

CS848

Background: crypto and db basics

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One-time pad



- Let $\{0, 1\}^\lambda$ denote the set of λ -bit binary strings

KeyGen:

$k \leftarrow \{0, 1\}^\lambda$
return k

Enc($k, m \in \{0, 1\}^\lambda$):

return $k \oplus m$

Dec($k, c \in \{0, 1\}^\lambda$):

return $k \oplus c$

A 'good' encryption scheme

"an encryption scheme is a good one if its ciphertexts look like random junk to an attacker"

Let CTXT be a function callable by an adversary who can choose m and who sees c

CTXT(m):

```
 $k \leftarrow \{0, 1\}^\lambda$  // KeyGen of OTP  
 $c := k \oplus m$  // Enc of OTP  
return  $c$ 
```

vs.

CTXT(m):

```
 $c \leftarrow \{0, 1\}^\lambda$  // C of OTP  
return  $c$ 
```

Real vs
Random

"an encryption scheme is a good one if encryptions of m_L look like encryptions of m_R to an attacker"

Let EAVESDROP be a function callable by an adversary who chooses m_L & m_R and sees c

EAVESDROP(m_L, m_R):

```
 $k \leftarrow \{0, 1\}^\lambda$  // KeyGen of OTP  
 $c := k \oplus m_L$  // Enc of OTP  
return  $c$ 
```

EAVESDROP(m_L, m_R):

```
 $k \leftarrow \{0, 1\}^\lambda$  // KeyGen of OTP  
 $c := k \oplus m_R$  // Enc of OTP  
return  $c$ 
```

Left vs right

Basics of provable security: Interchangeability

Let \mathcal{L} be a library of functions that make random choices and let \mathcal{A} be a calling program (callable by an adversary)

\mathcal{L}
$\text{CTXT}(m):$
$k \leftarrow \{0, 1\}^\lambda$
$c := k \oplus m$
return c

$\mathcal{A}:$
$m \leftarrow \{0, 1\}^\lambda$
$c := \text{CTXT}(m)$
return $m \stackrel{?}{=} c$

then $\Pr[\mathcal{A} \diamond \mathcal{L} \Rightarrow \text{true}] = 1/2^\lambda$

Let $\mathcal{L}_{\text{left}}$ and $\mathcal{L}_{\text{right}}$ be two libraries that have the same interface. We say that $\mathcal{L}_{\text{left}}$ and $\mathcal{L}_{\text{right}}$ are **interchangeable**, and write $\mathcal{L}_{\text{left}} \equiv \mathcal{L}_{\text{right}}$, if for all programs \mathcal{A} that output a boolean value,

$$\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{left}} \Rightarrow \text{true}] = \Pr[\mathcal{A} \diamond \mathcal{L}_{\text{right}} \Rightarrow \text{true}].$$

Basics of provable security: proving 'insecurity'

For OTP (and \mathcal{A} from last slide), $\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{left}} \Rightarrow \text{true}] = \Pr[\mathcal{A} \diamond \mathcal{L}_{\text{right}} \Rightarrow \text{true}] = 1/2^\lambda$

```

 $\mathcal{L}_{\text{otp-real}}$ 
-----
EAVESDROP( $m \in \{0, 1\}^\lambda$ ):
   $k \leftarrow \{0, 1\}^\lambda$  // OTP.KeyGen
  return  $k \oplus m$  // OTP.Enc(k, m)
    
```

=

```

 $\mathcal{L}_{\text{otp-rand}}$ 
-----
EAVESDROP( $m \in \{0, 1\}^\lambda$ ):
   $c \leftarrow \{0, 1\}^\lambda$  // OTP.C
  return  $c$ 
    
```

Just one calling program enough to prove insecurity

What about??

```

 $\mathcal{L}_{\text{ots-real}}^\Sigma$ 
-----
CTXT( $m \in \{0, 1\}^\lambda$ ):
   $k \leftarrow \{0, 1\}^\lambda$  //  $\Sigma$ .KeyGen
   $c := k \& m$  //  $\Sigma$ .Enc
  return  $c$ 
    
```

```

 $\mathcal{L}_{\text{ots-rand}}^\Sigma$ 
-----
CTXT( $m \in \{0, 1\}^\lambda$ ):
   $c \leftarrow \{0, 1\}^\lambda$  //  $\Sigma$ .C
  return  $c$ 
    
```

```

 $\mathcal{A}$ :
-----
 $c := \text{CTXT}(0^\lambda)$ 
return  $c \stackrel{?}{=} 0^\lambda$ 
    
```

