CS 763 Fall 2022
A. Lubiw

due Wed. Nov. 23 if you are presenting Nov. 28, 30, or Dec. 5,
due Wed. Nov. 30 if you are presenting Nov. 21 or 23.

ASSIGNMENT 5

ACKNOWLEDGE YOUR SOURCES.

1. [10 marks] The **Smallest Triangle Problem** is to find, given a set S of n points in the plane, three points of S that determine the smallest area triangle. This is a generalization of the problem of testing if three points are collinear (because collinear points give a triangle of area 0). In this question you will use duality and arrangements to solve the Smallest Triangle Problem in $O(n^2)$ time. Assume that no two points of S have the same x-coordinate.

   (a) First suppose that two points a and b in S are fixed. The goal is to find the point $c \in S$ to minimize the area of triangle abc. (Yes, the problem can then trivially be solved in linear time, but we'll still look at the dual.) Let ℓ be the line through a and b. For any point $p \in S$, let ℓ_p be the line through p parallel to ℓ.

   Prove that c is the point such that ℓ_c is closest to ℓ. (Don't belabour this, it's high-school geometry.)

   Express this in terms of the dual, with lines a^*, b^*, c^* and points ℓ^*, ℓ_c^*. (Remember what happens to parallel lines when you dualize. Draw a figure.)

   Describe how to find c^* in the dual arrangement.

   (b) Give an $O(n^2)$ time algorithm to solve the Smallest Triangle Problem by constructing the dual arrangement. Give a high-level description of your algorithm, not detailed pseudo-code. **Hint:** You will need to revisit the algorithm that constructs the arrangement so you can collect the information that was useful in part (a).

2. [10 marks] Design a polynomial time algorithm to find a path from point s to point t among disjoint disc obstacles in the plane. Do not invest too much energy in the best run time, but do be sure to justify correctness. Give a high-level description of your algorithm, not detailed pseudo-code. You may assume some geometric primitives for pairs of discs without giving details—but be sure to say what geometric primitives you assume.