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Reinstating Combinatorial Protections for Manipulation and Bribery in Single-Peaked and Nearly Single-Peaked Electorates

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Abstract

Understanding when and how computational complexity can be used to protect elections against different manipulative actions has been a highly active research area over the past two decades. A recent body of work, however, has shown that many of the NP-hardness shields, previously obtained, vanish when the electorate has single-peaked or nearly single-peaked preferences. In light of these results, we investigate whether it is possible to reimpose NP-hardness shields for such electorates by allowing the voters to specify partial preferences instead of insisting they cast complete ballots. In particular, we show that in single-peaked and nearly single-peaked electorates, if voters are allowed to submit top-truncated ballots, then the complexity of manipulation and bribery for many voting rules increases from being in P to being NP-complete.

1 Introduction

Collective decision making problems abound in human as well as multiagent contexts and they typically proceed by using a mechanism that aggregates the preferences of the participating agents. Voting is one such mechanism, and is, in fact, one of the most widely used ones. For instance, it has been proposed as a mechanism for web spam reduction (Dwork et al. 2001), for collaborative filtering and recommender systems (Pennock, Horvitz, and Giles 2000), and for multiagent planning (Ephrati and Rosenschein 1993). As a result of its importance, voting has been extensively studied, and the theory of social choice has a number of impossibility results surrounding fundamental issues that arise in running elections. Among these, one aspect that has attracted considerable attention is the impact of different manipulative actions (bribery, control, and manipulation) on elections. Although the Gibbard-Satterthwaite theorem states that all reasonable voting rules are manipulable, starting with the seminal work of Bartholdi III, Tovey, and Trick (1989), there has been much work that has looked into how computational complexity can be used as a barrier to protect elections against different manipulative actions (see (Faliszewski, Hemaspaandra, and Hemaspaandra 2010) for a survey).

While there have been a lot of results in computational social choice that has obtained NP-hardness shields for different voting rules using constructions on combinatorially rich structures such as partitions and covers, a recent body of work, which was mainly inspired from the work of Walsh (2007), has shown that such combinatorial protections vanish when the voters have structured preferences. In particular, in single-peaked electorates it was observed by Faliszewski et al. (2011) for control and manipulation and by Brandt et al. (2015) for bribery that many of the previously known NP-hardness results fall into polynomial time. Subsequently, there has also been work done on the notion of nearly single-peaked preferences by Faliszewski, Hemaspaandra, and Hemaspaandra (2014) where similar, although not as stark, observations have been made.

In the context of the above results, this paper aims to take this line of research in a new direction by looking at the impact of partial preferences on manipulative actions in single-peaked and nearly single-peaked electorates. In particular, we consider top-truncated ballots, which are natural in settings where an agent is certain about his most preferred candidates, but is unsure or indifferent among the remaining ones, and we look at their impact on manipulative actions in single-peaked and nearly single-peaked settings. In doing so, we arrive at a number of interesting results, which in turn form the theme of our paper – of reinstating combinatorial protections by allowing top-truncated voting.

Our contributions include the following:

1. We, for the first time, systematically study the impact of partial voting on manipulative actions in structured preference profiles. In particular, we look at the problem of manipulation and bribery in single-peaked and nearly single-peaked settings when top-truncated ballots are allowed.
2. Under the assumption that the voters submit complete ballots, we first provide polynomial-time algorithms for manipulation and weighted-bribery for certain voting rules in single-peaked and nearly single-peaked settings, thus extending the works of Faliszewski et al. (2011) for manipulation, Brandt et al. (2015) for bribery, and Faliszewski, Hemaspaandra, and Hemaspaandra (2014) for nearly single-peaked electorates. We then show how these polynomial-time problems become NP-complete when top-truncated ballots are allowed.
3. We show an example of a natural voting rule where, contrary to intuition, the complexity of manipulation actually

increases when moving from the general case (i.e. when there is no restriction on the preferences) to the single-peaked case. In particular, in Theorem 5 we show how the complexity of manipulating eliminate(veto), when top-truncated ballots are allowed, moves from being in P in the general case to being NP-complete in the single-peaked case.