$\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{ots-real}}^\Sigma \Rightarrow \text{true}] = 1.$ $\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{ots-rand}}^\Sigma \Rightarrow \text{true}] = 1/2^\lambda$

Indistinguishability

Two libraries $\mathcal{L}_{\text{left}}$ and $\mathcal{L}_{\text{right}}$ are *indistinguishable* if for all polynomial-time calling programs \mathcal{A} ,

$$\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{left}} \Rightarrow 1] \approx \Pr[\mathcal{A} \diamond \mathcal{L}_{\text{right}} \Rightarrow 1] \quad \rightarrow \quad \mathcal{L}_{\text{left}} \approx \mathcal{L}_{\text{right}}$$

Advantage or bias of an adversary: $|\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{left}} \Rightarrow 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_{\text{right}} \Rightarrow 1]|$

Two libraries are indistinguishable if the adversary's advantage is negligible (some small number like $1/2^{128}$)

Pseudorandom generators

- A pseudorandom generator (PRG) is a deterministic function G whose outputs are longer than its inputs

Let $G : \{0, 1\}^\lambda \rightarrow \{0, 1\}^{\lambda+\ell}$ be a deterministic function with $\ell > 0$. We say that G is a **secure pseudorandom generator (PRG)** if $\mathcal{L}_{\text{prg-real}}^G \approx \mathcal{L}_{\text{prg-rand}}^G$, where:

$\mathcal{L}_{\text{prg-real}}^G$
<u>QUERY():</u> $s \leftarrow \{0, 1\}^\lambda$ return $G(s)$

$\mathcal{L}_{\text{prg-rand}}^G$
<u>QUERY():</u> $r \leftarrow \{0, 1\}^{\lambda+\ell}$ return r

The value ℓ is called the **stretch** of the PRG. The input to the PRG is typically called a **seed**.

Pseudorandom Functions (PRFs)

- PRFs are functions of the form

for $x \in \{0, 1\}^{in}$:
$T[x] \leftarrow \{0, 1\}^{out}$
LOOKUP($x \in \{0, 1\}^{in}$):
<hr/>
return $T[x]$

Let $F : \{0, 1\}^\lambda \times \{0, 1\}^{in} \rightarrow \{0, 1\}^{out}$ be a deterministic function. We say that F is a secure **pseudorandom function (PRF)** if $\mathcal{L}_{\text{prf-real}}^F \approx \mathcal{L}_{\text{prf-rand}}^F$, where:

$\mathcal{L}_{\text{prf-real}}^F$
$k \leftarrow \{0, 1\}^\lambda$
LOOKUP($x \in \{0, 1\}^{in}$):
<hr/>
return $F(k, x)$

$\mathcal{L}_{\text{prf-rand}}^F$
$T := \text{empty assoc. array}$
LOOKUP($x \in \{0, 1\}^{in}$):
<hr/>
if $T[x]$ undefined:
$T[x] \leftarrow \{0, 1\}^{out}$
return $T[x]$

Pseudorandom Permutations (PRPs) or block ciphers

- Let K be the keyspace, X the message or input space and Y the output space.

- A PRF, F :

$$F : K \times X \rightarrow Y$$

- A PRP, E :

$$E : K \times X \rightarrow X$$

- A PRP is required to be bijective, and to have an efficient inversion function, PRP^{-1} .
- E is also called block cipher: E corresponds to encryption, E^{-1} corresponds to decryption, and all outputs of E look pseudorandom

Security Against Chosen Plaintext Attacks

Let Σ be an encryption scheme. We say that Σ has **security against chosen-plaintext attacks (CPA security)** if $\mathcal{L}_{\text{cpa-L}}^\Sigma \approx \mathcal{L}_{\text{cpa-R}}^\Sigma$, where:

$\mathcal{L}_{\text{cpa-L}}^\Sigma$
$k \leftarrow \Sigma.\text{KeyGen}$
$\text{EAVESDROP}(m_L, m_R \in \Sigma.\mathcal{M}):$
<hr/>
$c := \Sigma.\text{Enc}(k, m_L)$
return c

$\mathcal{L}_{\text{cpa-R}}^\Sigma$
$k \leftarrow \Sigma.\text{KeyGen}$
$\text{EAVESDROP}(m_L, m_R \in \Sigma.\mathcal{M}):$
<hr/>
$c := \Sigma.\text{Enc}(k, m_R)$
return c

Deterministic encryption like block ciphers are not CPA-secure!

