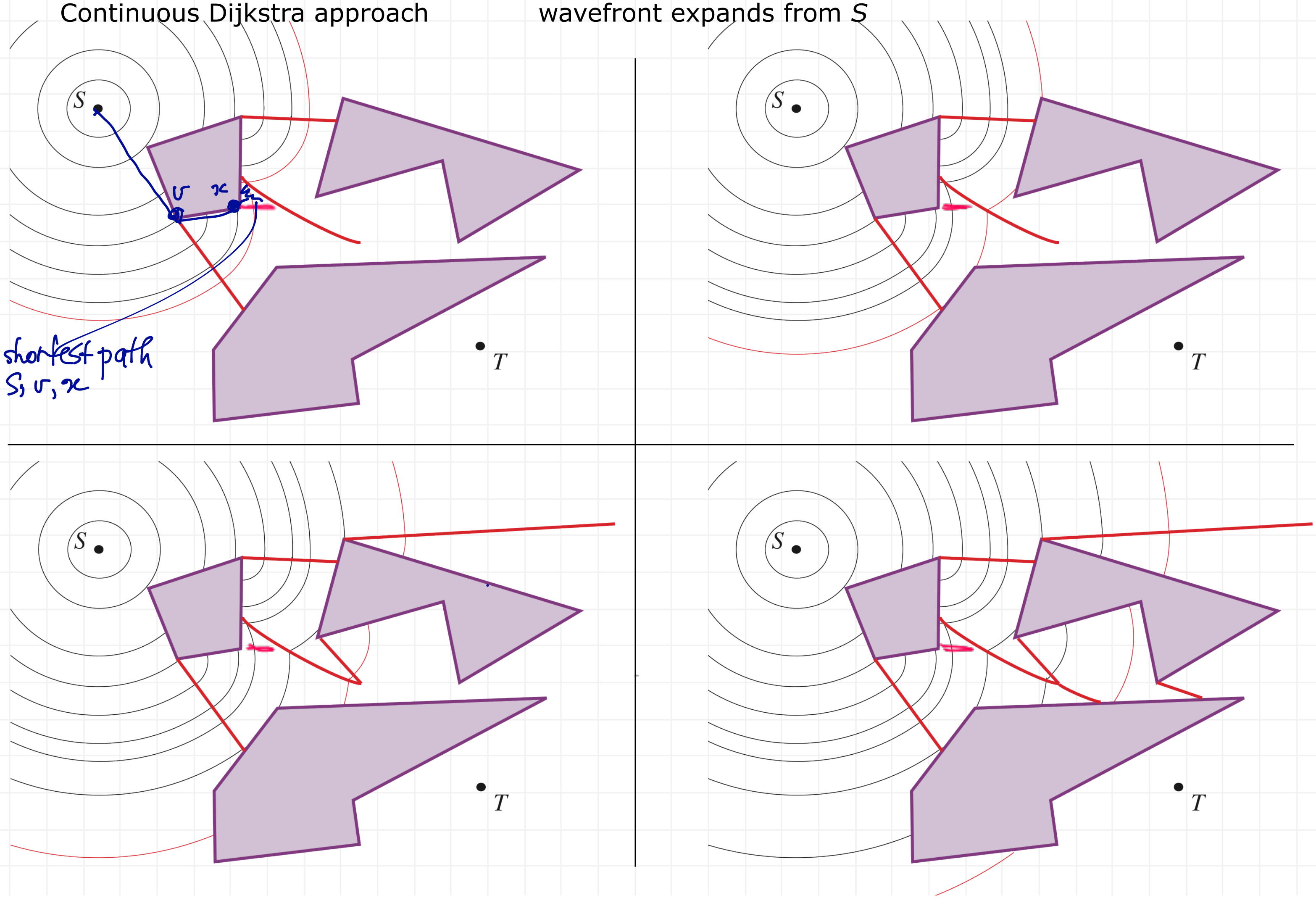
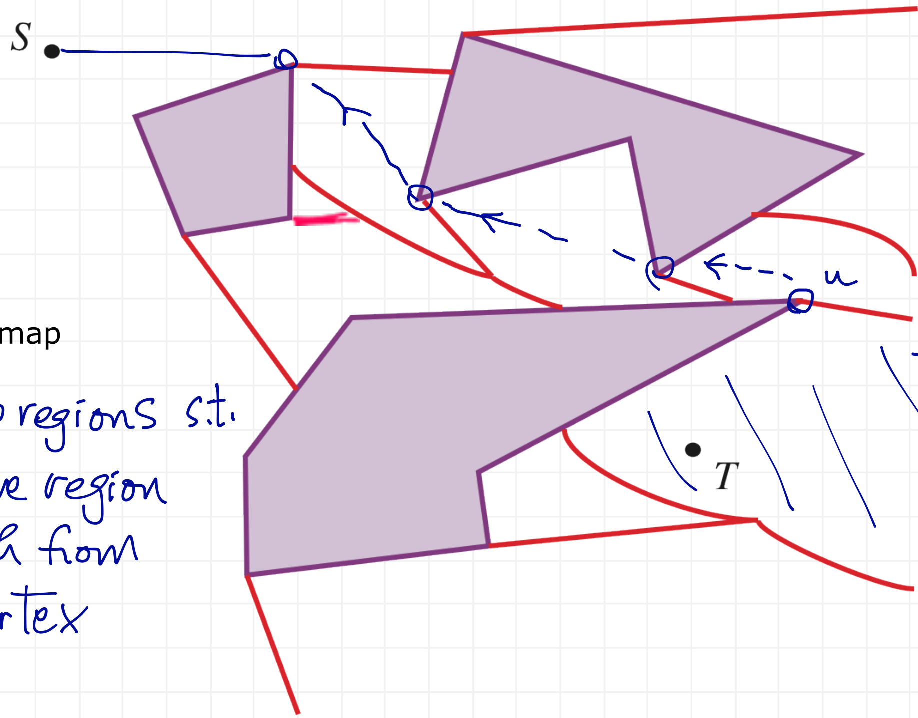


Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

Given a polygonal domain, two points S, T, find the shortest path from S to T

Continuous Dijkstra approach wavefront expands from S



shortest path map

Divide plane into regions s.t. $\forall$ points in same region the shortest path from S uses same vertex sequence

This region comes from u

Trace back the shortest path to S

Basic geometric shortest path algorithms — shortest paths in 2D polygonal domain

Given a polygonal domain, two points *S*, *T*, find the shortest path from *S* to *T*

Continuous Dijkstra approach   wavefront expands from *S*

implementation issues - keep track of future events where wavefront changes combinatorially

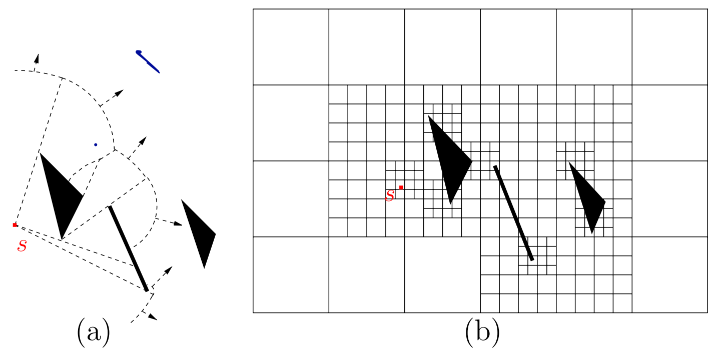
- need to know - what are events, time
- need to update wavefront

achieving O(*n* log *n*) preprocessing

Complicated

Subdivide space into cells

approximate wavefront cell by cell (but exact algorithm)



Schreiber & Sharir

query O(log *n*) to given shortest path length, O(log *n* + *k*) to give path, *k* = number of edges

use planar point location (with curves)

