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Algorithms for Polynomials in Legendre-Sobolev Bases

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Earlier work [1] has shown how handwritten characters may be represented as plane curves with $x(\lambda)$ and $y(\lambda)$ polynomial functions, and how efficient recognition can be achieved when the polynomials are written in a Legendre-Sobolev (LS) basis. It is therefore interesting to be able to perform various symbolic-numeric polynomial operations directly in LS bases. We find it is sufficient to work with basis polynomials orthogonal with respect to inner products of the form studied by Althammer [2],

$$\langle f, g\rangle = \int_{-1}^{1} f(\lambda)g(\lambda)\mathrm{d}\lambda + \mu \int_{-1}^{1} f'(\lambda)g'(\lambda)\mathrm{d}\lambda,\ \mu \geq 0$$

We show how functional approximations may be constructed from moments integrated in real time, how to compute derivatives, roots and polynomial GCD in LS bases by linear algebra methods.

Keywords
symbolic-numeric algorithms, polynomial algebra, Legendre-Sobolev polynomials, mathematical handwriting recognition

References
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