Recipes for using closure properties in proofs

Proving a language L is in a class C:

- 1. Start with a set of languages known to be in C.
- 2. Use operations under which C is closed to create L from languages in the set.
- 3. As all languages in the set were known to be in C and C is closed under the operations used to construct L, we can conclude that L must be in C.



Proving a language L is NOT in a class C:

- 1. Start with a set (possibly empty) of languages known to be in C.
- 2. Choose a language K known to NOT be in C.
- 3. Use operations under which C is closed to create K from L and other languages in the set.
- 4. Using proof by contradiction, suppose that L were in C. Then since L and all languages in the set are in C and C is closed under the operations used to construct K, we can conclude that K must be in C. But K is not in C, and hence our assumption that Lwas in C must have been incorrect.

