| Surname: | Circle time/room of your tutorial for return of your paper, or "do not return": |
|---|---|
| Personal name: | TUT 103:  11:30-12:20F in MC 4042 |
| | TUT 104:  03:30-04:20F in MC 4042 |
| ID #: | TUT 105:  04:30-05:20F in MC 4042 |
| | TUT 106:  02:30-03:20M in OPT 309 |
| | TUT 101:  03:30-04:20M in MC 4042 |
| Mark:          Marker: | TUT 102:  04:30-05:20M in MC 4042 |
| |    do not return in tutorial |

**Question 1** (20pt)
Show that all axioms of the Hilbert proof system $H$, namely axioms given by the following axiom schemes,

$$\alpha \to (\beta \to \alpha),$$
$$(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma)), \text{ and}$$
$$(\neg\alpha \to \neg\beta) \to (\beta \to \alpha)$$

are valid formulæ (tautologies).

**Question 2** (27pt)
Prove, by giving an explicit proof in the Hilbert system, the following:

(a) $\{\neg q\} \vdash_H (p \to q) \to \neg p$

(b) $\vdash_H ((\neg p) \to p) \to p$

(c) $\vdash_H p \to (\neg\neg p)$

**Question 3** (30pt) Prove the *"law of substituting equivalent formula for equivalent formula"*, namely that for every pair of logically equivalent formulæ $\psi_1$, $\psi_2$, an arbitrary formula $\varphi$, and an atomic proposition $p$, we have

$$\models \theta_1(\varphi) \text{ if and only if } \models \theta_2(\varphi),$$

where $\theta_1 = \{p \mapsto \psi_1\}$ and $\theta_2 = \{p \mapsto \psi_2\}$.

**Question 4** (23pt) Show that given formulæ $\psi_1$ and $\psi_2$ such that $\vdash_H \psi_1 \to \psi_2$, an arbitrary formula $\varphi$, and a set of formulæ $\Sigma$ the following holds:

$$\text{if } \Sigma \cup \{\psi_2\} \vdash_H \varphi \text{ then } \Sigma \cup \{\psi_1\} \vdash_H \varphi.$$