The overarching theme in this work is that top-truncated voting is useful in reinstating combinatorial protections in single-peaked and nearly single-peaked electorates. We believe that the results form a *win-win* scenario: allowing voters to specify top-truncated ballots (or partial preferences, in general) is extremely useful and often necessary in many multi-agent systems applications and even in real-world elections, and allowing this additional flexibility in turn gives us what we want in terms of making the complexity of many manipulative-action problems hard.

Proofs omitted due to space constraints can be found in the full version of this paper (Menon and Larson 2015b).

Related Work There are two lines of research that are closely related to our work. First is the work on structured preference profiles. This line of research has mainly looked at single-peaked preferences and more recently at nearly single-peaked preferences. The notion of single-peaked preferences was introduced by Black (1948) and subsequently there has been a lot of work in social choice literature on the same. Among these, in particular, we note the work of Cantala (2004) who introduced the concept of "single-peaked with outside options" which is similar to the notion of single-peaked with top-truncated ballots that we study here, and the work of Barberá (2007) who discussed how properties of different variants of single-peaked preferences change for varying amounts of indifference permitted.

In computational social choice, three papers that are most related to our work are (Faliszewski et al. 2011), (Brandt et al. 2015), and (Faliszewski, Hemaspaandra, and Hemaspaandra 2014). The first two papers discuss manipulation and control, and bribery, respectively, and show how most of the NP-hardness shields for these manipulative actions vanish in single-peaked settings. The third paper studies the complexity of manipulative actions in nearly single-peaked electorates and shows how in many cases the hardness results evaporate. Our paper, in contrast, follows the theme of reinstating these combinatorial protections.

Although the above mentioned papers are by far the most related to our work, it is worth noting that this line of research has mainly focused on the problem of single-peaked consistency where, informally, the task is to determine if a given set of preferences is single-peaked or otherwise (see (Bartholdi and Trick 1986; Doignon and Falmagne 1994; Escoffier, Lang, and Öztürk 2008; Erdélyi, Lackner, and Pfandler 2013; Lackner 2014)).

The second line of research that is related to this paper is the work on election problems when partial preferences are allowed. Here, the papers that are most related are those of Narodytska and Walsh (2014), Fitzsimmons and Hemaspaandra (2015), and Menon and Larson (2015a). Narodytska and Walsh (2014) were the first to look at complexity of constructive manipulation under top-truncated voting and they provided an analysis for three particular voting protocols. Subsequently, Fitzsimmons and Hemaspaandra (2015) looked into how the complexity of bribery, control, and manipulation is affected when ties are allowed, and Menon and Larson (2015a) generalized the complexity of constructive and destructive manipulation with top-truncated ballots for broader classes of voting rules and also looked at the impact on complexity when there is uncertainty about the non-manipulators' votes. While all three papers discuss results in the general setting (i.e. when there is no restriction on the structure of the preferences), in contrast to this, in this paper, we look at the complexity of manipulation and bribery with top-truncated ballots when the preferences are restricted to being single-peaked or nearly single-peaked.

Additionally, we also note that there has been work on other election problems when preferences are only partially specified. Notable among them are the works of Konczak and Lang (2005) and Xia and Conitzer (2011) on the possible and necessary winners problem, and the work of Baumeister et al. (2012) which discusses planning various kinds of campaigns in settings where the ballots can be truncated at the top, bottom or both. The extension-bribery problem they introduce in that paper is closely related to the manipulation problem with top-truncated ballots.

2 Preliminaries

Elections, Voting Rules, and Preferences

Elections An election is modeled as a pair $E = (C, V)$, where C is the set of candidates and V is the set of voter preferences. For every voter v_i, $\succ_i$ denotes their preference order over C. $\succ_i$ is said to be a complete order (or a complete vote) if it is antisymmetric, transitive, and a total ordering on C. Here, we also consider incomplete orders in the form of top-truncated ballots. $\succ_i$ is said to be a top-truncated order (or simply, a top order) if it is a complete order over any non-empty $C' \subseteq C$ and if all candidates in $C \setminus C'$ are assumed to be tied and are ranked below the candidates in C'. For simplicity, we sometimes use $(c_1, \cdots, c_m)$ to denote a preference order $c_1 \succ \cdots \succ c_m$. Since this paper looks at weighted elections, additionally, every voter v_i has a weight w_i associated with them.