Summary of shortest Euclidean paths in the plane

concepts
- locally shortest paths
- polygon vs polygonal domain
- 3 problem versions: Given s, t; Given s, query t; Query s, t.
- shortest path map

approaches
- model as a graph (discretize)
- wavefront expansion (continuous Dijkstra)
- shortest path map + planar point location

Open question: for polygonal domain, can we achieve $O(n + h \log h)$?

h = # holes

n = # vertices

Have $O(n \log n)$

would match lower bound.

http://cs.smith.edu/~orourke/TOPP/P21.html

is achievable for L1 metric

Further geometric shortest path algorithms (papers to present)

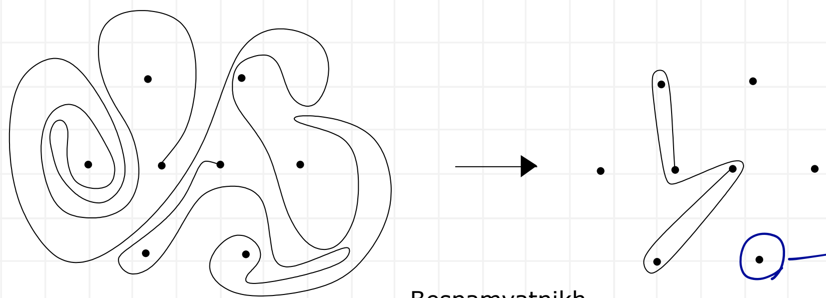
extensions of above:
- homotopy given
- dependence on number of holes
- two-point query

geodesic diameter, center
link distance
L1 distance
curved obstacles

polyhedral surfaces
3D
weighted region, etc.

Further geometric shortest path algorithms (papers to present)

homotopy given



— really ?

not around here?

Bespamyatnikh

[HTML] **Computing minimum length paths of a given homotopy class**
J Hershberger, J Snoeyink - **Computational** geometry, 1994 - Elsevier
Cited by 140 Related articles All 2 versions Web of Science: 62 CiteSave
From: http://scholar.google.ca/scholar?q=Computing+minimum+length+paths+of+a+given+homotopy+class&btnG=&hl=en&as_sdt=0%2C5

— this is paper with (improved) funnel alg.
– solves above – run time ~ # triangles entered by input path (counting repeats)

[HTML] **Computing homotopic shortest paths efficiently**
A Efrat, SG Kobourov, A Lubiw - **Computational** Geometry, 2006 - Elsevier
Cited by 33 Related articles All 5 versions CiteSave
From: http://scholar.google.ca/scholar?hl=en&q=Computing+homotopic+shortest+paths+efficiently&btnG=&as_sdt=1%2C5&as_sdtp=

run time sensitive to input & output size.

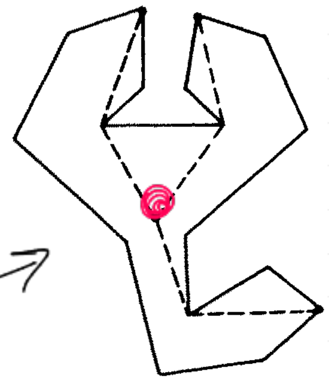
two-point query

Two-point Euclidean shortest path queries in the plane
YJ Chiang, JSB Mitchell - Proceedings of the tenth annual ACM-SIAM …, 1999 - dl.acm.org
Cited by 44 Related articles All 8 versions CiteSave
From: http://scholar.google.ca/scholar?q=Two-point+Euclidean+shortest+path+queries+in+the+plane&btnG=&hl=en&as_sdt=0%2C5

Shortest path queries in polygonal domains
H Guo, A Maheshwari, JR Sack - Algorithmic Aspects in Information and …, 2008 - Springer
Cited by 7 Related articles All 7 versions CiteSave
From: http://scholar.google.ca/scholar?q=Shortest+path+queries+in+polygonal+domains&btnG=&hl=en&as_sdt=2005&sciodt=0%2C5&cites=2172980975228742329&scipsc=

Geodesic Diameter: Given a polygon / polygonal region, what is the maximum distance between two points?

