Self-Approaching Graphs

CS 860 Fall 2014, Anna Lubiw

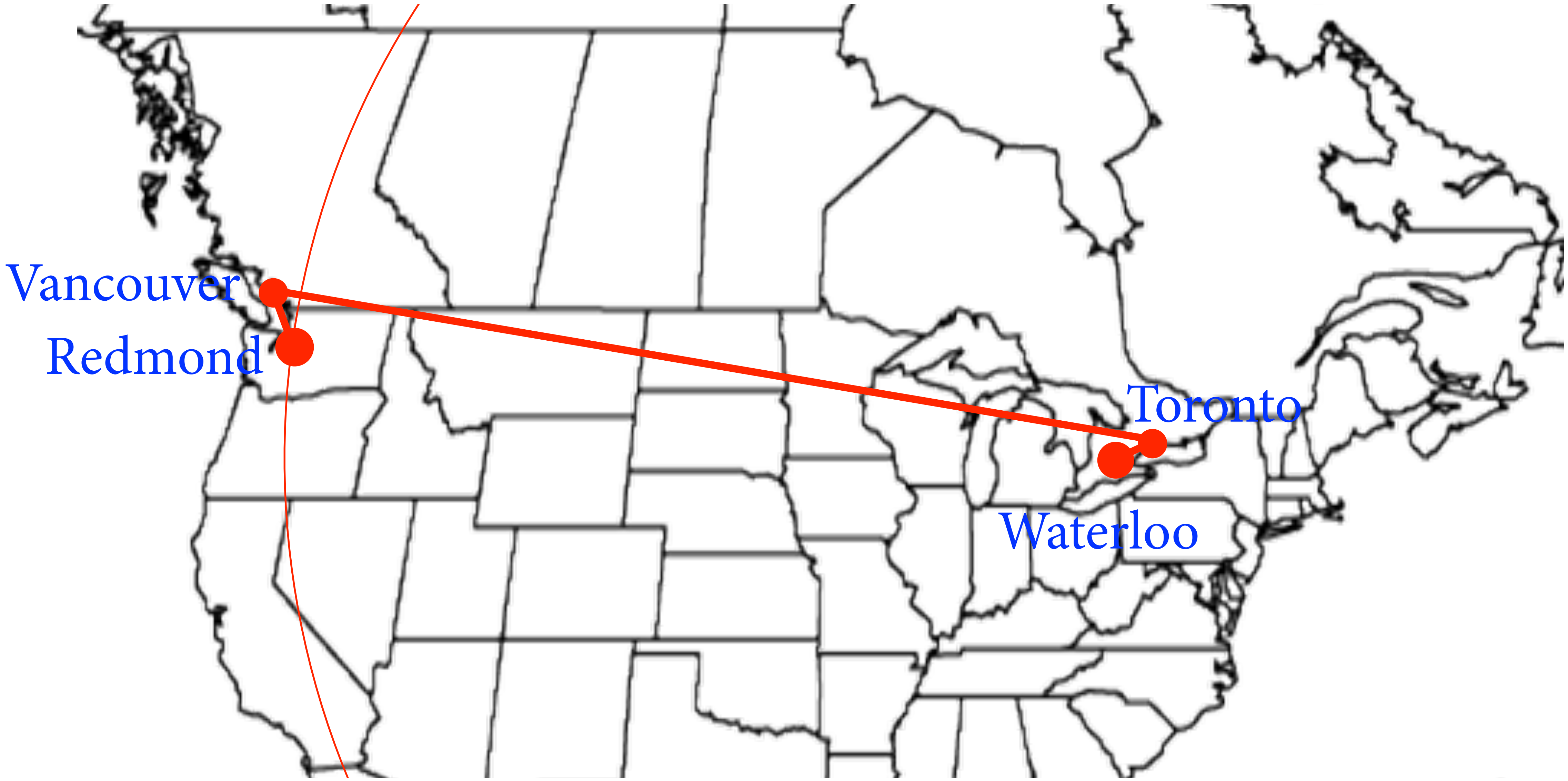
Papers

Self-Approaching Graphs.
Soroush Alamdari, Timothy M. Chan, Elyot Grant, Anna Lubiw, Vinayak Pathak.
Graph Drawing 2012.

On Self-Approaching and Increasing-Chord Drawings of 3-Connected Planar Graphs.
Martin Nollenburg, Roman Prutkin, and Ignaz Rutter.
Graph Drawing 2014.

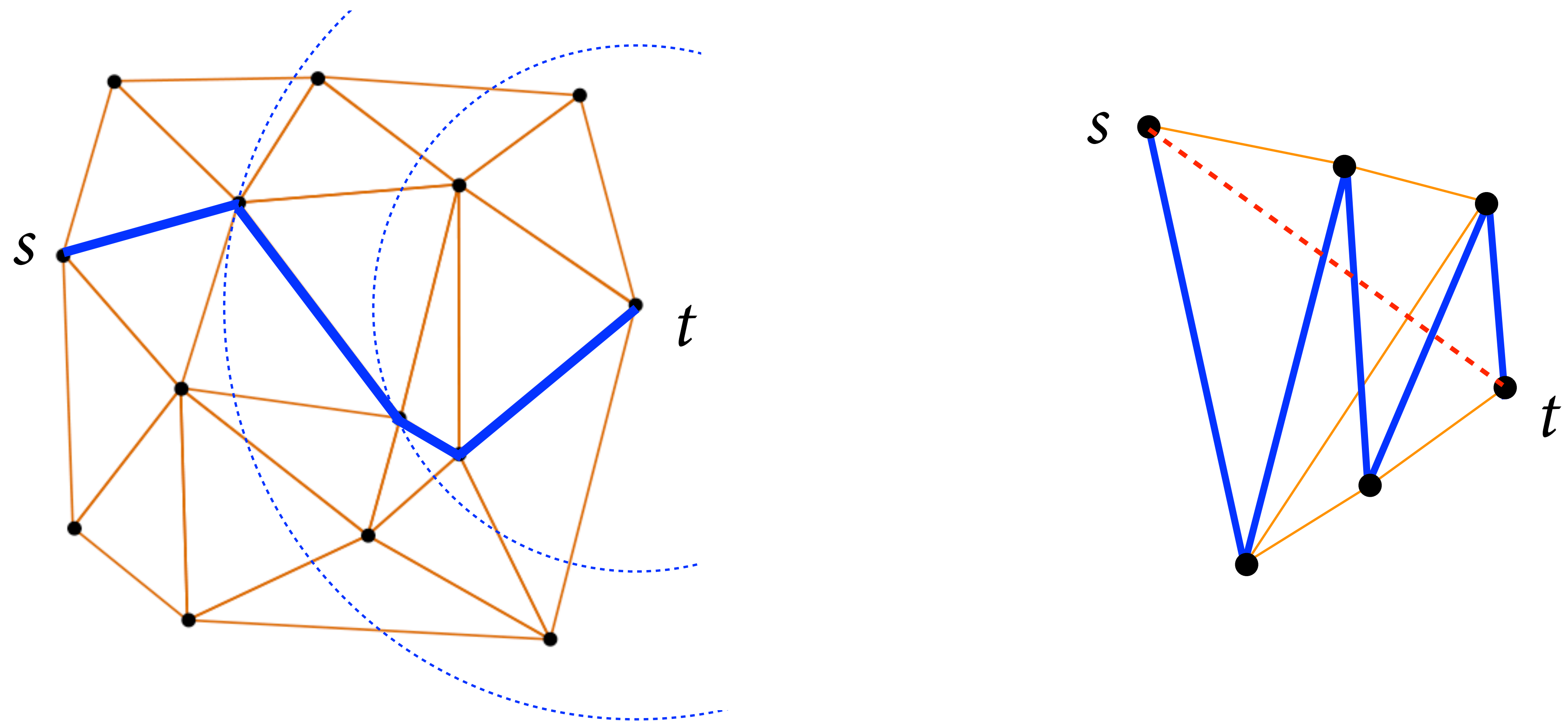
Increasing-Chord Graphs On Point Sets.
Hooman Reisi Dehkordi, Fabrizio Frati, Joachim Gudmundsson.
Graph Drawing 2014.

