

Problem Session 3

The problems below are a rough indication of the material to be covered in the session. The amount of time spent on each topic will depend on the needs of those attending. For discussion of solutions, attend the session or talk to course personnel in office hours.

1. [**Construction of a CFG**] Construct a CFG for the language $L = \{a^i b^j \mid i \leq 2j\}$; demonstrate the derivation of $aaaaabbb$ using your grammar.

2. [**Chomsky Normal Form**] Find a grammar G' in Chomsky normal form equivalent to the one given below. Show the results of each step taken.

$$S \rightarrow ABD \mid BCD$$

$$A \rightarrow aA \mid bB \mid \epsilon$$

$$B \rightarrow c \mid Ab \mid AA \mid D$$

$$C \rightarrow AB \mid aba$$

$$D \rightarrow aAB \mid aCB \mid d$$

3. [**Construction of a PDA**] Construct a PDA that accepts the language L by empty stack, where $L = \{a^i b^j c^k \in \{a, b, c\}^* \mid i = 2j \text{ or } j = 2k \text{ and } i, j, k \geq 1\}$. Demonstrate how the string $x = a^3 b^4 c^2$ is accepted by your PDA.

4. [**Proving $L(G) = L$**] Prove that $L(G) = \{w \in \{0, 1\}^* \mid n_0(w) = n_1(w)\}$, for G as defined below:

$$S \rightarrow AB \mid BA \mid \epsilon$$

$$A \rightarrow S0S$$

$$B \rightarrow S1S$$