Voting Rules An election system or a voting protocol takes the set of votes V as input and it outputs a collection $W \subseteq C$ called the winner set. A candidate is said to be a Condorcet winner if it is preferred over every other candidate by a strict majority of the voters, while it is said be a weak-Condorcet winner if it is preferred over every other candidate by at least half of the voters. In this paper, we consider the following voting rules. We first present their original definitions which is on complete orders and then discuss how top orders can be handled.

1. **Positional scoring rules:** A positional scoring rule is defined by a scoring vector $\alpha = \langle \alpha_1, \cdots, \alpha_m \rangle$, where $\alpha_1 \geq \cdots \geq \alpha_m$. For each voter v, a candidate receives α_i points if it is ranked in the ith position by v. Some examples of scoring rules are the *plurality rule* with $\alpha = \langle 1, 0, \cdots, 0 \rangle$,

the *Borda rule* with $\alpha = \langle m-1, m-2, \cdots, 0\rangle$, and the *veto rule* with $\alpha = \langle 1, \cdots, 1, 0\rangle$.

2. **Scoring elimination rules:** Let X be any scoring rule. Given a complete ordering, eliminate(X) is the rule that successively eliminates the candidate with the lowest score according to X. Once a candidate is eliminated, the rule is then repeated with the reduced set of candidates until there is a single candidate left. In this paper, we mainly consider two scoring elimination rules: eliminate(Borda) – which is also known as Baldwin's rule or Fishburn's version of Nanson's rule (Niou 1987) – and eliminate(veto).
3. **Copeland$^\alpha$:** Let $\alpha \in \mathbb{Q}$, $0 \leq \alpha \leq 1$. In Copeland$^\alpha$, introduced by Faliszewski, Hemaspaandra, and Schnoor (2008), for each pair of candidates, the candidate preferred by the majority receives one point and the other one receives a 0. In case of a tie, both receive α points.

A voting rule which, on every input that has a weak-Condorcet winner, outputs the set of all weak-Condorcet winners as the set of winners is said to be weak-Condorcet consistent.

To deal with top-truncated orders in positional scoring rules where a voter ranks only k out of the m candidates ($k < m$), we use the following three schemes that were used by Narodytska and Walsh (2014) in their preliminary work on manipulation with top orders. Emerson (2013) also used the same schemes for the Borda rule.

1. **Round-up:** A candidate ranked in the ith position ($i \leq k$) receives a score of α_i, while all the unranked candidates receive a score of α_m. For any positional scoring rule X, we denote this by $X_\uparrow$.
2. **Round-down:** A candidate ranked in the ith position ($i \leq k$) receives a score of $\alpha_{m-(k-i)-1}$, while all the unranked candidates receive a score of α_m. For any positional scoring rule X, we denote this by $X_\downarrow$.
3. **Average score:** A candidate ranked in the ith position ($i \leq k$) receives a score of α_i, while all the unranked candidates receive a score of $\frac{\sum_{k<j\leq m} \alpha_j}{m-k}$. For any positional scoring rule X, we denote this by X_{av}.

In scoring elimination rules, we deal with top-truncated votes by using the round-up scheme described above. Here, we consider a vote to be valid only until at least one of the candidates listed in it is remaining in the election. In other words, we simply ignore a vote once all the candidates listed in it are eliminated. In the case of Copeland$^\alpha$, top orders are dealt by just keeping to the definition which assumes that all the unranked candidates are tied and are ranked below the ranked candidates.

Single-Peaked Preferences The notion of single-peaked preferences, first introduced by Black (1948), captures settings where the preferences of a voter are based on a one-dimensional axis. The basic idea here is that every voter has a peak (their most preferred alternative) and that their utility for an alternative decreases the further it is away from this peak. Below, we provide the formal definition of single-peaked preferences on complete orders. We use the definition of single-peakedness found in (Faliszewski et al. 2011).

Definition 1. A collection of votes, V, is said to be single-peaked if there exists a linear order L over C such that for every triple of candidates a, b, and c it holds that:

$$(aLbLc \vee cLbLa) \implies (\forall v \in V)\,[a \succ_v b \implies b \succ_v c]\,.$$

When voters are allowed to present top-truncated ballots, this notion of single-peakedness essentially captures those scenarios where they have a continuous range over L over which their preferences are single-peaked and outside of which they are indifferent among the alternatives. In social choice theory, this notion of single-peaked preferences has been captured as single-peaked with outside options in the context of choosing a level of public good by Cantala (2004).

