CS 360 Winter 2016
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Assignment 4

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

1. *[15 marks]* Prove that $L = \{s\textsf{doubleback}(s)s \mid s \in \{a,b\}^*\}$ is not context-free. Use the pumping lemma for context-free languages. Recall that $\textsf{doubleback}(s)$ is formed by reversing and doubling the characters in s.

2. *[15 marks]* Show that $\textsf{twopiece}(L_1, L_2)$ is context-free whenever L_1 is context-free and L_2 is regular, where $\textsf{twopiece}(L_1, L_2) = \{xy \mid x \in L_1 \setminus L_2, y \in L_2\}$.

   Give a high-level overview and a detailed construction, and explain why your construction works (that is, why the language of what you construct is what you claim it to be).

   Hint: Use the fact that there exist a PDA accepting L_1 and a DFA accepting L_2.

3. *[15 marks]* Give the transition diagram of a one-tape deterministic Turing machine that accepts the language $L = \{s\textsf{doubleback}(s) \mid s \in \{a,b\}^*\}$. Demonstrate how your Turing machine works by showing the sequence of transitions taken to accept the string $abbbaa$.

4. *[15 marks]* Give both a high-level overview and a detailed description of a nondeterministic multitape Turing machine $L = \{sx\textsf{doubleback}(s)y \mid s, x, y \in \{a,b\}^*, |s| > 0\}$. Notice that this is not the same language as in the previous question. For full marks, make use of nondeterminism, and explain how it is being used. It may be easier if you break your detailed description into subtasks.

5. *[15 marks]* For any languages L_1 and L_2, we define $\textsf{split}(L_1, L_2) = \{xy \mid x \in L_1 \setminus L_2, y \in L_2, |x| = |y|\}$. Notice that this is not the same as $\textsf{twopiece}$, as it has another condition.

   (a) *[12 marks]* Show that if L_1 and L_2 are recursive, then $\textsf{split}(L_1, L_2)$ also recursive. Justify your answer by constructing a Turing machine T such that $L(T) = \textsf{split}(L)$ and T halts on all inputs.

   (b) *[3 marks]* Can you use the same construction to show that $\textsf{split}(L_1, L_2)$ is r.e. when L_1 and L_2 are both r.e.? If so, explain why. If not, give the largest classes C_1 and C_2 of languages for L_1 and L_2 such that if L_1 is in C_1 and L_2 is in C_2, then $\textsf{split}(L_1, L_2)$ is r.e.. Justify your answer.