Algorithms for Shortest Paths

course web page: https://cs.uwaterloo.ca/~alubiw/CS860.html

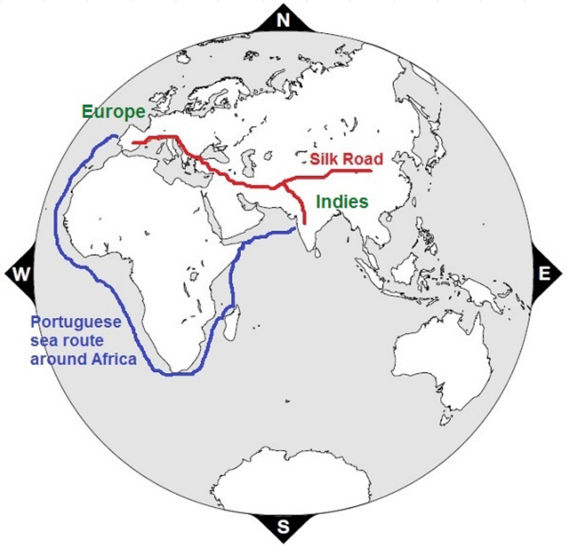
piazza: https://piazza.com/uwaterloo.ca/fall2014/cs860/home

Learn - for marks

desire path

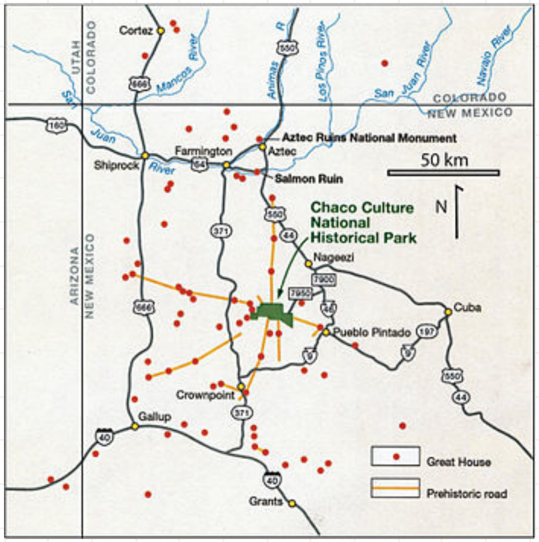
Historical

Columbus



CHRISTOPHER COLUMBUS' MAIDEN JOURNEY - 1492

Anasazi



Topics

- Intro and Dijkstra's algorithm
- basic algorithms for geometric shortest paths
- basic algorithms for graph shortest paths

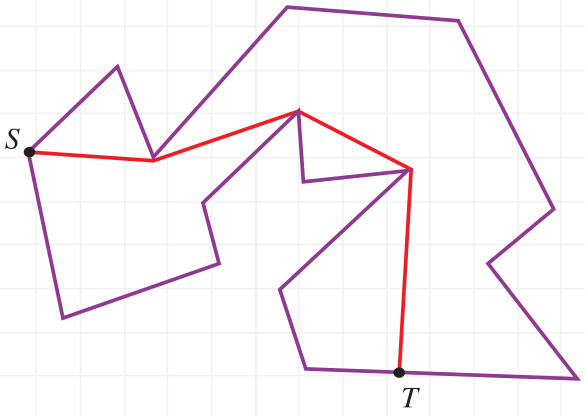
- more graph shortest path algorithms: all pairs, kth shortest, planar graphs, forbidden pairs, etc.
- more geometric shortest path algorithms: link distance, polyhedral surfaces, 3D, weighted region, etc.

- further topics
    - spanners
    - network routing
    - reconfiguration problems
    - touring (TSP)
    - and etc.

Topics

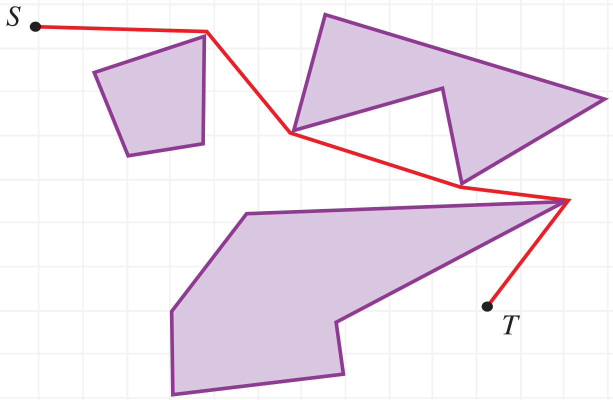
- basic algorithms for geometric shortest paths

