**CS 245 — Fall 2012**
**Assignment 4**

**Due November 19, at 23:55,**

**in the CS 245 drop box assigned to your tutorial section**

**Attach this page as a cover page on your submission**

| | |
|---|---|
| Surname: | Circle time/room of your tutorial for return of your paper, or "do not return": |
| Personal name: | `TUT 103:  11:30-12:20F in MC 4042`<br>`TUT 104:  03:30-04:20F in MC 4042`<br>`TUT 105:  04:30-05:20F in MC 4042` |
| ID #: | `TUT 106:  02:30-03:20M in OPT 309`<br>`TUT 101:  03:30-04:20M in MC 4042` |
| Mark: Marker: | `TUT 102:  04:30-05:20M in MC 4042`<br>do not return in tutorial |

**Question 1** (20pt) Without using the soundness and completeness theorems, prove the following statements:

(a) $\models \varphi \to \forall x.\varphi$ for all well-formed first-order formulae $\varphi$ for which $x \notin FV(\varphi)$,

(b) $\not\models \varphi \to \forall x.\varphi$ in general.

**Question 2** (20pt)

(a) Show that if $\models \varphi \to \psi$ then $\models (\forall x.\varphi) \to (\forall x.\psi)$.

(b) Show that $\{\varphi \to \psi\} \not\models (\forall x.\varphi) \to (\forall x.\psi)$.

**Question 3** (25pt) Formally prove $\vdash (\forall x.\varphi) \rightarrow (\exists x.\varphi)$.

**Question 4** (35pt) Let $G = (V, E)$ be a graph, where $V$ is a set of vertices and $E$ is a set of (directed) edges. A set $C \subseteq V$ is a *vertex cover* of $G$ if for every edge $(a, b) \in E$, either $a \in C$ or $b \in C$. The size of a vertex cover is its number of vertices.

Each individual graph gives an interpretation whose the universe (domain) is the set of vertices and $R$ denotes the edge relation. A first-order sentence that uses a single binary predicate symbol $R$ can (be taken to) describe a property of a graph.

(a) Write down a first order sentence that defines the set of all interpretations $I$ in which the graph defined by $R^I$ has a vertex cover of size 2;

(b) Write down a first order sentence that defines the set of all interpretations $I$ in which the graph defined by $R^I$ does not have a finite vertex cover; and

(c) Prove that the set of all interpretations that do have a finite vertex cover is not definable;

all in a first order language that contains binary predicate symbol $R$.