Geodesic Center: Given a polygon / polygonal region, find the point that minimizes the maximum distance to any other point.



*

Matrix searching with the **shortest-path metric**

O(n) for diameter

J Hershberger, S Suri - SIAM Journal on Computing, 1997 - SIAM
We present an O (n) time algorithm for computing row-wise maxima or minima of an implicit, totally monotone n*n **matrix** whose entries represent **shortest-path** distances between pairs of vertices in a simple polygon. We apply this result to derive improved algorithms for ...
Cited by 20 Related articles All 4 versions Web of Science: 7 Cite Save

From: http://scholar.google.ca/scholar?hl=en&q=matrix+searching+with+the+shortest+path+metric&btnG=&as_sdt=1%2C5&as_sdtp=

Computing the **geodesic center** of a **simple polygon**

O(n log n)

R Pollack, M Sharir, G Rote - Discrete & **Computational** Geometry, 1989 - Springer
Abstract The **geodesic center** of a **simple polygon** is a point inside the **polygon** which minimizes the maximum internal distance to any point in the **polygon**. We present an algorithm which calculates the **geodesic center** of a **simple polygon** with n vertices in time ...
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From: http://scholar.google.ca/scholar?q=computing+the+geodesic+center+of+a+simple+polygon&btnG=&hl=en&as_sdt=0%2C5

OPEN: O(n)

new!

The geodesic diameter of polygonal domains

SW Bae, M Korman, Y Okamoto - Discrete & Computational Geometry, 2013 - Springer
Abstract This paper studies the **geodesic diameter** of **polygonal domains** having $ h $ holes and $ n $ corners. For simple **polygons** (ie, $ h= 0$), the **geodesic diameter** is determined by a pair of corners of a given **polygon** and can be computed in linear time, as shown by ...
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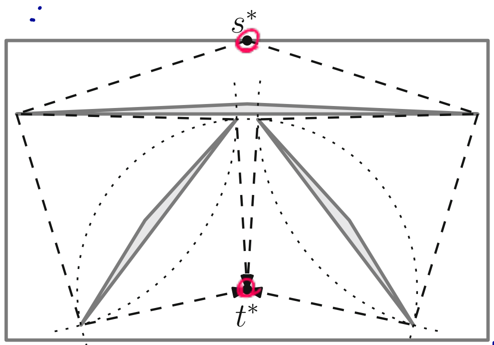


diameter not realized at vertices (in general)

From: http://scholar.google.ca/scholar?q=The+geodesic+diameter+of+polygonal+domains&btnG=&hl=en&as_sdt=2005&sciodt=0%2C5&cites=16376144327624502384&scipsc=

