CS 245 — Fall 2012
Assignment 5

Due December 3, at 23:55,

in the CS 245 drop box assigned to your tutorial section

Attach this page as a cover page on your submission

Surname:
Personal name:
ID #:
Mark: Marker:

The availability of the marked papers will be posted on the class web site.

Question 1 (30pt)
Let t_i be a term of the form $s(\cdots s(0)\cdots)$ that represents a natural number n_i written in unary (e.g., $s(s(s(0)))$ represents the number 3).

- Form a set of clauses Σ such that $\Sigma \vdash \mathsf{TIMES}(t_1, t_2, t_3)$ whenever $n_1 \cdot n_2 = n_3$;
- Show a resolution refutation of $\mathsf{TIMES}(s(s(0), s(0), s(s(0))))$ w.r.t. Σ from above;
- Is the set of terms $\{f(t_1, t_2, t_3) \mid n_1 \cdot n_2 = n_3\}$ recursive?

For the first two parts you may use the clauses that define PLUS given in class.

Question 2 (10pt)
Let P be a ternary predicate symbol. Show that if $\forall x.\forall y.\exists z.P(x, y, z)$ is satisfiable then also $\forall x.\forall y.P(x, y, f(x, y))$ is satisfiable.

Question 3 (20pt)
Let Λ be a non-empty finite alphabet (set of symbols). Show that

(a) the set Λ^* of all finite strings over Λ is *countable*; and

(b) the set of all *countably-infinite sequences* of symbols from Λ is *not* countable.

Question 4 (10pt; 10pt bonus)
Show that satisfiability of first order formulæ is undecidable.

For extra 10pt bonus show that satisfiability of first order formulæ is not recursively enumerable.