One simple (nonce-based) randomized encryption scheme using PRFs

Let F be a secure PRF with $\text{in} = \lambda$. Define the following encryption scheme based on F :

$$\begin{aligned}\mathcal{K} &= \{0, 1\}^\lambda \\ \mathcal{M} &= \{0, 1\}^{\text{out}} \\ \mathcal{C} &= \{0, 1\}^\lambda \times \{0, 1\}^{\text{out}}\end{aligned}$$

KeyGen:

$$\begin{aligned}k &\leftarrow \{0, 1\}^\lambda \\ \text{return } k\end{aligned}$$

Enc(k, m):

$$\begin{aligned}r &\leftarrow \{0, 1\}^\lambda \\ x &:= F(k, r) \oplus m \\ \text{return } (r, x)\end{aligned}$$

Dec($k, (r, x)$):

$$\begin{aligned}m &:= F(k, r) \oplus x \\ \text{return } m\end{aligned}$$

This scheme is also **'symmetric'**: same secret key is used for encryption and decryption

Public-key or asymmetric encryption

- Goal: make encryption key public, so that anyone can send an encryption to the owner of that key, even if the two users have never spoken before and have no shared secrets.
- The decryption key is private, so that only the designated owner can decrypt.

Public-key or asymmetric encryption

Let Σ be a public-key encryption scheme. Then Σ is **secure against chosen-plaintext attacks (CPA secure)** if $\mathcal{L}_{\text{pk-cpa-L}}^{\Sigma} \approx \mathcal{L}_{\text{pk-cpa-R}}^{\Sigma}$, where:

$\mathcal{L}_{\text{pk-cpa-L}}^{\Sigma}$
$(pk, sk) \leftarrow \Sigma.\text{KeyGen}$
<u>GETPK():</u> return pk
<u>CHALLENGE($m_L, m_R \in \Sigma.\mathcal{M}$):</u> return $\Sigma.\text{Enc}(pk, m_L)$

$\mathcal{L}_{\text{pk-cpa-R}}^{\Sigma}$
$(pk, sk) \leftarrow \Sigma.\text{KeyGen}$
<u>GETPK():</u> return pk
<u>CHALLENGE($m_L, m_R \in \Sigma.\mathcal{M}$):</u> return $\Sigma.\text{Enc}(pk, m_R)$

Example – ElGamal encryption

Let m be the message we want to encrypt

Encryption key: (s, p) such that $p = g^s$

Encryption procedure:

$E(p, m)$:

Pick a random number r

Return $(g^r, m * p^r)$



Call it A



Call it B

Decryption procedure:

$$D(sk, E(p, m)) = B (A^{sk})^{-1}$$

$$= D(s, (g^r, m * p^r))$$

$$= (m * p^r) * ((g^r)^s)^{-1}$$

$$= (m * (g^s)^r * (g^{rs})^{-1})$$

$$= m$$

Homomorphic encryption

- Allows computing – addition and multiplication – on encrypted data and result is also encrypted

ElGamal encryption is partially homomorphic – supports homomorphic **multiplication**

p is public key



$$\begin{aligned} E(m1) * E(m2) &= (g^{r1}, m1 * p^{r1}) * (g^{r2}, m2 * p^{r2}) \\ &= (g^{r1 + r2}, (m1 * m2) * p^{r1 + r2}) \\ &= E(m1 * m2) \end{aligned}$$

Homomorphic encryption (additive)

Modified El Gamal encryption procedure:

$E(p, m)$:

Pick a random number r

Return $(g^r, g^m * p^r)$

This version supports homomorphic additions

p is public key



$$\begin{aligned} E(m1) * E(m2) &= (g^{r1}, g^{m1} * p^{r1}) * (g^{r2}, g^{m2} * p^{r2}) \\ &= (g^{r1 + r2}, g^{(m1 + m2)} * p^{r1 + r2}) \\ &= E(m1 + m2) \end{aligned}$$