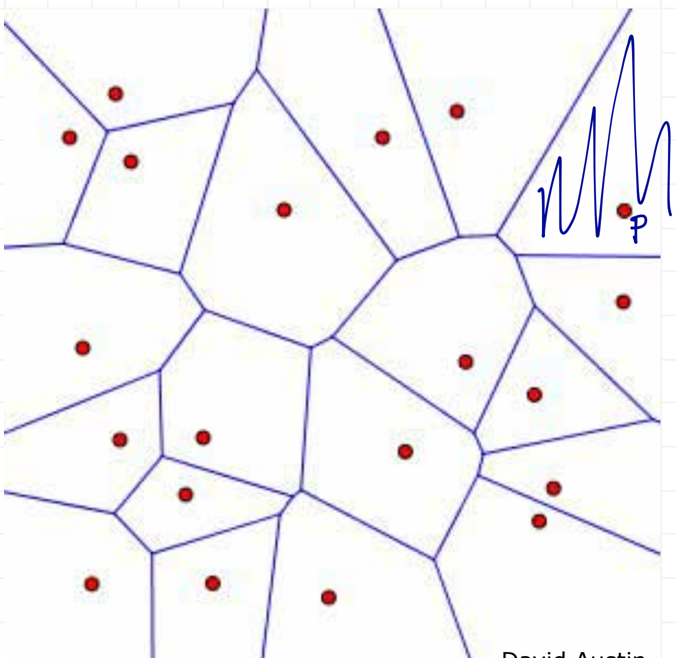
? center of polygonal domain

Aside: Nearest neighbour

Given a set S of n points in the plane, preprocess to handle query: Given a query point q, what is the closest point in S?

Preprocessing $O(n \log n)$, Space $O(n)$, Query $O(\log n)$ ← this matches soln in 1D (sort + binary search)

Vononoi diagram



David Austin

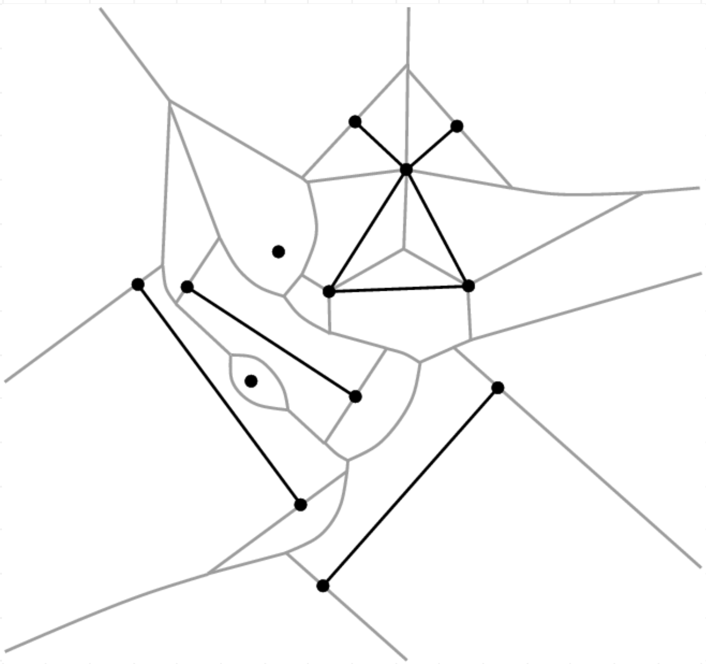
set of all points whose closest neighbour is p.

Compute Voronoi diagram size $O(n)$

+ Planar Point Location

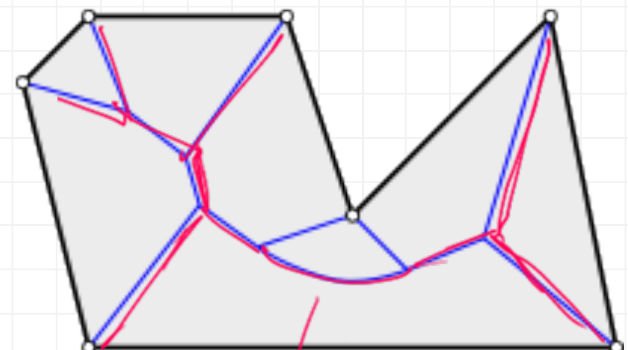
See comp. geom. texts

Voronoi diagram of line segments



CGAL - Computational Geometry Algorithms Library

Voronoi diagram of edges and vertices of a polygon

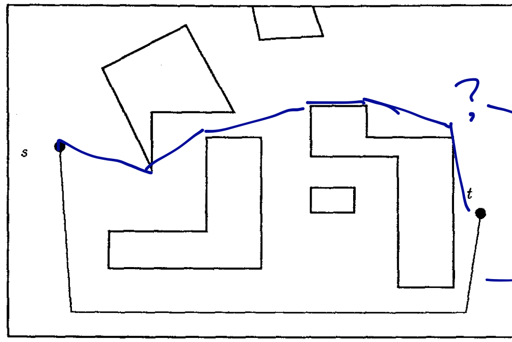


Stefan Huber

medial axis

= pts with ≥ 2 closest neighbours on polygon boundary

Link distance
counts # segments on path



[On some link distance problems in a simple polygon](http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=88124) — O(n)
S Suri - IEEE transactions on Robotics and Automation, 1990 - cat.inist.fr
Cited by 73
http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=88124