Throughout this paper, following the model proposed by Walsh (2007), we assume that the societal order L is given as part of the input.

Example 1. Let $C = \{c_1, c_2, c_3, c_4\}$ and $c_2Lc_4Lc_3Lc_1$ be a linear order over C. Then, the preference orders $c_4 \succ c_3 \succ c_2 \succ c_1$ and $c_4 \succ c_2 \succ c_3 \succ c_1$ are both valid complete single-peaked orders, while the preference order $c_4 \succ c_1 \succ c_2 \succ c_3$ is not a valid single-peaked order. Also, with respect to the given linear order, $c_4 \succ c_3$ and $c_4 \succ c_2$ are both valid top-truncated single-peaked orders, while $c_4 \succ c_1$ is not.

Nearly Single-Peaked Preferences Although single-peaked preferences are an interesting domain to study, it is often the case that real-world electorates are not truly single-peaked, but are only very close to being single-peaked. The notion of "near" single-peakedness was first raised by Conitzer (2009) and Escoffier, Lang, and Öztürk (2008), and was subsequently systematically studied by Faliszewski, Hemaspaandra, and Hemaspaandra (2014) and Erdélyi, Lackner, and Pfandler (2013). In this paper, we look at only one notion of "nearness", namely the maverick notion which is defined below.

Definition 2 (k-maverick SP Electorate)**.** A collection of votes V is called a k-maverick SP electorate if all but at most k of the voters are single-peaked consistent with the societal order L.

Manipulative Actions

In this paper, we consider two manipulative action problems: manipulation and bribery. In particular, we study the Constructive Coalitional Weighted Manipulation (CWCM) problem and the Weighted-bribery problem. CWCM was first studied by Conitzer, Sandholm, and Lang (2007). and is described below.

Definition 3 (CWCM)**.** Given a set of candidates C, a set of weighted votes, S (preferences of the non-manipulators), the weights for a set of votes, T (manipulators' votes), and a preferred candidate, p, we are asked if there exists a way to cast the votes in T so that p is a winner in the election $E = (C, S \cup T)$.

The complexity-theoretic study of the bribery problem was first introduced by Faliszewski, Hemaspaandra, and Hemaspaandra (2009). Here we mainly look at the weighted version of the bribery problem which is described below.

Definition 4 (Weighted-bribery)**.** Given a set of candidates, C, the set of weighted votes, V, a preferred candidate, p, and a limit, $k \in \mathbb{N}$, we are asked if there exists a way to change the votes of at most k of the voters in V so that it results in p being a winner.

Throughout this paper, unless otherwise specified, we use the non-unique winner model (where the objective is to make the preferred candidate a winner) as our standard model.

3 Manipulation

In this section we study CWCM with top-truncated votes in both single-peaked and nearly single-peaked electorates. Since the theme of this paper is the reinstatement of combinatorial protections by top-truncated voting, for all the voting rules considered in this section, we present both the "easiness" result (if not already known from previous work) as well as the subsequent "hardness" result that arises as a consequence of allowing top-truncated ballots.

Single-Peaked Electorates

Walsh (2007) was the first to consider manipulation with single-peaked preferences and he showed that STV remains NP-hard to manipulate for 3 candidates. Subsequently, Faliszewski et al. (2011) showed that for many voting protocols which are usually hard to manipulate, restricting the preferences to being single-peaked makes them easy. In particular, they showed that any 3-candidate scoring rule with $(\alpha_1 - \alpha_3) \leq 2(\alpha_2 - \alpha_3)$ is easy to manipulate. This result was then extended to obtain a complete characterization for any m-candidate scoring rule in (Brandt et al. 2015). Here, we look at 3-candidate scoring rules again and we study the impact on complexity of manipulation when top-truncated voting is allowed. The following results for the case of 3-candidate Borda rule was also shown by Fitzsimmons and Hemaspaandra (2015).

