ASSIGNMENT 2

ACKNOWLEDGE YOUR SOURCES.

1. [10 marks] Convex Hull in 2D.
   
   (a) [5 marks] Give a linear-time algorithm to find the convex hull of the union of two convex polygons in the plane. Your algorithm should handle any pair of convex polygons, disjoint, nested, etc.
   
   **Hint.** Think about which of the convex hull algorithms we covered in class can be used for this purpose. Your algorithm should be high-level and brief, but be sure you address correctness and runtime.
   
   (b) [5 marks] Given a set $S$ of points in the plane, its “onion peeling” consists of a sequence $H_1, \ldots, H_k$ of convex polygons, where $H_1$ is the convex hull of $S$, $H_2$ is the convex hull of $S$ with the points of $H_1$ removed, etc. Draw yourself a picture. The inner-most points are sometimes used as the “centers” of the point set.
   
   Give an $O(n^2)$ time algorithm to find the onion peeling using gift-wrapping (Jarvis’s algorithm). Your algorithm should be high-level and brief, but be sure you address correctness and runtime.
   
   [For your information, but irrelevant for this assignment: There is an $O(n \log n)$ time onion peeling algorithm due to Chazelle.]

2. [10 marks] The *sharpness* of a vertex $v$ of a convex polyhedron in $\mathbb{R}^3$ is $2\pi$ minus the sum of the face angles at $v$. For example, a vertex of a cube has 3 face angles each of $\pi/2$, so its sharpness is $2\pi - 3\pi/2 = \pi/2$. Prove that the sum of the sharpness of all the vertices of a convex polyhedron is $4\pi$.
   
   You will need Euler’s formula and one other ingredient.