CS 360
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Pumping Lemma notes

In class we derived the Pumping Lemma as a statement about regular languages, of the form "If L is regular, then property P holds."

We observe that this is a statement of the form "If A, then B". When such a statement is true, its contrapositive ("If not B, then not A") is also true, but not necessarily its converse ("If B, then A"). Here the contrapositive of the statement about regular languages is "If property P does not hold, then L is not regular".

Thus, we have two statements that are contrapositives of each other and that are both true: the statement about regular languages (which we use in decision problems) and the Pumping Lemma (which we use to prove that a language is not regular).

For completeness, here are the two versions in detail. One can be derived from the other by observing that the negation of a universal statement is an existential statement and the negation of an existential statement is a universal statement.

Statement about regular languages: For any regular language L, there exists an integer $n > 0$ (depending on L) such that for any $w \in L$, $|w| \geq n$, there exists a decomposition $w = xyz$ with $|xy| \leq n$, $|y| > 0$ such that for any integer $k \geq 0$, $xy^kz \in L$.

Pumping Lemma: If for all integers $n > 0$ there exists a $w \in L$, $|w| \geq n$ such that for all decompositions $w = xyz$ satisfying the conditions $|xy| \leq n$ and $|y| > 0$, there exists an integer $k \geq 0$ such that $xy^kz \notin L$, then L is not regular.