Theorem 1. For any 3-candidate scoring rule X that is not isomorphic to plurality or veto, in single-peaked electorates, CWCM with top-truncated votes in $X_\downarrow$ is NP-complete.

Theorem 2. For any 3-candidate scoring rule X that is not isomorphic to plurality, in single-peaked electorates, CWCM with top-truncated votes in X_{av} is NP-complete.

From Theorem 1 and Theorem 2 we can see that a relaxation of the complete votes assumption by additionally allowing top-truncated votes actually increases the complexity of CWCM for all 3-candidate scoring rules with $(\alpha_1 - \alpha_3) \leq 2(\alpha_2 - \alpha_3)$ from being in P (Faliszewski et al. 2011) to being NP-complete when either the round-down or average score evaluation schemes are used. However, with the round-up evaluation scheme manipulation become easy for all m-candidate scoring rules as shown below.

Theorem 3. In single-peaked electorates, computing if a coalition of manipulators can manipulate plurality$_\downarrow$, veto$_\downarrow$, plurality$_{av}$, and $X_\uparrow$, for any scoring rule X, with weighted top-truncated votes takes polynomial time (for any number of candidates).

Another interesting point to note here is that Theorem 1, Theorem 2, and Theorem 3 together also imply that the restriction of preferences to being single-peaked has no effect on the complexity of manipulation with top-truncated ballots, since the same results were obtained by Menon and Larson (2015a) in the general case as well.

Next, we look at CWCM in Copeland$^\alpha$ and we present both the "easiness" and the "hardness" result.

Theorem 4. In single-peaked electorates, for 3-candidate Copeland$^\alpha$, $\alpha \in \mathbb{Q}$, $0 \leq \alpha < 1$,

1. CWCM with **complete votes** is in P.
2. CWCM with **top-truncated votes** is NP-complete.

We note that for $\alpha = 1$ both CWCM with complete votes and top-truncated votes can be shown to be in P.

Our final result for manipulation under single-peaked preferences is the very interesting case of eliminate(veto).

Theorem 5. In single-peaked electorates, in the unique winner model, for eliminate(veto),

1. CWCM with **complete votes** is in P when the number of candidates is bounded.
2. CWCM with **top-truncated votes** is NP-complete for even three candidates.

Theorem 5 is most interesting not because of the fact that it follows the theme of our paper, but for the following other reasons. First is the very unusual behavior that it is showing here. Eliminate(veto), when there are only a bounded number of candidates and in the unique winner model, is known to be in P for practically everything – from CWCM with complete votes in the general case (Coleman and Teague 2007), to CWCM with top-truncated votes in the general case (Menon and Larson 2015a), and to even when there is only partial information (in the form of top-truncated votes) on the non-manipulators' votes (Menon and Larson 2015a). However, here, with single-peaked preferences and with top-truncated votes, it is NP-complete even when there are only three candidates. Second, what makes Theorem 5 even more interesting is the fact that this actually serves as a counterexample (with the only caveat that, to be fair, they had considered only complete votes in that paper) to a conjecture stated by Faliszewski et al. (2011) where they say that they do not expect the complexity of manipulation for "any existing, natural voting system" to increase when moving from the general case (where there is no restriction on the preferences) to the single-peaked case. But this is exactly what we are seeing here.

Nearly Single-Peaked Preferences

For nearly single-peaked electorates, Faliszewski, Hemaspaandra, and Hemaspaandra (2014) were the first to look at the complexity of manipulation, bribery, and control. In that paper, they introduced several notions of nearness and among them was the k-maverick-SP-society where all but at most k of the voters are consistent with the societal order L. As noted before, we only consider this notion of "nearness" in this paper. We start off by looking at 3-candidate scoring rules and we show the impact of top-truncated voting on

CWCM. Note that Faliszewski et al. showed that for all 3-candidate scoring rules that are not isomorphic to plurality CWCM for 1-maverick-SP-societies was NP-complete (Faliszewski, Hemaspaandra, and Hemaspaandra 2014).

Theorem 6. In 1-maverick-SP societies, for any 3-candidate scoring rule X that is not isomorphic to plurality or veto, CWCM with top-truncated votes in $X_{\downarrow}$ is NP-complete.

Theorem 7. In 1-maverick-SP societies, for any 3-candidate scoring rule X that is not isomorphic to plurality, CWCM with top-truncated votes in X_{av} is NP-complete.

