

# Fundamentals of Physical Design

## Query Processing: First-Order Queries

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## ... the Story so Far:

- ① Integrity constraints and *simple index declarations* capture a variety of logical-to-physical mappings and physical designs: *physical data independence*,
- ② Most of *physical design* issues (including appropriate costs) can be captured in such a framework, and
- ③ *Conjunctive queries* can be compiled to low-level query plans.  
⇒ complex queries: “select block at a time” approach.

# Shortcomings

- ① General first-order queries: not satisfactory
- ② Complex schema: what to do with negations et al.?  
⇒ additional “rewriting rules”? (when is it enough?)
- ③ Completeness (can we find a rewriting if one exists)?  
⇒ conjunctive query over conjunctive materialized views  
... may need *negation* in the plan.

# Non-conjunctive Rewriting for CQ over CQ views

- Query:

$$Q(x, y) \equiv \exists z, v, u. R(z, x), R(z, v), R(v, u), R(u, y)$$

- Views (with “`index Vi () (x, y)`”):

$$V_1(x, y) \equiv \exists z, v. R(z, x), R(z, v), R(v, y)$$

$$V_2(x, y) \equiv \exists z. R(x, z), R(z, y)$$

$$V_3(x, y) \equiv \exists z, v. R(x, z), R(z, v), R(v, y)$$

- Rewriting (and a `plan`, assuming indices for the views):

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# First-order (FO) Query Language

## Syntax:

$Q ::= A \vee$	class access
$v.Pf1 = u.Pf2$	equation
$\text{true}$	singleton
$\text{from } Q_1, Q_2$	natural join
$\text{elim } v_1, \dots, v_k \ Q$	selection (distinct)
$\text{empty } v_1, \dots, v_k$	empty set
$Q_1 \cup Q_2$	union (union-compatible)
$Q_1 \setminus Q_2$	set difference (union-compatible)

... essentially an alternative syntax for First-order Formulae  
... restricted to *domain-independent* queries.

# Beth Definability

- (1)  $\mathcal{T}$  a schema (theory), (2)  $Q$  a (FO) query, and
- (3)  $A_1, \dots, A_k$  data: indices with  $BP(A_j) = (\{\}, V_j)$ .

- ① What does it mean for a query to be *defined* by the *data*?
- ② Can we *test* for this?
- ③ Does it mean we have a *rewriting* (a plan)?

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A query  $Q$  is *implicitly definable in  $A_j$ s* if  $Q(M_1) = Q(M_2)$  for all  $M_1 \models \mathcal{T}$  and  $M_2 \models \mathcal{T}$  two databases such that

$$(\text{select } v. V_j (A_j v)) (M_1) = (\text{select } v. V_j (A_j v)) (M_2).$$

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- ② Can we *test* for this? YES:

$\mathcal{T} \cup \mathcal{T}^* \models Q \leftrightarrow Q^*$ , where  $(.)^*$  renames all symbols but  $A_j \& V_j$ .

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... this unfortunately fails in *finite models*

# Craig Interpolation

## Theorem (Craig, 1957)

For  $\varphi \rightarrow \psi$  a valid FO formula there is a FO formula  $\zeta \in \mathcal{L}(\varphi) \cap \mathcal{L}(\psi)$ ,  
an **interpolant**, such that  $\varphi \rightarrow \zeta$  and  $\zeta \rightarrow \psi$  are valid.

How do we use this? Convert

$$\mathcal{T} \cup \mathcal{T}^* \models Q \leftrightarrow Q^* \quad \text{to} \quad \models ((\bigwedge \mathcal{T}) \wedge Q) \rightarrow ((\bigwedge \mathcal{T}^*) \rightarrow Q^*)$$

and then apply Craig's interpolation theorem to find an interpolant  $\zeta$  such that  $(\bigwedge \mathcal{T}) \wedge Q \rightarrow \zeta$  and  $\zeta \rightarrow (\bigwedge \mathcal{T}^*) \rightarrow Q^*$ .

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... note that  $\zeta$  contains only the “required” symbols.

# How to Get Interpolants? Biased Tableaux

Prove  $\varphi \rightarrow \psi$  by refuting  $\{L(\varphi), R(\neg\psi)\}$  using tableaux rules.