Getting (closer?) to your destination



Greedy drawing

For every pair of vertices s and t, there is a greedy s,t path



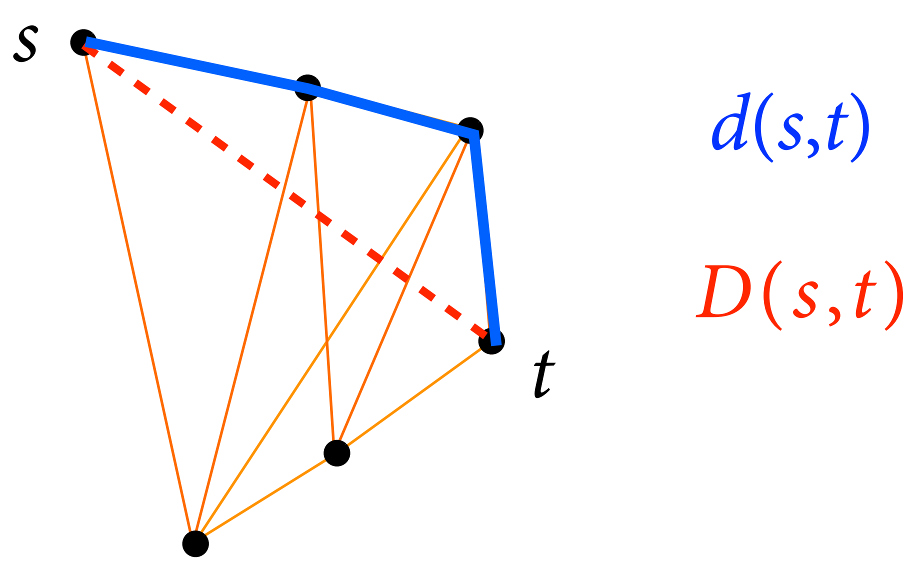
A greedy drawing permits local greedy routing.

Any 3-connected planar graph has a greedy drawing [Leighton and Moitra, 2008; Angelini, Frati, and Grilli, 2009], with few bits [Goodrich and Strash, 2009].

A greedy s,t path can be long compared to $d(s,t)$.

Dilation and spanners

dilation: $\max_{\text{vertices } s,t} \{d(s,t) \;/\; D(s,t)\}$



spanner: remove many edges while keeping dilation small

detour: $\sup_{\text{points } p,q} \{d(p,q) \;/\; D(p,q)\}$ i.e. we care about points on edges too

crossing edges ⇒ detour is infinite

Background

Questions:

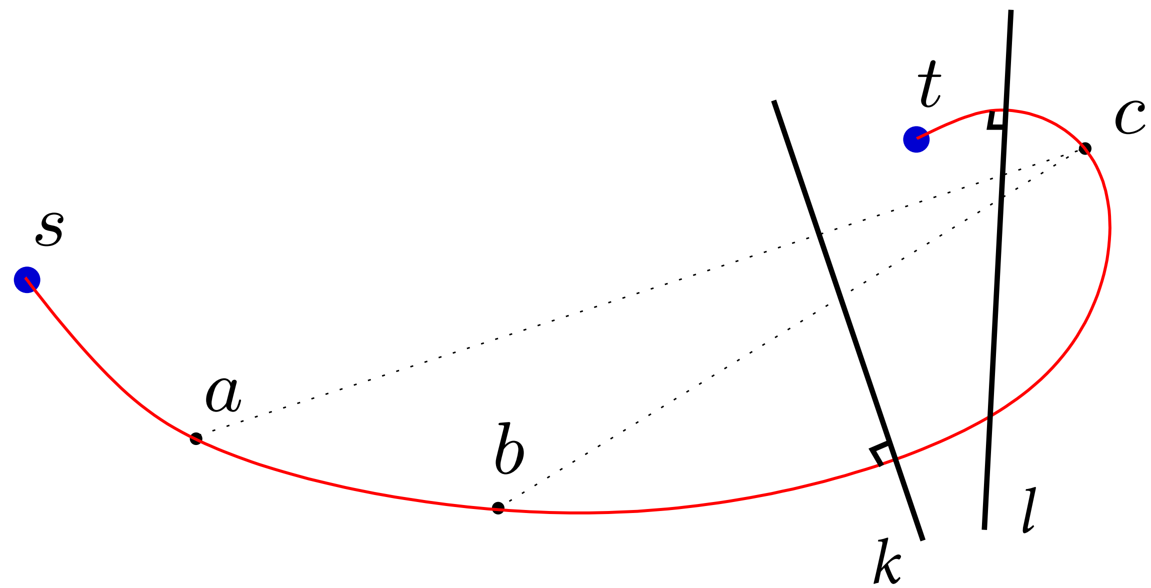
- Given a graph, find a drawing that is greedy or . . .
- Given a set of points, connect them with a graph that is a spanner or . . .

The Delaunay triangulation is greedy and is a spanner, but greedy paths do not have good dilation.