✱ **Minimum-link paths among obstacles in the plane** — polygonal domain
JSB Mitchell, G Rote, G Woeginger - Algorithmica, 1992 - Springer
Cited by 79
From: http://scholar.google.ca/scholar?hl=en&q=Minimum-Link+Paths+Among+Obstacles+in+the+Plane&btnG=&as_sdt=1%2C5&as_sdtp=

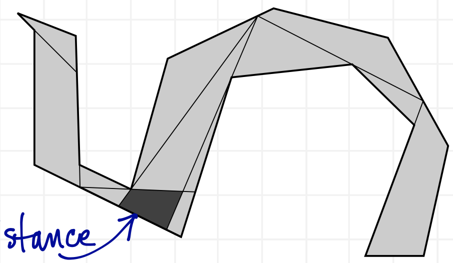
On the bit complexity of minimum link paths: Superquadratic algorithms for problem solvable in linear time
S Kahan, J Snoeyink - Computational Geometry, 1999 - Elsevier
All of the linear-time algorithms that have been developed for minimum-link paths use the real RAM model of computation. If one considers bit complexity, however, merely representing a minimum-link path may require a superquadratic number of bits. This ...

— real RAM versus counting bits

Logarithmic-time link path queries in a simple polygon
EM Arkin, JSB Mitchell, S Suri - International Journal of …, 1995 - World Scientific
We develop a data structure for answering **link** distance **queries** between two arbitrary points in a **simple polygon**. The data structure requires O (n3) time and space for its construction and answers **link** distance **queries** in O (log n) time, after which a minimum-**link path** can ...
Cited by 11 Related articles All 2 versions Cite Save

From: http://scholar.google.ca/scholar?hl=en&q=Logarithmic-time+link+path+queries+in+a+simple+polygon.&btnG=&as_sdt=1%2C5&as_sdtp=

link center



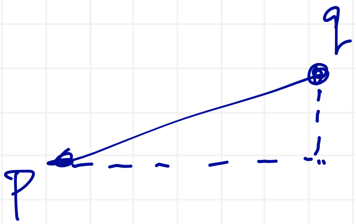
link distance from here is ≤ 3 for all points

O'Rourke

[AnO (**n logn**) **algorithm** for **computing** the **link center** of a **simple polygon**](http://scholar.google.ca/scholar?q=An+O%28n+log+n%29+algorithm+for+computing+the+link+center+of+a+simple+polygon.&btnG=&hl=en&as_sdt=0%2C5)
HN Djidjev, A Lingas, JR Sack - Discrete & **Computational** Geometry, 1992 - Springer
Cited by 33
From: http://scholar.google.ca/scholar?q=An+O%28n+log+n%29+algorithm+for+computing+the+link+center+of+a+simple+polygon.&btnG=&hl=en&as_sdt=0%2C5

faster for orthogonal polygon, NP-hard for polygonal domain

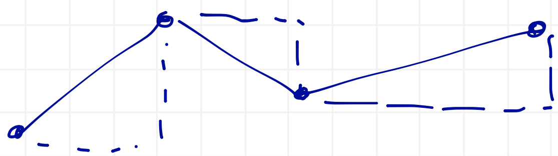
L1 metric



$$d_1(p,q) = |p_x - q_x| + |p_y - q_y|$$

"Manhattan distance"