## Closed Branches

$$S \cup \{L(\varphi), L(\neg\varphi)\} \xrightarrow{\text{int}} \perp$$

$$S \cup \{L(\varphi), R(\neg\varphi)\} \xrightarrow{\text{int}} \varphi$$

$$S \cup \{R(\varphi), R(\neg\varphi)\} \xrightarrow{\text{int}} \top$$

$$S \cup \{R(\varphi), L(\neg\varphi)\} \xrightarrow{\text{int}} \neg\varphi$$

## Propositional Rules (only the “interesting” ones)

$$S \cup \{L(\varphi_1 \wedge \varphi_2)\} \xrightarrow{\text{int}} \zeta$$

$$S \cup \{L(\varphi_1), L(\varphi_2)\} \xrightarrow{\text{int}} \zeta$$

$$S \cup \{R(\varphi_1 \wedge \varphi_2)\} \xrightarrow{\text{int}} \zeta$$

$$S \cup \{R(\varphi_1), R(\varphi_2)\} \xrightarrow{\text{int}} \zeta$$

$$S \cup \{L(\neg(\varphi_1 \wedge \varphi_2))\} \xrightarrow{\text{int}} \zeta_1 \vee \zeta_2$$

$$S \cup \{L(\neg\varphi_1)\} \xrightarrow{\text{int}} \zeta_1 \quad S \cup \{L(\neg\varphi_2)\} \xrightarrow{\text{int}} \zeta_2$$

$$S \cup \{R(\neg(\varphi_1 \wedge \varphi_2))\} \xrightarrow{\text{int}} \zeta_1 \wedge \zeta_2$$

$$S \cup \{R(\neg\varphi_1)\} \xrightarrow{\text{int}} \zeta_1 \quad S \cup \{R(\neg\varphi_2)\} \xrightarrow{\text{int}} \zeta_2$$

## Quantifiers&Equality Rules ... similar

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## Propositional Rules (only the “interesting” ones)

$$\begin{array}{c} \frac{S \cup \{L(\varphi_1 \wedge \varphi_2)\} \xrightarrow{\text{int}} \zeta}{S \cup \{L(\varphi_1), L(\varphi_2)\} \xrightarrow{\text{int}} \zeta} \quad \frac{S \cup \{L(\neg(\varphi_1 \wedge \varphi_2))\} \xrightarrow{\text{int}} \zeta_1 \vee \zeta_2}{S \cup \{L(\neg\varphi_1)\} \xrightarrow{\text{int}} \zeta_1 \quad S \cup \{L(\neg\varphi_2)\} \xrightarrow{\text{int}} \zeta_2} \\ \frac{S \cup \{R(\varphi_1 \wedge \varphi_2)\} \xrightarrow{\text{int}} \zeta}{S \cup \{R(\varphi_1), R(\varphi_2)\} \xrightarrow{\text{int}} \zeta} \quad \frac{S \cup \{R(\neg(\varphi_1 \wedge \varphi_2))\} \xrightarrow{\text{int}} \zeta_1 \wedge \zeta_2}{S \cup \{R(\neg\varphi_1)\} \xrightarrow{\text{int}} \zeta_1 \quad S \cup \{R(\neg\varphi_2)\} \xrightarrow{\text{int}} \zeta_2} \end{array}$$

## Quantifiers&Equality Rules ... similar

... and then extract the interpolant  $\zeta$  s.t.  $\varphi \rightarrow \zeta \rightarrow \psi$ .

## Example: Tableaux for $\{x \mid \exists y, z. E(x, y, z)\}$

$$\forall x, y, z. I_1(x, y) \wedge I_2(x, z) \rightarrow E(x, y, z)$$

$$\forall x, y, z. E(x, y, z) \rightarrow I_1(x, y)$$

$$\forall x, y, z. E(x, y, z) \rightarrow I_2(x, z)$$

$$LE(c, a, b), R\neg E^*(c, y, z)$$

$$L(\neg E(c, a, b) \vee I_1(c, a))$$

$$L\neg E(c, a, b)$$

$$LI_1(c, a)$$

$\times_{LL}$

$$L(\neg E(c, a, b) \vee I_2(c, b))$$

$$L\neg E(c, a, b)$$

$$LI_2(c, b)$$

$\times_{LL}$

$$R(E^*(c, y, z) \vee \neg I_1(c, y) \vee \neg I_2(c, z))$$

$$RE^*(c, y, z)$$

$$R(\neg I_1(c, y) \vee \neg I_2(c, z))$$

$\times_{RR}$

$$R\neg I_1(c, y)$$

$$R\neg I_2(c, z)$$

$\times_{LR}$

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Plan:  $\exists y, z. I_1(x, y) \wedge I_2(x, z)$