Simon's presentation: Competitive routing in the half-θ_6-graph,
Bose, Fagerberg, van Renssen, Verdonschot, 2012
Alternative triangulation that allows local routing with bounded dilation

Self-approaching curve

self-approaching s,t curve: $\forall\, a,b,c$ (in order) $D(b,c) \le D(a,c)$

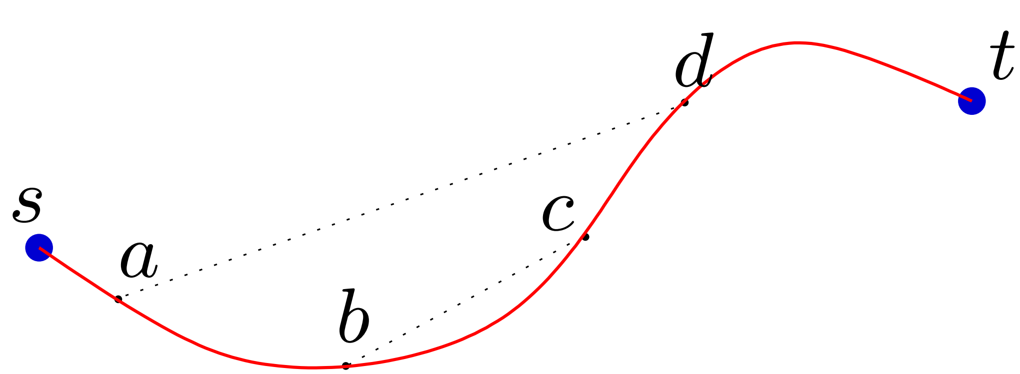


detour
5.3332

[Icking, Klein, Langetepe, 1995]

Equivalently, perpendiculars to the curve do not intersect the curve later on.

self-approaching in both directions = increasing-chord: $\forall\, a,b,c,d$ (in order) $D(b,c) \le D(a,d)$

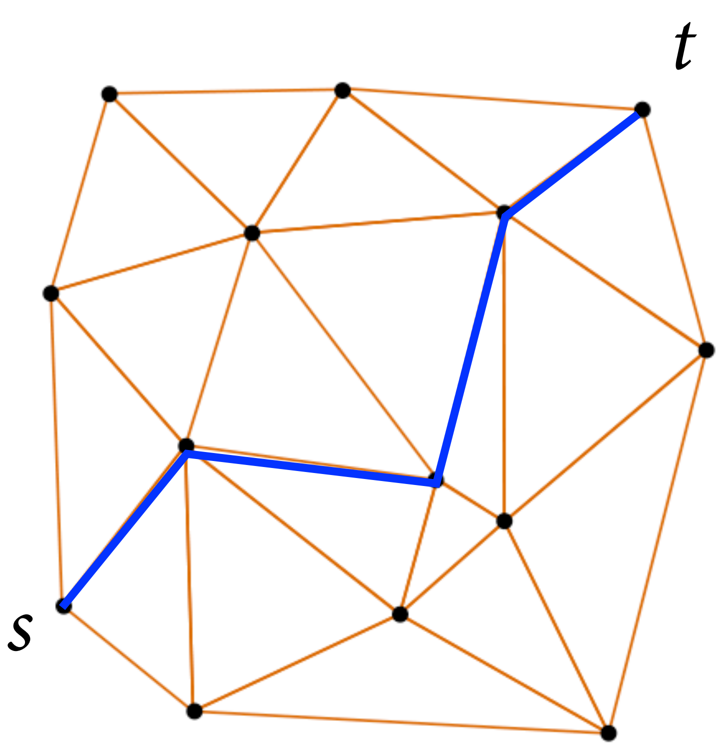


detour
2.094
[Rote 1994]

Self-approaching graph

For every pair of vertices *s*, *t*, there is a self-approaching *s*,*t* path.

increasing chord graph: For every pair of vertices *s*, *t*, there is an *s*,*t* path that is self-approaching in both directions.



Self-approaching graph

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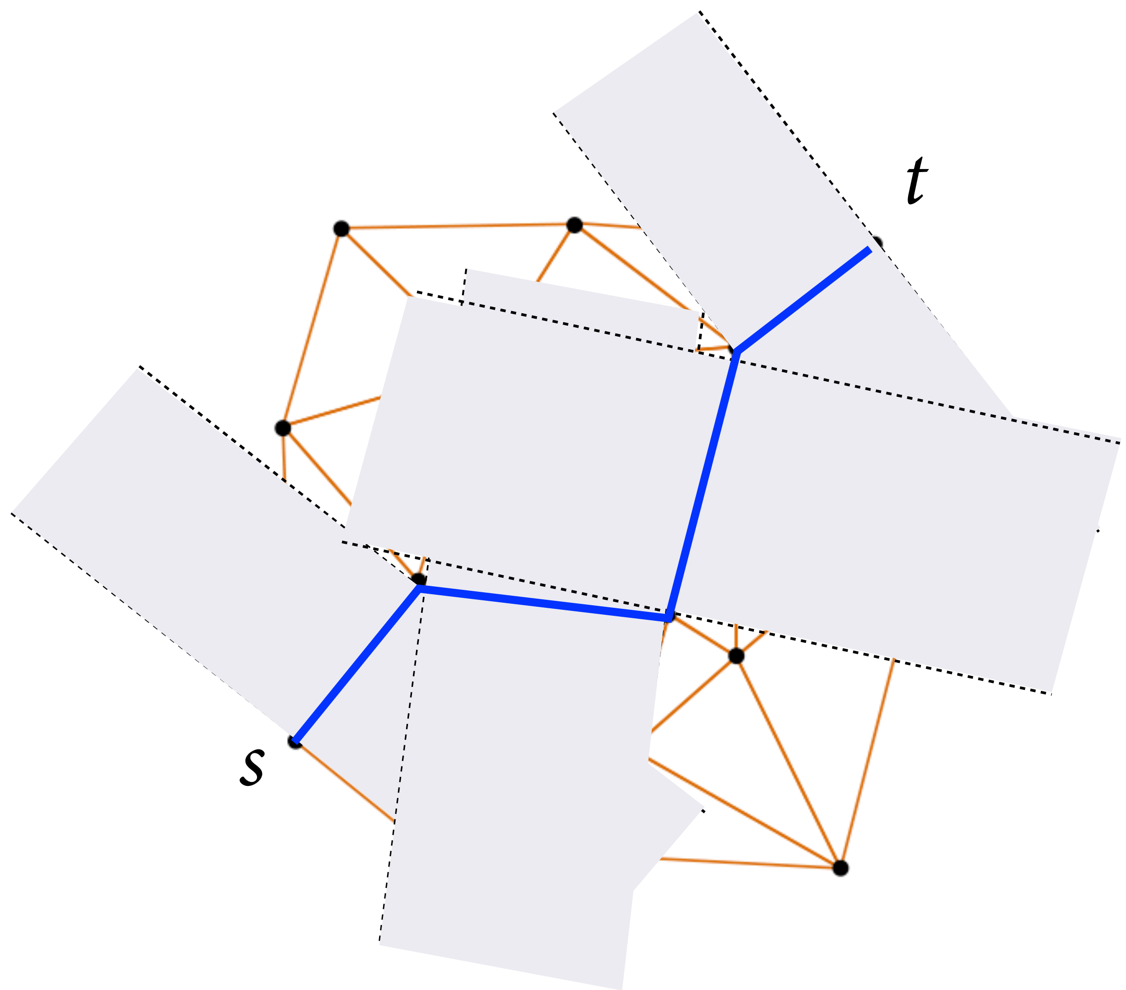
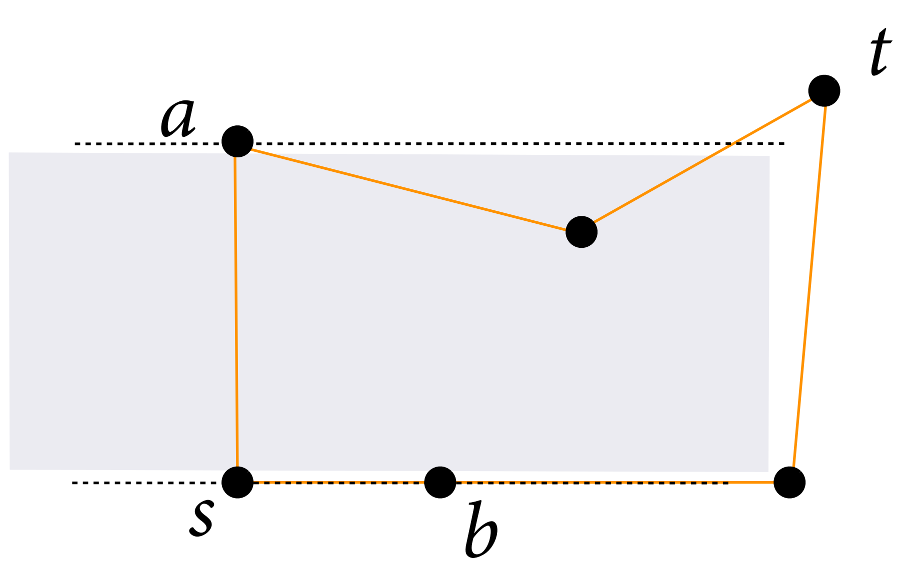