Polygon



$O(n)$

Polygonal Domain



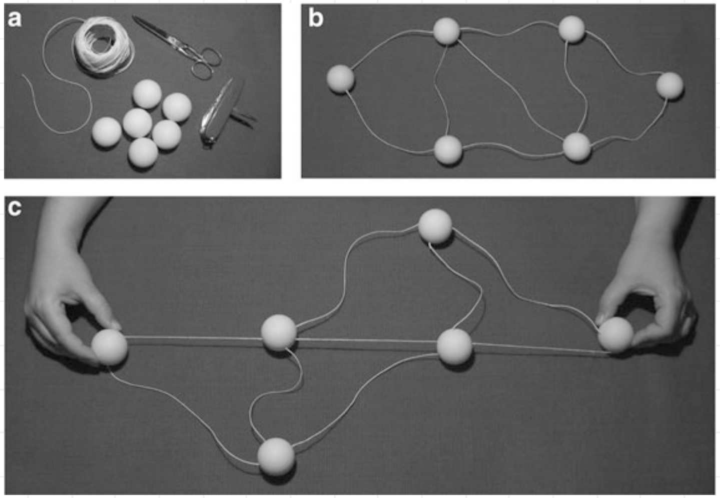
$\Theta(n \log n)$

fixed target versus query versions

Topics

- basic algorithms for graph shortest paths

  Dijkstra, Bellman-Ford



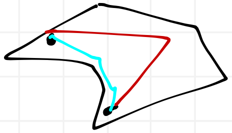
Camil Demetrescu and Giuseppe F. Italiano

- more graph shortest path algorithms:
  - all pairs Timothy Chan
  - kth shortest David Eppstein, Finding the **k shortest** paths, cited by 1293
  - planar graphs highway graphs
  - forbidden pairs
  - etc. maybe disjoint paths