Order preserving encryption (OPE)

- If in plaintext, $x < y$, then encrypting these values using an OPE ensures that for any secret key k ,

$$OPE_k(x) < OPE_k(y)$$

- OPEs help serve range queries on encrypted data but it leaks more information than non-deterministic encryption schemes

Database basics

Basic db queries

- Simple key-value stores support single key GET and PUT queries

More complex queries

- Selection

`SELECT * from T1 where age = 20`

- Range

Select name, salary from T1 where `age > 20` and `age < 60`

- Aggregates

- Select `AVG(salary)` from T1 where `age > 20`
- AVG, MIN, MAX, COUNT, SUM

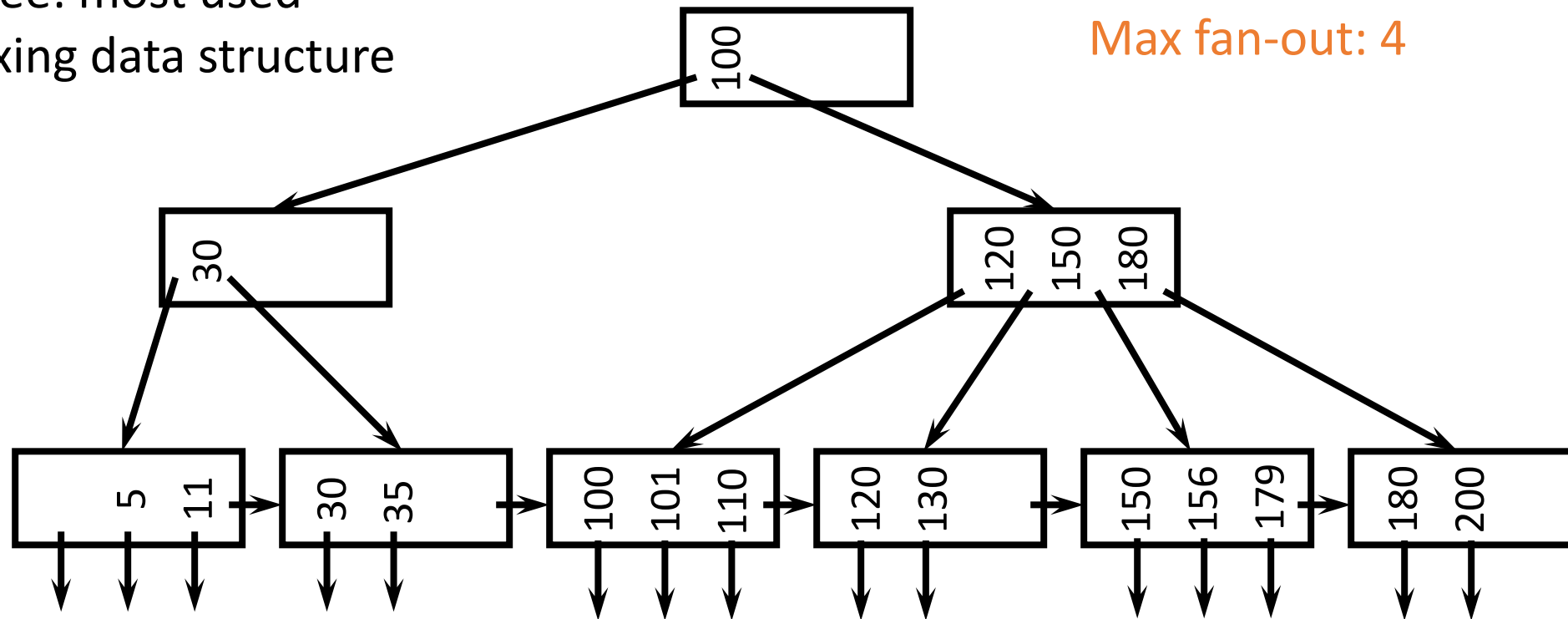
- Join

Select * from T1 `JOIN` T2 `ON` T1.<column_name> = T2.<column_name>

Indexes

data structures that help find records faster by pointing to loc. where a record is stores