note that a greedy strategy fails

Questions

1. given a graph drawing, is it self-approaching?

2. given a graph, does it have a self-approaching drawing?

3. given points in the plane, connect them with a self-approaching network

Questions Results

1. given a graph drawing, is it self-approaching?

   open, but some partial results

2. given a graph, does it have a self-approaching drawing?

   open, but we can test trees

3. given points in the plane, connect them with a self-approaching network

   yes, $O(n)$

1. Given a graph drawing, is it self-approaching?

A natural (harder) problem:

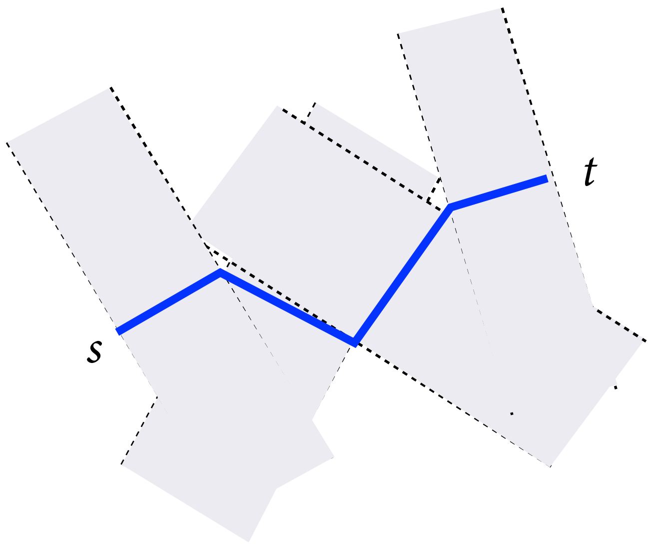
(✶) Given a graph and vertices s and t, is there a self-approaching s,t path?

Results:

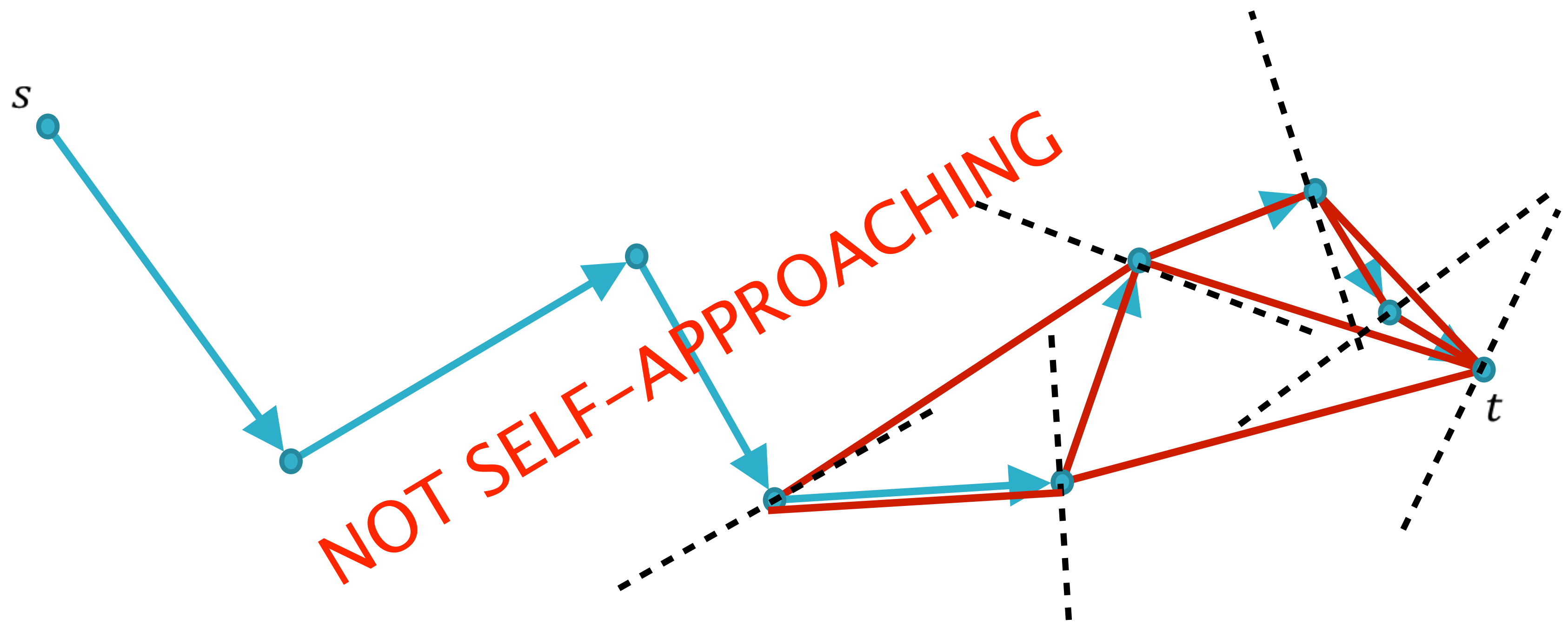
- Can test a given 2D path in $O(n)$ time.
- Can test a given 3D path in $O(n \operatorname{polylog}(n))$ time.
- (✶) is NP-complete in 3D.