computing diameter

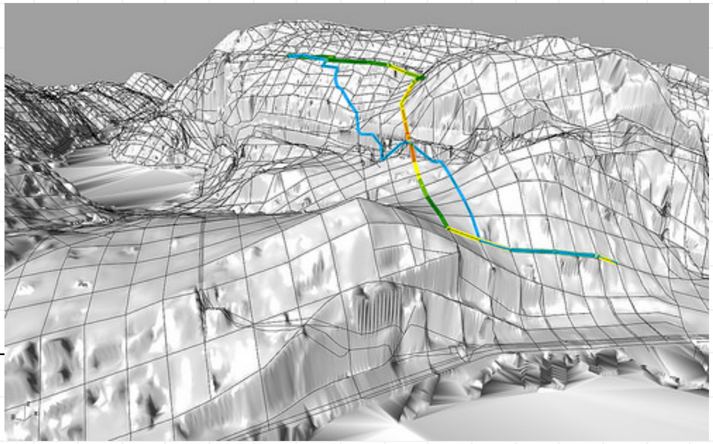
Topics

- more geometric shortest path algorithms:
  - link distance



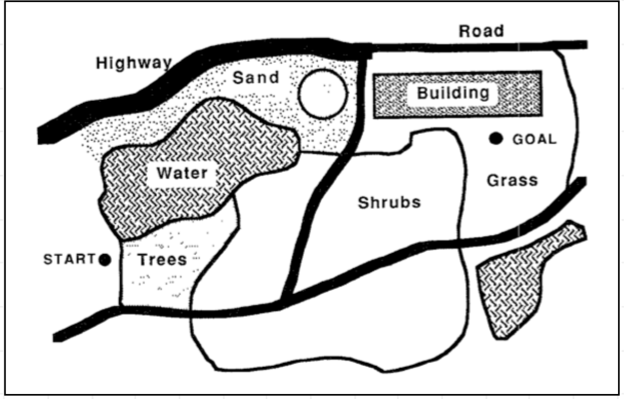
  - polyhedral surfaces

$2\frac{1}{2}$D



  - 3D

  - weighted region

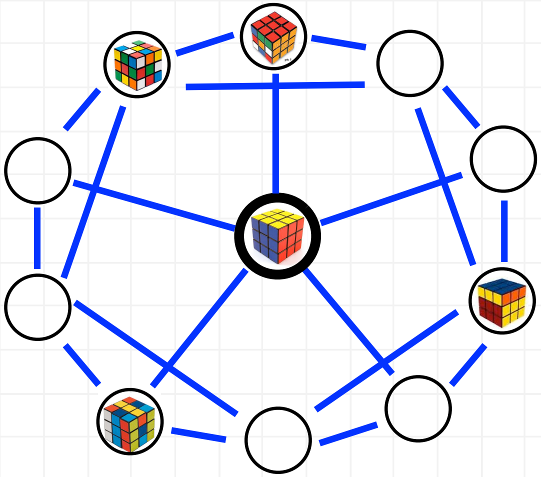


  - etc.

Topics

- further topics
    - spanners - make graph less dense while approximately preserving distances
    - network routing

      greedy routing
    - reconfiguration problems, diameter
    - touring (TSP)
    - and etc.



Credit requirements

- 2 assignments (20%)
- presentation of 1 paper (30%) - 30 minutes + discussion
- exploration of open question (20%) - written report, approx. 2 pages. Formulate a good open question and suggest how to tackle it. BONUS if you succeed.
- mini reviews of presented papers (30%) - half page reports, submitted through Piazza.

Shortest Paths in Graphs

directed $|V| = n$ $|E| = m$

Input: a graph $G=(V,E)$ with weights $w:E \longrightarrow R$ on edges, vertices s, t
Find the shortest [simple] path from s to t.
do not repeat vertices

NP-complete in general — reduction from Ham. path from s to t
Set all edge weights to -1
Ask for path of weight $\leq -(n-1)$

Difficulty is negative weight cycles.

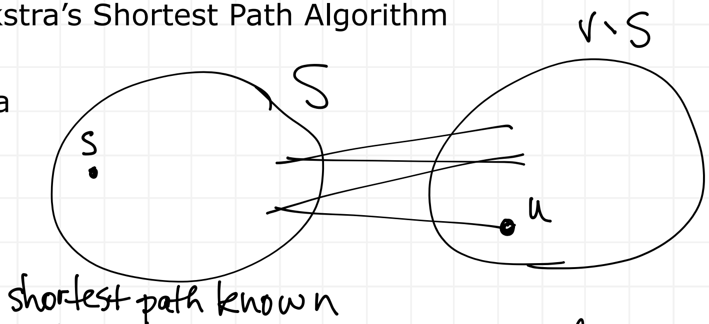
polynomial time for

- directed acyclic graphs — topological sort
- non-negative weights — Dijkstra
- graphs with no negative weight cycle
  —Bellman-Ford. dynamic programming.

Dijkstra's Shortest Path Algorithm

for non-negative weights.

Idea



$d(u)$ – shortest path using edges in S + one edge to u

update step: initialize $S = \emptyset$ $d(s) = 0$ $d(u) = \infty$ $\forall$ other u

pick $v \in V \setminus S$ of min $d(v)$

add v to S

update $u \in V \setminus S$ $d(u) = \min\{d(u), d(v) + w(v,u)\}$

for edge (v,u)

total work on updates $O(m)$

Correctness

Implementations

basic $O(n^2 + m)$

heap ~~$O(n\log n + m)$~~ $O(m \log n)$ — each update costs $\log n$

Fibonacci heap — "decrease-key" operation – $O(1)$ amortized

$O(n \log n + m)$ – theoretical

Dijkstra's
paper

A **note** on **two problems** in connexion with graphs
EW **Dijkstra** - Numerische mathematik, 1959 - Springer

Cited by 13626 Related articles All 13 versions

From: http://scholar.google.ca/scholar?hl=en&q=dijkstra+a+note+on+two+problems&btnG=&as_sdt=1%2C5&as_sdtp=

for some history, see:

The **Quest** for the **Shortest Route**
C Demetrescu, GF Italiano - The Power of Algorithms, 2013 - Springer

From: http://scholar.google.ca/scholar?q=the+quest+for+the+shortest+route&btnG=&hl=en&as_sdt=0%2C5

Dijkstra:

> At the time, algorithms were hardly considered a scientific topic. I wouldn't have known where to publish it. . . . The mathematical culture of the day was very much identified with the continuum and infinity. Could a finite discrete problem be of any interest? The number of paths from here to there on a finite graph is finite; each path is a finite length; you must search for the minimum of a finite set. Any finite set has a minimum – next problem, please. It was not considered mathematically respectable. . .