L1 length of path = sum of lengths of segments



Danny Z. Chen, Haitao Wang: **L_1 Shortest Path Queries among Polygonal Obstacles in the Plane.** STACS 2013: 293-304

From: http://www.informatik.uni-trier.de/~ley/pers/hd/c/Chen:Danny_Ziyi

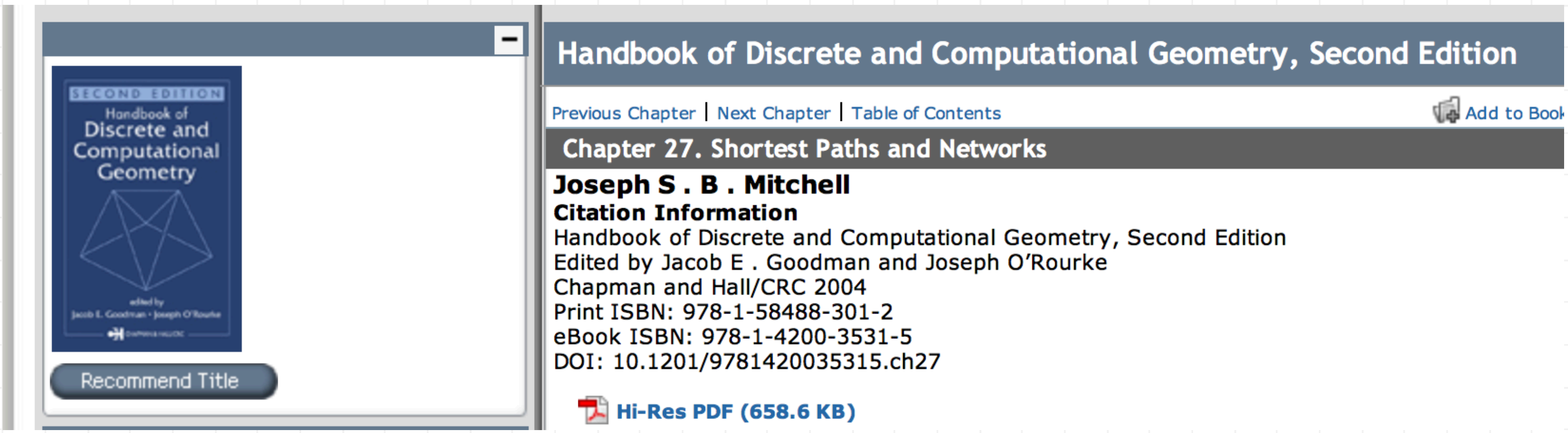
curved obstacles

Computing shortest paths among curved obstacles in the plane
DZ Chen, H Wang - … annual symposium on Symposuim on **computational** …, 2013 - dl.acm.org
Abstract In this paper, we study the problem of finding Euclidean **shortest paths among curved obstacles** in the **plane**. We model **curved obstacles** as splinegons. A splinegon can be viewed as replacing each edge of a polygon by a convex **curved** edge, and each **...**
Cited by 7Related articlesAll 5 versionsCiteSave

From: http://scholar.google.com/scholar?q=Computing+shortest+paths+among+curved+obstacles+in+the+plane.

see Joe Mitchell's survey for many more papers

J.S.B. Mitchell, Shortest Paths and Networks, Chapter 27 in Handbook of Discrete and Computational Geometry, 2nd edition, 2004



available electronically from Trellis

another paper about Dijkstra's algorithm to present

Combining speed-up techniques for shortest-path computations
M Holzer, F Schulz, D Wagner, T Willhalm - Journal of Experimental …, 2005 - dl.acm.org
Abstract In practice, computing a shortest path from one node to another in a directed graph is a very common task. This problem is classically solved by Dijkstra's algorithm. Many **techniques** are known to **speed up** this algorithm heuristically, while optimality of the **...**
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From: http://scholar.google.ca/scholar?hl=en&q=combining+speedup+techniques&btnG=&as_sdt=1%2C5&as_sdtp=

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