B+ tree: most used indexing data structure



Query processing

- 3 most common ways: scan based, index based, sort based.
- Example query: Select *name* from T1 where *age* < 40 and *age* > 20
- Scan: Assumes no indexes. Linearly scan T1 and check condition on each row. If true, add to output
- Index: Assumes index on *age*. Use the index to fetch all primary keys (pk) satisfying the condition. Then just query those pks from table
- Sort: Assumes data is stored in a sorted order on *age*. Scan only the relevant blocks (stop when conditions are satisfied)

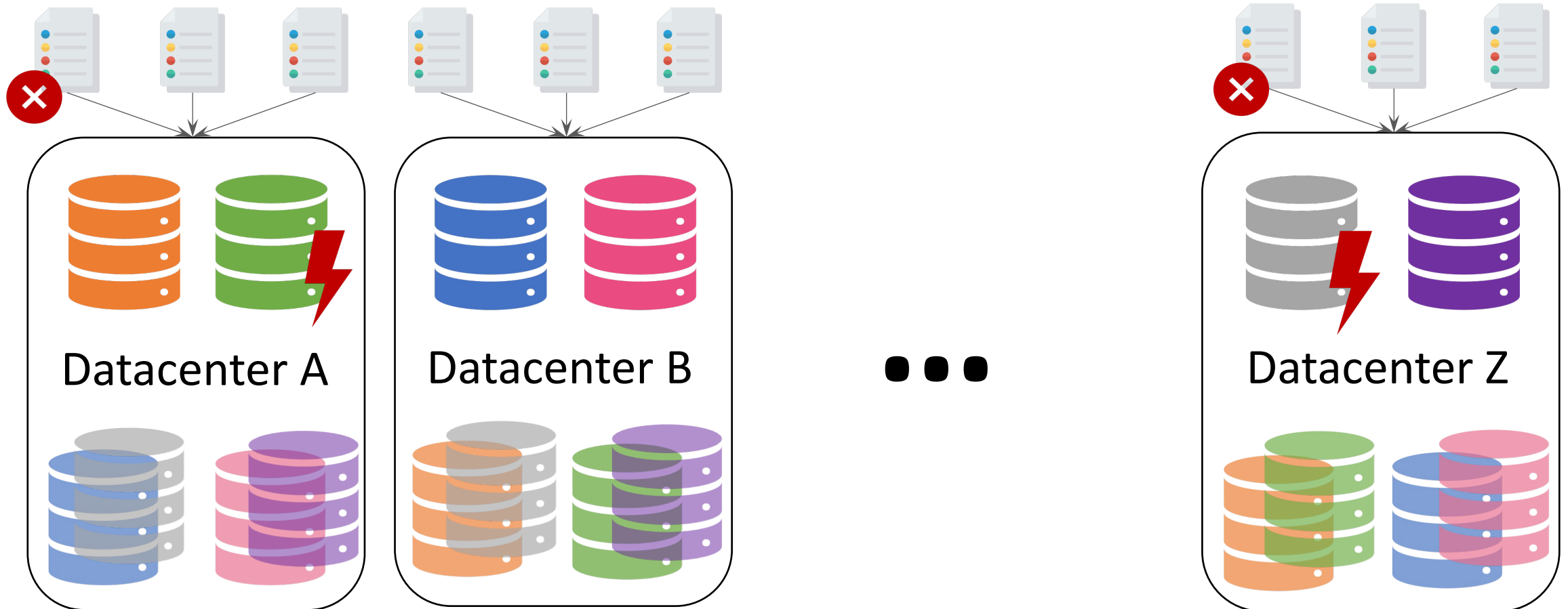
Transactions

- Encapsulates a number of operations – such as select, update, insert – in a single logical unit
- Database systems must ensure **ACID**
 - **Atomicity**: TX's are either completely done or not done at all
 - **Consistency**: TX's should leave the database in a consistent state
 - **Isolation**: TX's must behave as if they are executed in isolation
 - **Durability**: Effects of committed TX's are resilient against failures

Scalability and fault tolerance

By sharding and replicating the data

Application Access Tier




Conclusion

We looked at some basics of cryptography and database concepts

One or more of these techniques will be employed in each paper we will study

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Questions?