read his paper — on webpage and Piazza
write a mini-review — what is 2nd problem ?
— optional — find your favorite presentation of Dijkstra's alg.

other implementations of Dijkstra's algorithm

Dial's method — for integer weights in $[0, C]$

DIAL, R. Algorithm 360: Shortest path forest with topological ordering. Commun. ACM 12 (1969), 632-633.

use buckets $0 \cdot\cdot \underbrace{nC}_{\text{max length of shortest paths}}$

to store $d(u)$

just scan once through buckets $O(\underbrace{nC}_{\text{total work for finding min}} + \underbrace{m}_{\text{updates}})$.

other implementations of Dijkstra's algorithm

Dial's method

reducing space from nC to C+1

At any point in alg. $d(u)$ values, $u \in V \setminus S$
lie in range min .. min + C — not counting u with $d(u) = \infty$

So use cyclic buckets — space $C+1$

Pf. true initially when $S = \{s\}$

update increases min

u with finite $d(u)$ — decreases, so still $\leq \min + C$

u with $d(u) = \infty$ $\quad d(u) = \min\{\underset{\infty}{d(u)}, \underset{\min}{\underset{\|}{d(v)}} + \underset{\leq C}{w(v,u)}\} \leq \min + C$

time: $O(m + nC)$

other implementations of Dijkstra's algorithm

double bucket method

DENARDO, E. V., AND Fox, B. L. Shortest-route methods: 1. Reaching, pruning, and buckets. Oper.Res. 27(1979), 161-186.

"lower buckets" – K of them, initially $0 \ldots K-1$, K determined later
hold single values

"upper buckets" – each bucket holds K values

\# upper buckets $\frac{n \cdot C}{K}$

Find min

- if lowerbuckets all empty then
  search upper buckets
  move (distribute) contents of first non-empty bucket to lower bucket
- search lower buckets.

Time $O(\underbrace{\frac{nC}{K}}_{\text{searching upper buckets}} + \underbrace{nK}_{\text{searching lower buckets}} + m)$

min with $K = \sqrt{C}$

$O(n\sqrt{C} + m)$

related papers to present

Faster algorithms for the **shortest path problem**
RK Ahuja, K Mehlhorn, J Orlin, RE Tarjan - Journal of the ACM (JACM), 1990 - dl.acm.org
Cited by 551 Related articles All 19 versions CiteSave

From: http://scholar.google.ca/scholar?q=%EF%BF%BCFaster+Algorithms+for+the+Shortest+Path+Problem&btnG=&hl=en&as_sdt=0%2C5

— radix heaps for Dijkstra

Undirected single-source shortest paths with **positive integer weights** in **linear time**
M Thorup - Journal of the ACM (JACM), 1999 - dl.acm.org
Cited by 307 Related articles All 11 versions CiteSave

From: http://scholar.google.ca/scholar?q=Undirected+Single-Source+Shortest+Paths+with+Positive+Integer+Weights+in+Linear+Time&btnG=&hl=en&as_sdt=0%2C5

Shortest paths algorithms: **Theory** and **experimental evaluation**
BV Cherkassky, AV Goldberg, T Radzik - Mathematical programming, 1996 - Springer
Cited by 781 Related articles All 36 versions CiteSave

From: http://scholar.google.ca/scholar?q=Shortest+paths+algorithms%3A+Theory+and+experimental+evaluation&btnG=&hl=en&as_sdt=0%2C5

— bucket methods can be better than heaps

Computing the **shortest path**: A* **search meets graph theory**
AV Goldberg, C Harrelson - Proceedings of the sixteenth annual ACM- …, 2005 - dl.acm.org
Cited by 462 Related articles All 20 versions CiteSave

From: http://scholar.google.ca/scholar?q=Computing+the+Shortest+Path%3A+A%E2%88%97+Search+Meets+Graph+Theory&btnG=&hl=en&as_sdt=0%2C5

