CS 245 — Fall 2012

Assignment 1

Due Wed October 3, at 23:55

in the CS 245 drop box assigned to your tutorial section

Attach this page as a cover page on your submission

Surname:	Circle time/room of your tutorial for return of your paper, or "do not return":

Personal name:	`TUT 103: 11:30-12:20F in MC 4042`
	`TUT 104: 03:30-04:20F in MC 4042`
	`TUT 105: 04:30-05:20F in MC 4042`

UW ID #:	`TUT 106: 02:30-03:20M in OPT 309`
	`TUT 101: 03:30-04:20M in MC 4042`
	`TUT 102: 04:30-05:20M in MC 4042`

Mark: Marker:	do not return in tutorial

Question 1 (20pt) For each of the following, determine whether the given formula is valid (a tautology) and/or satisfiable. Justify your answers.

(a) $((p \vee q) \wedge \neg p) \wedge \neg q$

(b) $(p \vee q) \to (p \wedge q)$

(c) $\Big(p \to (q \to r)\Big) \to \Big((p \to q) \to (p \to r)\Big)$

Question 2 (20pt) Consider a binary propositional connective "$\star$" with the truth table

p	q	$p \star q$
0	0	1
0	1	0
1	0	0
1	1	0

Show that the singleton set $\{\star\}$ is an adequate set of propositional connectives.

Question 3 (25pt) A *substitution* is a function mapping propositional symbols to formulæ. An *application of a substitution to a formula* φ is a (simultaneous) replacement of atomic propositions in φ by their images under the above mapping.

For example, consider a substitution

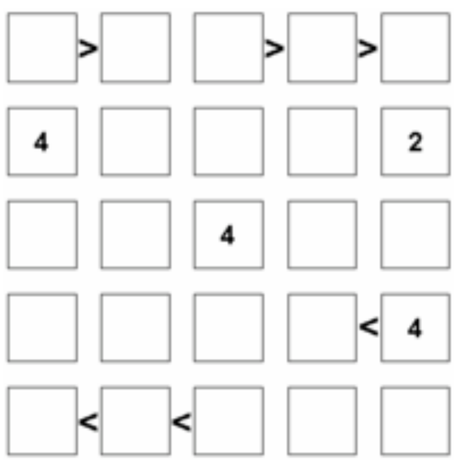
$$\theta = \{p \mapsto (q \wedge r),\ r \mapsto p\}.$$

Applying θ to the formula $\varphi =$ "$p \vee r$" yields the formula $\theta(\varphi) =$ "$(q \wedge r) \vee p$".

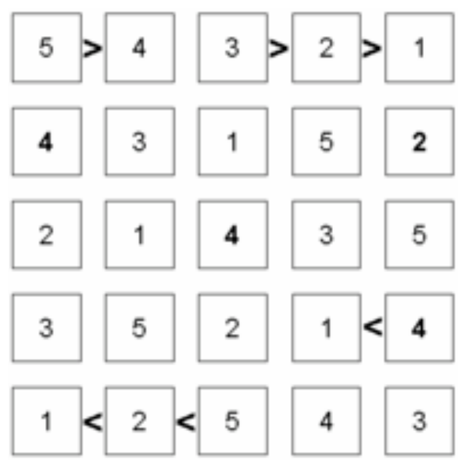
Using the above definition show that the formula $\theta(\varphi)$ is valid given a valid formula φ and an arbitrary substitution θ.

Question 4 (35pt) In this question, you will write propositional formulæ whose models represent the solutions to the *Futoshiki* puzzle.

How to Play: The puzzle is played on a square grid, such as 5 x 5. The objective is to place the numbers 1 to 5 in each row, ensuring that each column also contains each digit 1 to 5 only once. Some digits may be given at the start. In addition, comparison constraints are also initially specified between some of the squares, such that one must be higher or lower than its neighbour. These constraints must be honoured as the grid is filled out.



Initial puzzle



Solved puzzle

(a) Describe a propositional formula that represents that each cell contains a unique number, with each row and each column having all five different numbers. (You need not write out the exact formula in full detail, but describe it clearly and carefully.) Explain your answer.

(b) Describe a propositional formula that represents that each comparison constraint is met. (If you wish, you may assume that all comparisons lie in rows, as in the picture; however, you may find it interesting to consider other cases as well.)