Next, we look at eliminate(veto) and we show how top-truncated voting increases the complexity of manipulation for eliminate(veto) in 1-maverick-SP electorates and that it continues to portray the unusual behavior noted earlier.

Theorem 8. In 1-maverick-SP electorates, in the unique winner model, for eliminate(veto),

1. CWCM with **complete votes** is in P when the number of candidates is bounded.
2. CWCM with **top-truncated votes** is NP-complete for even three candidates.

4 Bribery

Faliszewski, Hemaspaandra, and Hemaspaandra (2009) were the first to look at the complexity of bribery in elections. Subsequently, the problem was studied by Brandt et al. (2015) in single-peaked settings and there they showed that many of the combinatorial protections for bribery vanish when the preferences are restricted to being single-peaked. Finally, Faliszewski, Hemaspaandra, and Hemaspaandra (2014) also studied the problem when the preferences are nearly single-peaked. Here, we revisit the problem of bribery in single-peaked and nearly-single peaked settings and we try and see if bribery too, like manipulation, fits into our theme of reinstating combinatorial protections in single-peaked and nearly single-peaked elections through top-truncated voting.

Weighted-Bribery in Scoring Rules

Here we first derive the results for 3-candidate scoring rules in single-peaked settings when only complete votes are allowed. Subsequently, we do the same when top-truncated ballots are allowed. The NP-completeness proofs use an idea that is similar to the one used by Faliszewski, Hemaspaandra, and Hemaspaandra (2009) in Theorem 4.9, where they use a reduction from a modified version of the weighted manipulation problem to show that α-weighted-bribery is NP-complete when it isn't the case that $\alpha_2 = \alpha_3 = \cdots = \alpha_m$.

Let us first define the modified version of manipulation that we will use to reduce to the problem of weighted-bribery. The modified problem defined here is similar to the one used by Faliszewski, Hemaspaandra, and Hemaspaandra (2009), with the only difference that in their problem all the manipulators need to have weights at least twice as much as the weight of the heaviest non-manipulator, while in our case we require that all the manipulators need to have weights at least thrice as much as the weight of the heaviest non-manipulator.

Definition 5 (CWCM'). CWCM' is the same problem as CWCM with the restriction that each manipulative voter has a weight at least thrice as much as the weight of the heaviest non-manipulator.

Next, we show that for all 3-candidate scoring rules with $(\alpha_1 - \alpha_3) > 2(\alpha_2 - \alpha_3)$, CWCM' is NP-complete. The proof here makes use of the corresponding result for CWCM given by Faliszewski et al. (2011) in Theorem 4.4.

Theorem 9. In single-peaked electorates, CWCM' with complete votes is NP-complete for 3-candidate scoring rules when $(\alpha_1 - \alpha_3) > 2(\alpha_2 - \alpha_3)$.

We now show the result for weighted-bribery in scoring rules.

Theorem 10. In single-peaked settings, weighted-bribery with complete votes is in P for 3-candidate scoring rules when $(\alpha_1 - \alpha_3) \leq 2(\alpha_2 - \alpha_3)$ and is NP-complete otherwise.

For the case of top-truncated ballots, we proved in Theorem 1 that, in single-peaked settings, CWCM with top-truncated votes is NP-complete for all 3-candidate scoring rules except plurality and veto when the evaluation scheme was round-down. Similarly, we also showed in Theorem 2 that CWCM with top-truncated votes is NP-complete for all 3-candidate scoring rules except plurality when the evaluation scheme was average-score. Based on these two theorems it is easy to see that we can make similar 'splitting' arguments as in Theorem 9 to prove these results hold true even for CWCM'. As a result, we can state the following results which can be proved by using a reduction from CWCM' similar to the one shown in case 2 of Theorem 10.

Theorem 11. For any 3-candidate scoring rule X that is not isomorphic to plurality or veto, in single-peaked electorates, weighted-bribery with top-truncated votes in $X_{\downarrow}$ is NP-complete.

Theorem 12. For any 3-candidate scoring rule X that is not isomorphic to plurality, in single-peaked electorates, weighted-bribery with top-truncated votes in X_{av} is NP-complete.

