CS 360 Winter 2016
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Assignment 1

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

When you are asked to create a machine, be sure to include each of the following:

- a brief explanation of your construction in English, detailing what states represent, how strings are accepted, and why the language accepted by the machine is L, and
- a transition diagram in which the initial state is clearly labeled and each accepting state is drawn with two concentric circles.

Your explanation should give the ideas and intuition behind the construction, not a detailed listing of the transitions (which would only be a repetition of what is represented in the diagram).

1. *[30 marks]* Consider the following language: $L = \{a^i b^j c^k \mid i, j, k \geq 0\} \cup \{c^i b^j a^k \mid i, j, k \geq 0\}$

   You will be creating a DFA that accepts L and also an NFA or ϵ-NFA that accepts L. Do not use the same automaton for the two parts of the question.

   (a) *[15 marks]* Create a DFA D such that $L(D) = L$.

   Be sure to draw a *complete* automaton, so that *every state has an arrow out for each symbol in the alphabet.*

   Try to keep the number of states in your DFA as small as possible.

   (b) *[12 marks]* Create an ϵ-NFA or NFA N such that $L(N) = L$.

   (c) *[3 marks]* Briefly discuss why you chose an NFA instead of an ϵ-NFA or why you chose an ϵ-NFA instead of an NFA in part (b).

2. *[15 marks]* Create an NFA N that accepts the language L of strings over $\{a, b\}$ that contain at least one occurrence of aaa but no occurrences of aab.

   For the string $x = aaba$, not in the language, show all possible sequences of states to confirm that the string is not accepted.

3. *[12 marks]* Let L be the language of strings w over $\{0, 1\}$ such that every non-empty even-length prefix of w ends with a 1, and let M be the language defined by the following recursive definition:

   Base case $\epsilon \in M$; $0 \in M$; $1 \in M$

   General case For any $w \in M$, $01w$, $11w$ are in M.

   Prove that $L = M$.

4. *[18 marks]* Given DFA $D_1 = (Q_1, \Sigma, q_1, \delta_1, F_1)$, create a DFA $D = (Q, \Sigma, q_1, \delta, F)$ that accepts the language $L = \{xy \mid x \in L(D_1), y \in \Sigma^*\}$. Then, prove that $L(D) = L$.