Testing if a path is self-approaching

naive



Check each edge's slab with the convex hull of the points ahead.
Use incremental convex hull algorithm: O(n).



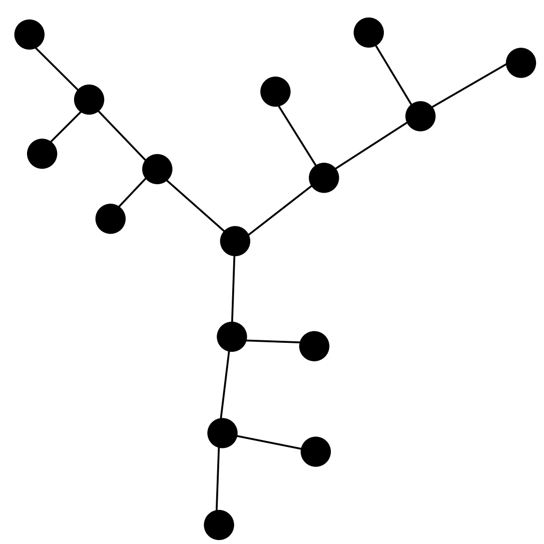
2. Given a graph, does it have a self-approaching drawing?

Theorem. A tree has a self-approaching drawing iff it is

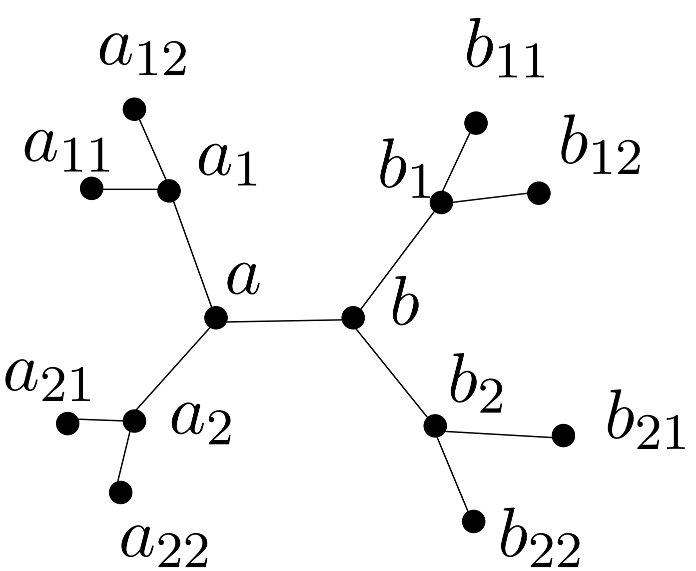
OR a subdivision of $K_{1,4}$

a subdivision of a windmill (= crab-free)

This can be testing in time $O(n)$.



a windmill



the crab

Open. Other graph classes, e.g. planar 3-connected.

Newer Results

On Self-Approaching and Increasing-Chord Drawings of 3-Connected Planar Graphs.
Martin Nollenburg, Roman Prutkin, and Ignaz Rutter.
Graph Drawing 2014.

Theorem. Every triangulation has an increasing chord drawing. If the triangulation is a planar 3-tree, the increasing chord drawing can be planar.

Ideas:

- Draw a subgraph of a triangulation (skinny angles)
- for planar 3-trees use Schnyder drawings

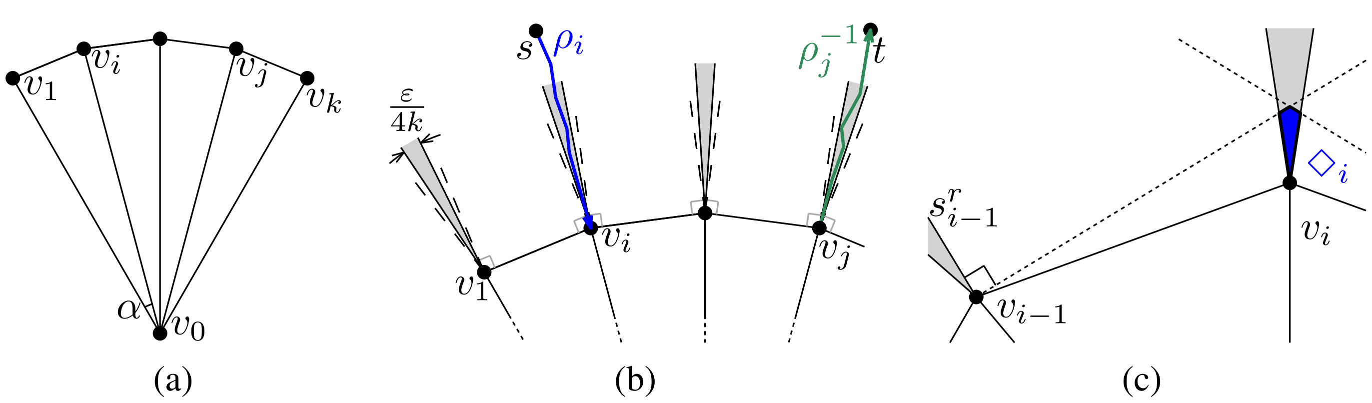


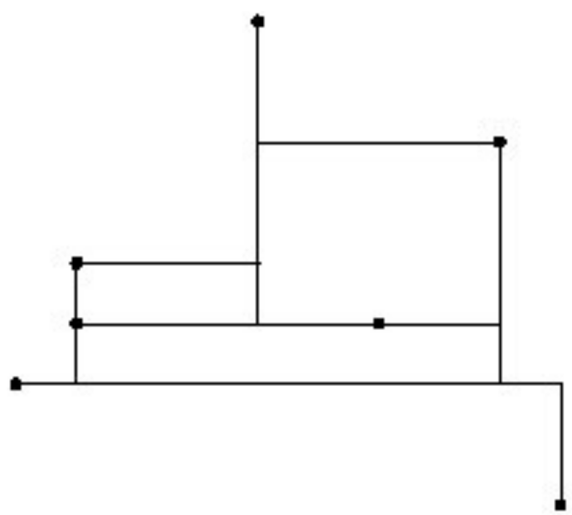
Fig. 2: Drawing a triangulated binary cactus with increasing chords inductively. The drawings $\Gamma_{i,\varepsilon'}$ of the subcactuses, $\varepsilon' = \frac{\varepsilon}{4k}$, are contained inside the gray cones.

3. Given points, construct a self-approaching network

Such a network will be a spanner.

