**CS 245 — Fall 2012**
**Assignment 3**

**Due November 07, at 23:55,**

**in the CS 245 drop box assigned to your tutorial section**

**Attach this page as a cover page on your submission**

| Surname: | Circle time/room of your tutorial for return of your paper, or "do not return": |
|---|---|
| Personal name: | TUT 103: 11:30-12:20F in MC 4042<br>TUT 104: 03:30-04:20F in MC 4042<br>TUT 105: 04:30-05:20F in MC 4042<br>TUT 106: 02:30-03:20M in OPT 309<br>TUT 101: 03:30-04:20M in MC 4042<br>TUT 102: 04:30-05:20M in MC 4042<br>do not return in tutorial |
| ID #: | |
| Mark: Marker: | |

**Question 1** (10pt)

(a) Show that the formula $(\Box p \wedge \Box q) \rightarrow \Box(p \wedge q)$ is valid.

(b) Show that the formula $(\Diamond p \wedge \Box q) \rightarrow \Diamond(p \wedge q)$ is valid.

**Question 2** (10pt)

(a) Describe a structure $I = (W, R, V)$ and a world $w \in W$ such that $I, w \models p \wedge \Diamond(\neg p \wedge \Box p)$.

(b) Give an example of a set $\Sigma$ and a formula $\varphi$ such that $\Sigma \models \varphi$, but $\Sigma \not\models \Box\varphi$. Explain.

Note that part (b) explains the need for restrictions on the use of assumptions in the inference rule $\dfrac{\vdash_K \varphi}{\vdash_K \Box\varphi}(Nec)$: applying it on a formula derived from an assumption is not sound.

**Do either Question 3 or Question 4.**

**Question 3** (20pt) Show that

$$\vdash_K (\Box p \wedge \Box q) \to \Box(p \wedge q).$$

You may use the axioms for the connective $\wedge$ (and $\vee$ if you like) as presented in class (or any of Ax1 to Ax9 on pages 109–110 of the text).

**Question 4** (20pt) The restriction on the use of rule Nec causes a problem when applying the Deduction Theorem in modal logic: the proof used in the propositional case no longer works quite as before.

(a) Show that if there is a proof of $\Sigma \vdash_K \varphi$, then there is a proof of $\Sigma \vdash_K \varphi$ in which every use of the rule Nec comes before every use of an assumption in $\Sigma$.

(b) Show that the Deduction Theorem holds for System K. That is, show that

$$\Sigma \vdash_K \varphi \to \psi \text{ iff } \Sigma \cup \{\varphi\} \vdash_K \psi.$$

You may use the result of part (a) whether or not you solved it. When appropriate, you may also use the results used in the propositional Deduction Theorem–but only when appropriate.