

## Assignment 2

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

1. [9 marks] Use any combination of regular expressions and closure properties to show that the language  $L$  is regular, where  $L = \{w \in \{a, b\}^* \mid w \text{ does not contain } aba \text{ as a substring, } w \text{ contains at least three } b\text{'s, and } w \text{ starts and ends with } a\}$ .
2. [9 marks] Give a regular expression that generates the language  $L = \{w \in \{a, b\}^* \mid w \text{ contains the substring } aab \text{ but does not contain the substring } baa\}$ . Briefly justify your answer.
3. [18 marks] Prove that each of the languages listed below is not regular, making use of the pumping lemma.

(a) [9 marks]  $L = \{u \in \{a, b\}^* \mid n_a(u) \leq 2n_b(u)\}$

(b) [9 marks]  $L = \{u\text{doubleback}(u) \mid u \in \{a, b\}^*\}$ , where  $\text{doubleback}(u)$  is formed by reversing and doubling the characters in  $u$ . For example  $\text{doubleback}(ab) = bbaa$ .

4. [14 marks] Suppose  $R_1$  and  $R_2$  are both regular languages over the alphabet  $\{a, b\}$ . For each subquestion, determine if the language defined is regular for all, some (but not all), or none of the possible choices for  $R_1$  and  $R_2$ . Justify your answer by providing proofs, examples, or counterexamples, as appropriate.

(a) [7 marks]  $S = \{w \in \{a, b\}^* \mid n_a(w) \leq 2n_b(w) \text{ and } w \in R_1\}$

(b) [7 marks]  $S = \{w \in \{a, b\}^* \mid w = xy, x \in R_1, y \in R_2, n_a(w) \text{ is even}\}$

5. [10 marks] The class of regular languages is closed under the operation  $\text{extend}$ , where  $\text{extend}(L) = \{xy \mid x \in L, y \in \Sigma^* \text{ for } L \text{ a language over } \Sigma\}$ . This can be proved by constructing a DFA, by constructing a regular expression, or by a combination of constructions and closure properties. Briefly outline how each of the three methods could be used. Formal proofs are not needed for this question.