Natural candidates:

- Delaunay triangulation no
- Manhattan network yes (an x-y monotone path is self-approaching)



[Benkert et al.]

size $\Theta(n \log n)$ [Gudmundsson, Klein, Knauer, and Smid, 2007]

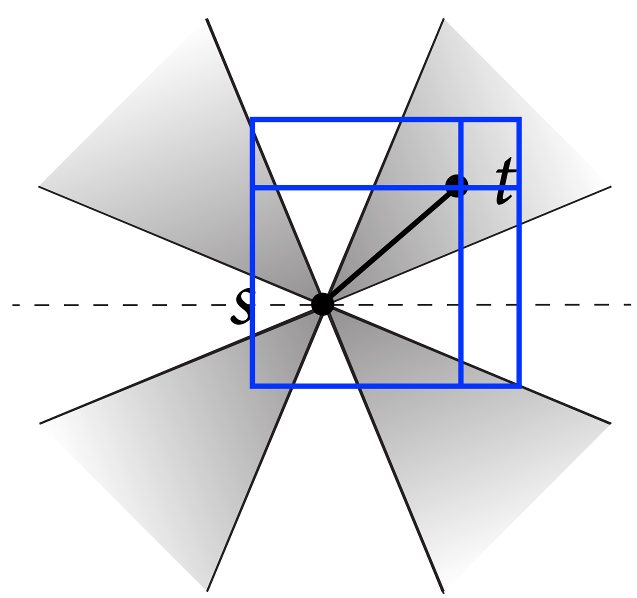
Theorem. Given a set P of n points in the plane, there exists an increasing-chord Steiner network with $O(n)$ vertices and edges.

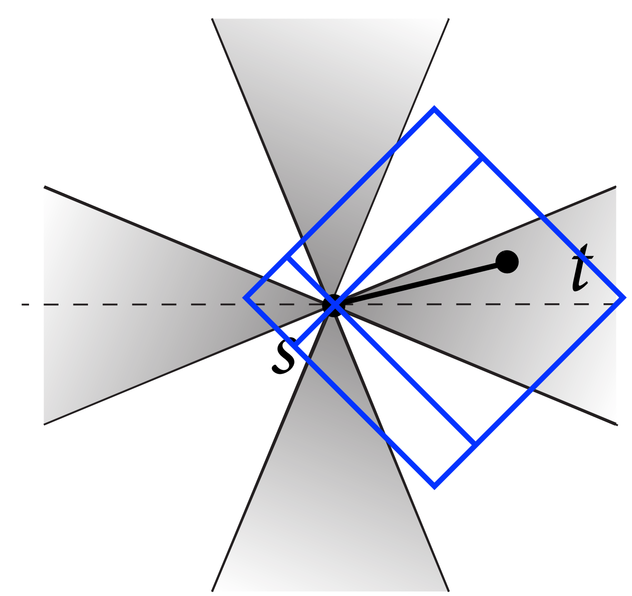
3. Given points, construct a self-approaching network

Theorem. Given a set P of n points in the plane, there exists an increasing-chord Steiner network with O(n) vertices and edges, and we can construct it in O(n log n) time.

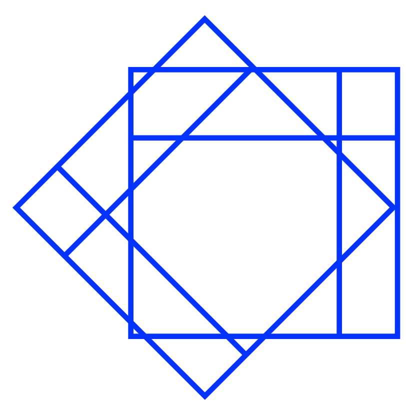
ingredients: compressed quad tree, well-separated pair decomposition

construct union of two networks for pairs s,t depending on angle to x-axis.



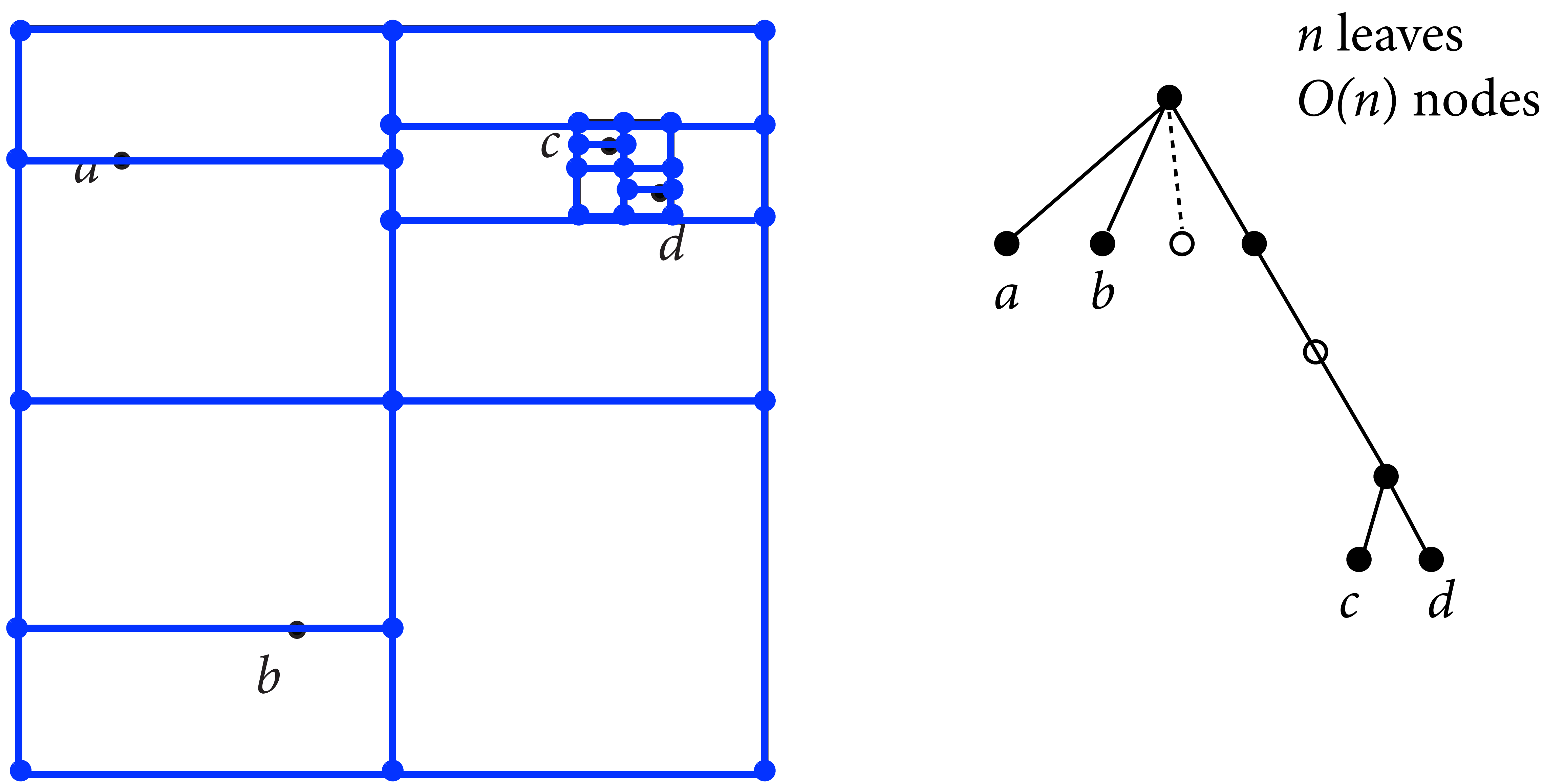


final network is octilinear



3. Given points, construct a self-approaching network

compressed quad tree



Every point can get to every corner of every enclosing square via an x-y monotone path.