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$$L(\neg E(c, a, b) \vee I_2(c, b))$$

$$\perp \vee (\top \wedge I_1(x, y) \wedge I_2(x, z))$$

$$L\neg E(c, a, b)$$

$$LI_2(c, b)$$

$\times_{LL}$

$$R(E^*(c, y, z) \vee \neg I_1(c, y) \vee \neg I_2(c, z))$$

$$\top \wedge I_1(x, y) \wedge I_2(x, z)$$

$$RE^*(c, y, z)$$

$$R(\neg I_1(c, y) \vee \neg I_2(c, z))$$

$$I_1(x, y) \wedge I_2(x, z)$$

$\times_{RR}$

$$R\neg I_1(c, y)$$

$$R\neg I_2(c, z)$$

$$I_1(x, y) \leftarrow |$$

$$I_2(x, z) \leftarrow |$$

$\times_{LR}$

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$\forall x, y, z. E(x, y, z) \rightarrow I_2(x, z)$

$LE(c, a, b), R\neg E^*(c, y, z)$

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$\times_{LL}$

$L(\neg E(c, a, b) \vee I_2(c, b))$

$L\neg E(c, a, b)$

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$R(E^*(c, y, z) \vee \neg I_1(c, y) \vee \neg I_2(c, z))$

$RE^*(c, y, z)$

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$R\neg I_2(c, z)$

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Plan:  $\exists y, z. I_1(x, y) \wedge I_2(x, z)$

# Example2: CQ Views (via Resolution&Vampire)

1	$Q(sk_0, sk_1)$	[Input]	21	$R(sk_{12}(sk_0, sk_1), sk_{13}(sk_0, sk_1))$	[Res:1,18]
2	$\neg Q'(sk_0, sk_1)$	[Input]	22	$\neg R(sk_{12}(sk_0, sk_1), X_1) \vee \neg R(sk_{13}(sk_0, sk_1), X_2) \vee V_1(X_1, X_2)$	[Res:13,21]
3	$V_2(X_0, X_1) \vee \neg R'(X_0, X_3) \vee \neg R'(X_3, X_1)$	[Input]	23	$\neg R(sk_{13}(sk_0, sk_1), X_1) \vee V_1(sk_0, X_1)$	[Res:20,22]
4	$R(sk_3(X_0, X_1), X_1) \vee \neg V_2(X_0, X_1)$	[Input]	24	$V_1(sk_0, sk_{14}(sk_0, sk_1))$	[Res:19,23]
5	$R(X_0, sk_3(X_0, X_1)) \vee \neg V_2(X_0, X_1)$	[Input]	25	$\neg R'(X_0, X_1) \vee \neg R'(X_0, sk_0) \vee \neg R'(X_2, sk_1) \vee \neg R'(X_1, X_2)$	[Res:2,14]
6	$R'(sk_4(X_0, X_1), sk_5(X_0, X_1)) \vee \neg V_1(X_0, X_1)$	[Input]	26	$\neg R'(sk_7(X_1, X_2), sk_1) \vee \neg R'(X_3, sk_6(X_1, X_2)) \vee \neg V_3(X_1, X_2) \vee \neg R'(X_3, sk_0)$	[Res:9,25]
7	$R'(sk_5(X_0, X_1), X_1) \vee \neg V_1(X_0, X_1)$	[Input]	27	$\neg R'(X_1, sk_6(X_2, sk_1)) \vee \neg V_3(X_2, sk_1) \vee \neg R'(X_1, sk_0)$	[Res:10,26]
8	$R'(sk_4(X_0, X_1), X_0) \vee \neg V_1(X_0, X_1)$	[Input]	28	$\neg V_3(X_1, sk_1) \vee \neg R'(X_1, sk_0)$	[Res:11,27]
9	$R'(sk_6(X_0, X_1), sk_7(X_0, X_1)) \vee \neg V_3(X_0, X_1)$	[Input]	29	$\neg R'(X_1, sk_5(X_2, X_3)) \vee V_2(X_1, X_3) \vee \neg V_1(X_2, X_3)$	[Res:3,7]
10	$R'(sk_7(X_0, X_1), X_1) \vee \neg V_3(X_0, X_1)$	[Input]	30	$V_2(sk_4(X_1, X_2), X_2) \vee \neg V_1(X_1, X_2)$	[Res:6,29]
11	$R'(X_0, sk_6(X_0, X_1)) \vee \neg V_3(X_0, X_1)$	[Input]	31	$R(sk_{14}(sk_0, sk_1), sk_1)$	[Res:1,16]
12	$V_3(X_0, X_1) \vee \neg R(X_0, X_4) \vee \neg R(X_5, X_1) \vee \neg R(X_4, X_5)$	[Input]	32	$\neg R(X_1, sk_3(X_2, X_3)) \vee \neg R(X_3, X_4) \vee \neg V_2(X_2, X_3) \vee V_3(X_1, X_4)$	[Res:4,12]
13	$V_1(X_0, X_1) \vee \neg R(X_4, X_0) \vee \neg R(X_5, X_1) \vee \neg R(X_4, X_5)$	[Input]	33	$\neg R(X_1, X_2) \vee \neg V_2(X_3, X_1) \vee V_3(X_3, X_2)$	[Res:5,32]
14	$Q'(X_0, X_1) \vee \neg R'(X_2, X_3) \vee \neg R'(X_2, X_0) \vee \neg R'(X_4, X_1) \vee \neg R'(X_3, X_4)$	[Input]	34	$\neg V_2(X_1, sk_{14}(sk_0, sk_1)) \vee V_3(X_1, sk_1)$	[Res:31,33]
15	$R(sk_{13}(X_0, X_1), sk_{14}(X_0, X_1)) \vee \neg Q(X_0, X_1)$	[Input]	35	$V_3(sk_4(X_1, sk_{14}(sk_0, sk_1)), sk_1) \vee \neg V_1(X_1, sk_{14}(sk_0, sk_1))$	[Res:30,34]
16	$R(sk_{14}(X_0, X_1), X_1) \vee \neg Q(X_0, X_1)$	[Input]	36	$\neg R'(sk_4(X_1, sk_{14}(sk_0, sk_1)), sk_0) \vee \neg V_1(X_1, sk_{14}(sk_0, sk_1))$	[Res:28,35]
17	$R(sk_{12}(X_0, X_1), X_0) \vee \neg Q(X_0, X_1)$	[Input]	37	$\square$	[Res:8,24,36 (w/forward subsumption)]
18	$R(sk_{12}(X_0, X_1), sk_{13}(X_0, X_1)) \vee \neg Q(X_0, X_1)$	[Res:1,15]			
19	$R(sk_{13}(sk_0, sk_1), sk_{14}(sk_0, sk_1))$	[Res:1,15]			
20	$R(sk_{12}(sk_0, sk_1), sk_0)$	[Res:1,17]			

