CS 763 Fall 2022
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due Wednesday October 5

ASSIGNMENT 2

ACKNOWLEDGE YOUR SOURCES.

1. [10 marks] Convex Hull in 2D.

   (a) [5 marks] Give a linear-time algorithm to find the convex hull of the union of two convex polygons in the plane. Your algorithm should handle any pair of convex polygons, disjoint, nested, etc.

   Hint. Think about which of the convex hull algorithms we covered in class can be used for this purpose. Your algorithm should be high-level and brief, but be sure you address correctness and runtime.

   (b) [5 marks] Given a set S of points in the plane, its "onion peeling" consists of a sequence $H_1, \ldots, H_k$ of convex polygons, where H_1 is the convex hull of S, H_2 is the convex hull of S with the points of H_1 removed, etc. Draw yourself a picture. The inner-most points are sometimes used as the "centers" of the point set.

   Give an $O(n^2)$ time algorithm to find the onion peeling using gift-wrapping (Jarvis's algorithm). Your algorithm should be high-level and brief, but be sure you address correctness and runtime.

   [For your information, but irrelevant for this assignment: There is an $O(n \log n)$ time onion peeling algorithm due to Chazelle.]

2. [10 marks] The *sharpness* of a vertex v of a convex polyhedron in $\mathbb{R}^3$ is 2π minus the sum of the face angles at v. For example, a vertex of a cube has 3 face angles each of $\pi/2$, so its sharpness is $2\pi - 3\pi/2 = \pi/2$. Prove that the sum of the sharpness of all the vertices of a convex polyhedron is 4π.

   You will need Euler's formula and one other ingredient.