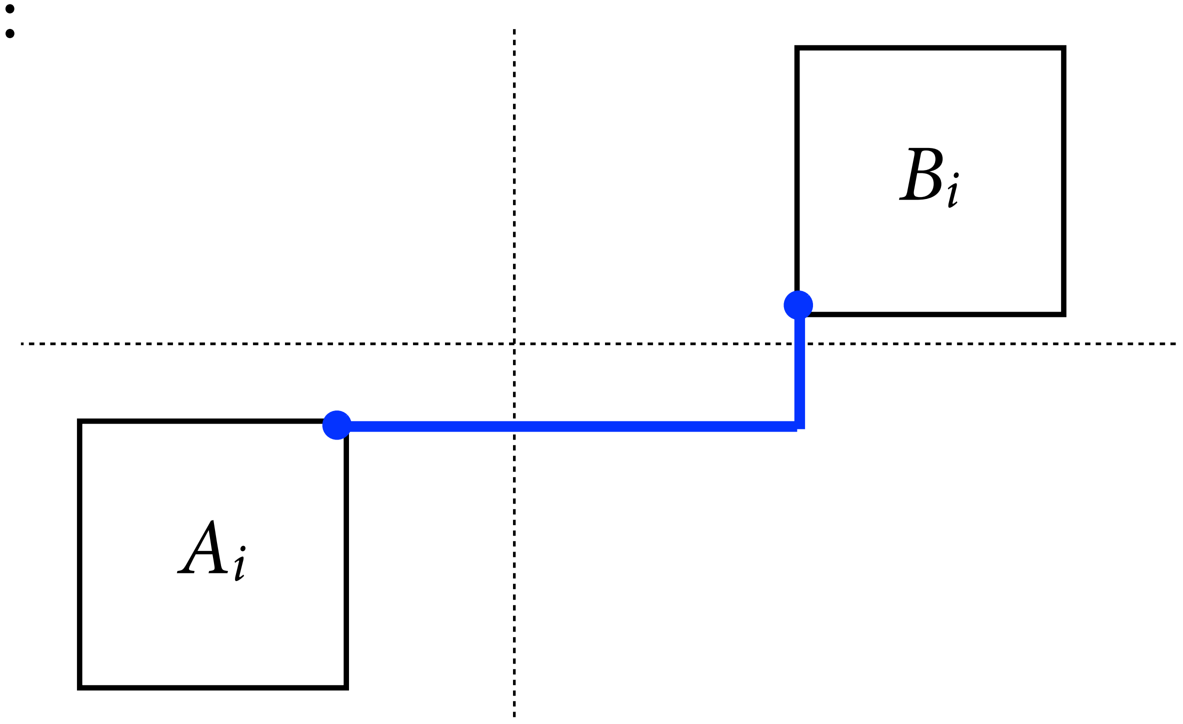
3. Given points, construct a self-approaching network

Given $\varepsilon > 0$, a well-separated pair decomposition of P is a collection of pairs of sets $\{A_1, B_1\}, \ldots, \{A_s, B_s\}$, such that

1. $\forall\, p,q \in P \;\exists$ unique i with (p,q) or $(q,p) \in A_i \times B_i$
2. A_i and B_i are well-separated: the diameters of A_i and B_i are $\leq \varepsilon d(A_i,B_i)$

There is a well-separated pair decomposition with s in $O(n/\varepsilon^2)$, and the A_i's and B_i's are squares of the compressed quad tree or points of P.

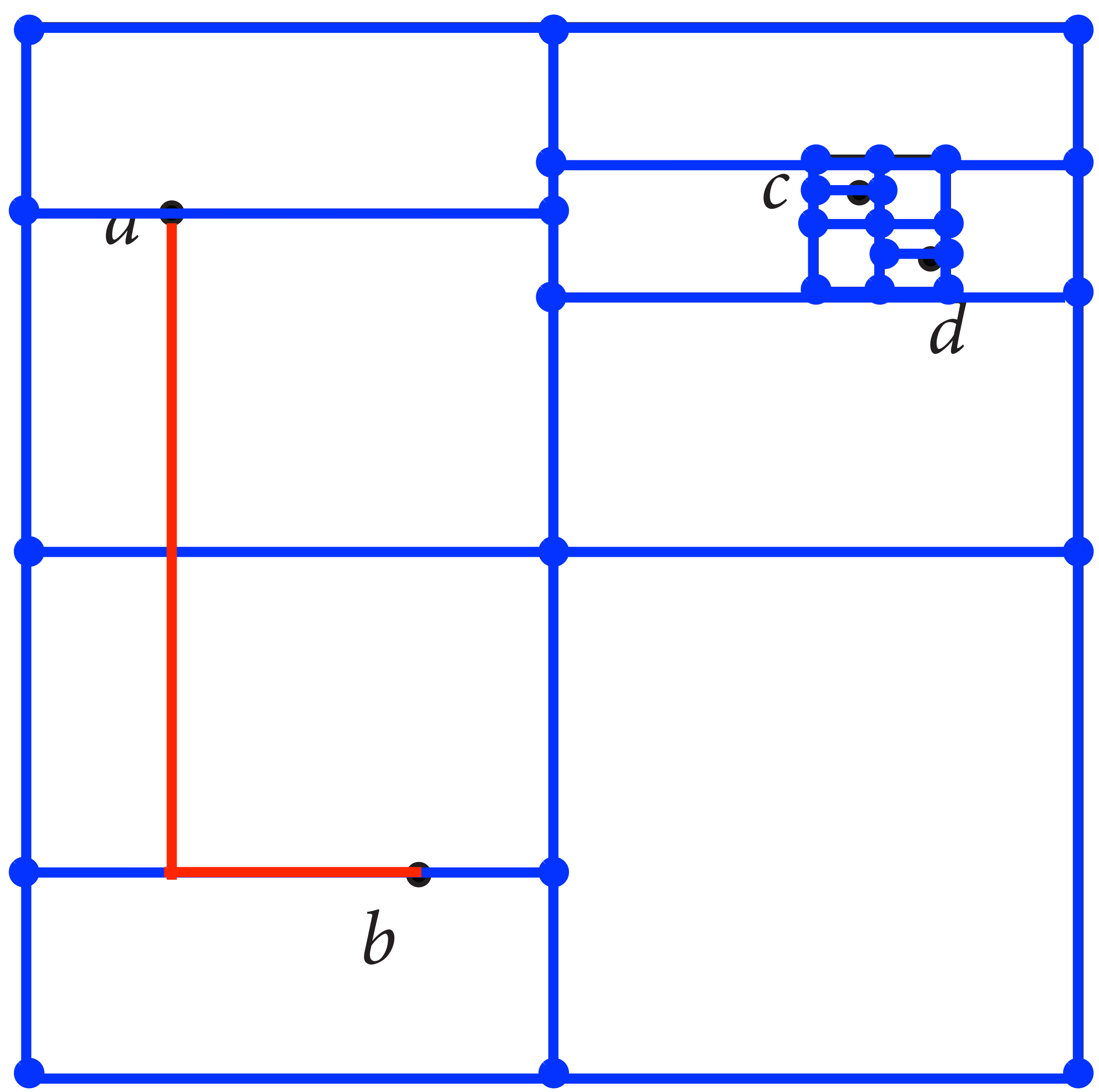
Final part of construction:



