CS 360 Winter 2016
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Assignment 3

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

1. *[20 marks]* For each of the languages given below, give a context-free grammar that generates the language. Briefly justify your answers, for example, by describing what each variable is intended to generate (formal proofs are not required). Demonstrate the use of your grammar by giving a derivation of the string listed.

   (a) *[10 marks]* $L = \{u\mathsf{doubleback}(u) \mid u \in \{a,b\}^*\}$, where $\mathsf{doubleback}(u)$ is formed by reversing and doubling the characters in u. For example, $\mathsf{doubleback}(ab) = bbaa$. Example string: $abbbbbbaa$.

   (b) *[10 marks]* $L = \{u \in \{a,b\}^* \mid$ no character in an even position is an a or no character in an odd position is a $b\}$, where the first position of a character in a string is defined to be 1 (and hence odd). Example string: $abbba$.

2. *[18 marks]* Prove that $L = L(G)$ where the grammar G is given below and $L = \{w \in \{a,b\}^* \mid |w| > 0$ and $n_a(w) = n_b(w)\}$.

$$\begin{aligned} S &\rightarrow Ab \mid Ba \\ A &\rightarrow a \mid Sa \mid AAB \\ B &\rightarrow b \mid Sb \mid BBA \end{aligned}$$

3. *[17 marks]* Each of the subquestions below specifies A and B, for which you are to determine whether all, some (but not all), or none of the possible items A satisfy condition B. Briefly justify each answer. For the answer "some", give two examples, one which satisfies the condition and one which does not. For answers "all" and "none", give arguments to support your claims.

   (a) *[8 marks]* A is a context-free grammar in Chomsky Normal Form and B is the condition "A is ambiguous"

   (b) *[5 marks]* A is a deterministic PDA that accepts by empty stack and B is the condition "the PDA formed from A by the algorithm that converts a PDA accepting by empty stack to a PDA accepting by final state is a determinstc PDA"

   (c) *[4 marks]* A is a PDA and B is the condition "there exists a PDA P such that $L(A) = L(P)$ and either P has at most three states or P is deterministic"

4. *[20 marks]* For each of the languages listed below, give a PDA that accepts the language. Include the transition diagram of your PDA and also an informal description of how your PDA works. Demonstrate how your PDA works by showing the sequence of transitions taken to accept the string listed. For full marks, be sure to use the type of PDA specified.

   (a) *[10 marks]* Create a PDA that accepts by final state for the language $L = \{w \in \{a, b\}^* \mid n_a(w) \bmod 3 \equiv 1 \text{ and } n_b(w) \text{ is even}\}$. Example string: *bbabaaba*.

   (b) *[10 marks]* Create a PDA that accepts by empty stack for the language $L = \{u\mathsf{doubleback}(u) \mid u \in \{a, b\}^*\}$, where $\mathsf{doubleback}(u)$ is formed by reversing and doubling the characters in u. For example, $\mathsf{doubleback}(ab) = bbaa$. Example string: *baaabb*.