# Domain Independence

## Question

Given a schema  $\mathcal{T}$ , set of indices  $\mathcal{I}$ , and a (range restricted) query  $Q$  such that a rewriting  $\zeta$  for  $Q$  over  $\mathcal{I}$  exists.

is  $\zeta$  domain independent (DI)?

- Not in general;
- To guarantee DI we define a *restriction* on constraints in  $\mathcal{T}$ :

Definition (Domain Independent Constraint)

A constraint is *domain independent* if its truth depends only on the interpretation of logical parameters (and not on the domain).



# Domain Independence

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Given a schema  $\mathcal{T}$ , set of indices  $\mathcal{I}$ , and a (range restricted) query  $Q$  such that a rewriting  $\zeta$  for  $Q$  over  $\mathcal{I}$  exists.

is  $\zeta$  domain independent (DI)?

- Not in general; consider

$$\mathcal{T} = \{\forall x.P(x) \vee R(x), \neg \exists x.P(x) \wedge R(x)\}$$

Then, for  $\{x \mid P(x)\}$ , the rewriting over  $\{R\}$  is  $\{x \mid \neg R(x)\}$ .

- To guarantee DI we define a *restriction* on constraints in  $\mathcal{T}$ :

Definition (Domain Independent Constraint)

A constraint is *domain independent* if its truth depends only on the interpretation of logical parameters (and not on the domain).

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... all usual database constraints are domain independent

### Theorem

Let  $\mathcal{T}$  be a schema,  $\mathcal{I}$  a set of indices, and  $Q$  a range restricted query for which a rewriting  $\zeta$  exists. Then  $\zeta$  is domain independent

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# Summary

Beth Definability and Interpolation provide  
a starting point for *query optimization* in first-order logic.

## Features:

- ⇒ handles full first order logic (constraints and queries)
- ⇒ builds on decades of research in *theorem proving*

## Drawbacks:

- ⇒ handles full first order logic (constraints and queries)
  - ... no decidability (of plan existence) in general
- ⇒ expensive&incomplete reasoning

## Open Issues:

- ⇒ how to handle “database extras”
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