3. Given points, construct a self-approaching network

well-separated pair decomposition

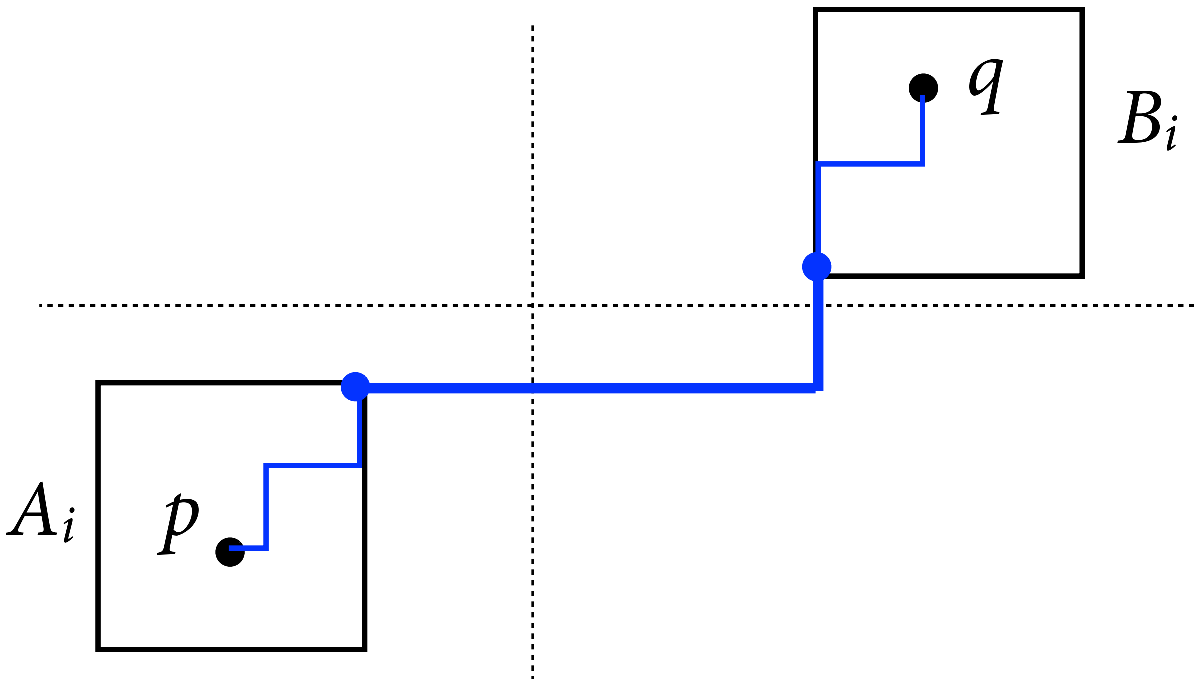
$\{a, b\}$, $\{c, d\}$, $\{a, \boxed{c,d}\}$, $\{b, \boxed{c,d}\}$



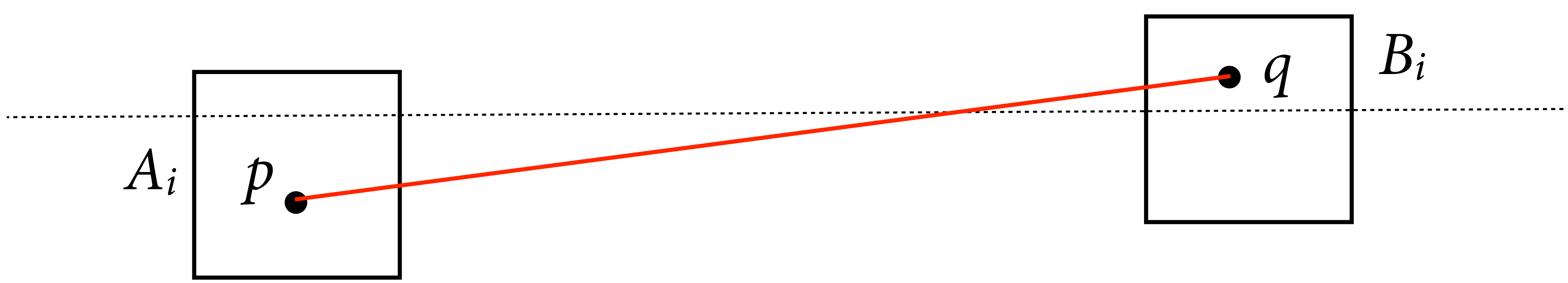
3. Given points, construct a self-approaching network

Why does this work?

case 1



case 2



slope(p,q) < O(ε)
so p,q handled by other network

Newer Results

Increasing-Chord Graphs On Point Sets.
Hooman Reisi Dehkordi, Fabrizio Frati, Joachim Gudmundsson.
Graph Drawing 2014.

Theorem 1. Given a set P of n points in the plane, there exists a planar increasing-chord Steiner network with $O(n)$ vertices and edges.

Theorem 2. Given a set P of n points in convex position in the plane, there exists an increasing-chord network without Steiner points with $O(n \log n)$ vertices and edges.

(ideas on blackboard)

Open Problems

1. given a graph drawing, is it self-approaching?
   - in P? NP-complete?
   - in 2D, given *s*,*t*, is there a self-approaching *s*,*t* path?
2. given a graph, does it have a self-approaching drawing?
   - in P?
   - 3-connected planar graphs? (traingulations always do)
   - drawing where local routing finds a self-approaching path?
3. given points in the plane, connect them with a self-approaching network
   - planar without Steiner points (open even for points in convex position)