CS 360 Winter 2016
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Problem Session 4

The problems below are a rough indication of the material to be covered in the session. The amount of time spent on each topic will depend on the needs of those attending. For discussion of solutions, attend the session or talk to course personnel in office hours.

1. [**Pumping lemma for CFLs**] Use the pumping lemma for CFLs to show that L is not context-free, for $L = \{a^i b^i c^j | j \leq i\}$.

2. [**Construction of a transition diagram of a TM**] Give a high-level description and transition diagram of a deterministic Turing machine that accepts the language $L = \{a^i b^{2i} \mid i \geq 0\}$. Demonstrate how your Turing machine works by showing the sequence of transitions taken to accept the string abb.

3. [**Construction of a high-level description of a TM**] Give an informal but detailed description of a *nondeterministic* Turing machine that on input $cw_1cw_2c \ldots cw_kdx$, where the w_i's and x are non-empty strings over $\{a, b\}$ and k is any positive integer, accepts if and only if $x = w_i$ for some i. You may use multiple tapes if you wish.

   Do not draw a transition diagram; just give a written description.

4. We define the *shuffle* of two strings x and y, denoted $x\mathcal{S}y$, to be the set of strings formed by interleaving the symbols of x and y, any number at a time. That is, $x\mathcal{S}y = \{x_1y_1 \ldots x_my_m \mid x = x_1 \ldots x_m, y = y_1 \ldots y_m\}$ and for each i, x_i and y_i are each strings of any length, zero or greater. For two languages L_1 and L_2, we define $L_1\mathcal{S}L_2 = \cup_{x \in L_1, y \in L_2} x\mathcal{S}y$. Prove that the class of r.e. languages is closed under shuffle.