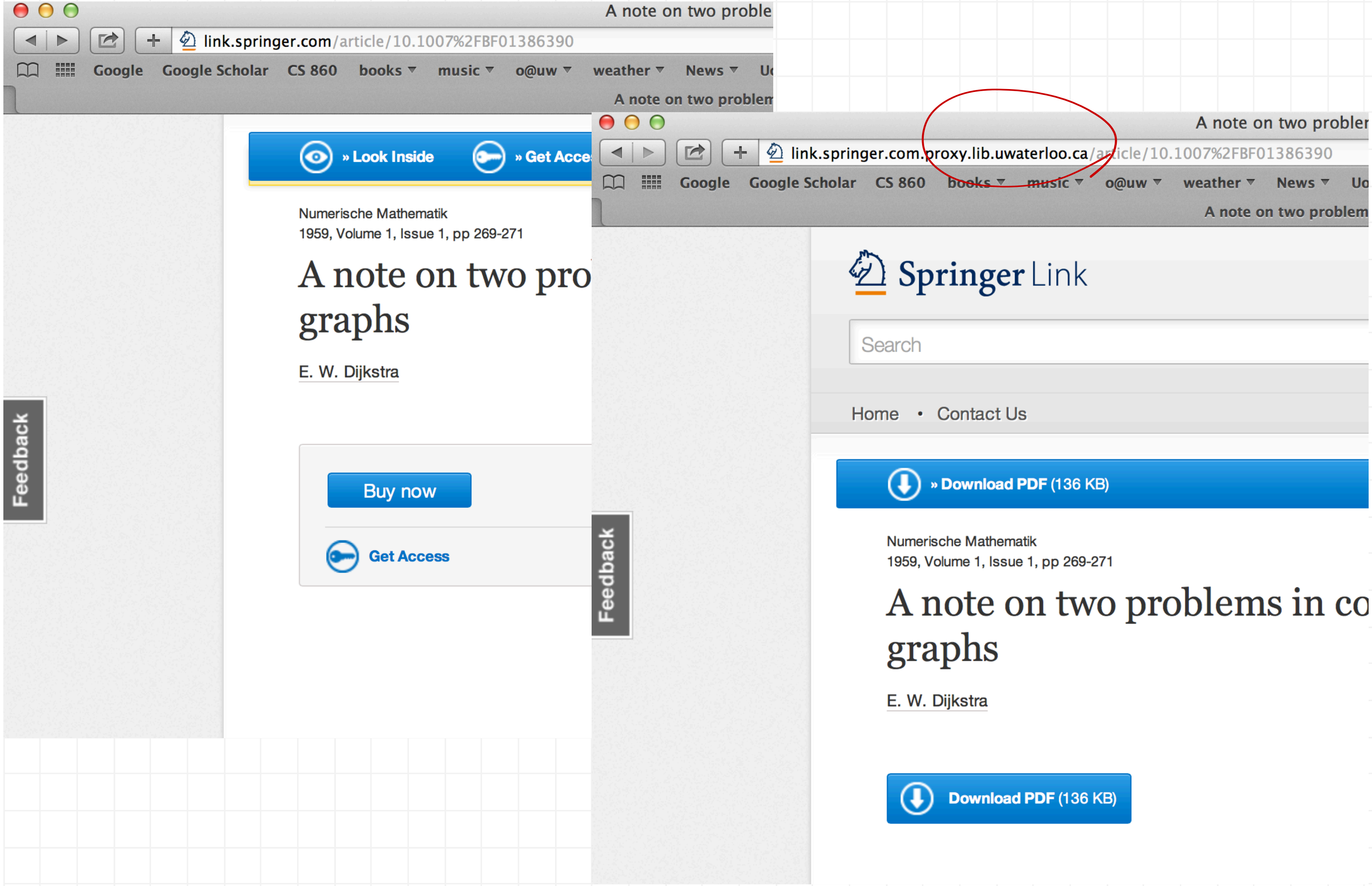
— builds on bi-directional Dijkstra

[HTML] **Trans-dichotomous algorithms** for **minimum spanning trees** and **shortest paths**
ML Fredman, DE Willard - Journal of Computer and System Sciences, 1994 - Elsevier
Cited by 282 Related articles All 4 versions CiteSave

From: http://scholar.google.ca/scholar?q=Trans-dichotomous+Algorithms+for+Minimum+Spanning+Trees+and+Shortest+Paths&btnG=&hl=en&as_sdt=0%2C5

— data structure
— beyond RAM model.

accessing papers from home



link.springer.com/article/10.1007%2FBF01386390

Numerische Mathematik
1959, Volume 1, Issue 1, pp 269-271

A note on two pro
graphs

E. W. Dijkstra

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