Note that we can prove the corresponding results for nearly single-peaked electorates as well.

Weighted-Bribery in Eliminate(veto)

Here we look at the problem of weighted-bribery in eliminate(veto). First, we study the problem in single-peaked electorates and following that we look at the nearly single-peaked case. In both cases, yet again, we observe that allowing top-truncated voting increases the complexity of weighted-bribery from being in P to being NP-complete.

Theorem 13. In single-peaked electorates, in the unique winner model, for 3-candidate eliminate(veto),

1. weighted-bribery with **complete votes** is in P.
2. weighted-bribery with **top-truncated votes** is NP-complete.

Next we look at the complexity of bribery for eliminate(veto) in 1-maverick single-peaked electorates.

Theorem 14. In 1-maverick single-peaked electorates, in the unique winner model, for 3-candidate eliminate(veto),

1. weighted-bribery with **complete votes** is in P.
2. weighted-bribery with **top-truncated votes** is NP-complete.

Is Weighted-Bribery for Weak-Condorcet Consistent Rules Always Easy?

Brandt et al. (2015) showed that in single-peaked electorates weighted-bribery is in P for all weak-Condorcet consistent voting rules (see (Brandt et al. 2015, Theorem 4.4) for a more general result). The P results in their theorem and the reason why it was possible to consider all weak-Condorcet consistent voting rules together was because of the well-known property of single-peaked electorates where it is guaranteed that there is always at least one weak-Condorcet winner (the top choices of the "median" voters are always weak-Condorcet winners). However, this property no longer holds when top-truncated votes are allowed. As has also been pointed out by Cantala (2004), it is not even necessary that a weak-Condorcet winner exists in such settings. We illustrate this with the following example.

Example 2. Let $C = \{a, b, c, d, e\}$ and $aLbLcLdLe$ the linear ordering. Let there be 5 voters with votes $a \succ_{v_1} b \succ_{v_1} c$, $b \succ_{v_2} c$, $c \succ_{v_3} d$, $d \succ_{v_4} e$, and $e \succ_{v_5} d$, respectively. Now, it is easy to see that in the pairwise majority relation, a and b lose to d, c loses to b, and both d and e lose to c. Since everyone loses at least once, there is no weak-Condorcet winner.

Because of the above we can no longer consider all weak-Condorcet consistent rules together like in (Brandt et al. 2015) and exploit the connection between the weak-Condorcet winner(s) and "median" voters to come up with polynomial time algorithms for weighted-bribery. In fact, next we show that for 3-candidate Baldwin's rule (also known as Fishburn's version of Nanson's rule (Niou 1987)), which is a weak-Condorcet consistent rule in single-peaked electorates (Brandt et al. 2015), weighted-bribery is NP-complete when top-truncated ballots are allowed. To show this we use an idea similar to the one used in Theorem 13.

Theorem 15. In single-peaked electorates, weighted-bribery with **top-truncated votes** is NP-complete for 3-candidate Baldwin's rule.

5 Is Allowing Top-truncated Voting in Single-Peaked Electorates Always Beneficial?

Although we have seen instances like in 3-candidate scoring rules with round-up evaluation scheme where the complexity of manipulation decreases as a result of moving from a purely single-peaked setting to a setting where top-truncated votes are allowed, we haven't really seen examples of any other voting rule which shows this behavior. Moreover, we also know that with a different evaluation scheme like round-down or average-score this behavior is no longer seen for even 3-candidate scoring rules. Therefore, a natural question one could ask is: "What role does the evaluation scheme play? Is it possible that given a voting rule one can always construct an evaluation scheme so that it will be beneficial to allow top-truncated voting in single-peaked electorates?". Alternatively, one could also ask: "Is there a voting system for which it is always easy to manipulate when top orders are allowed?". We answer the former question in the negative and the latter one in the affirmative. We show that, as long as all the unranked candidates are assumed to be tied and are assumed to be ranked below the ranked candidates (which is the natural definition of a top-truncated vote), there is at least one voting system for which, irrespective of how the top-truncated votes are dealt with, it is NP-hard to manipulate in purely single-peaked settings, but is easy to manipulate when top-truncated votes are allowed.

Theorem 16. There exists a voting system for which, in single-peaked settings,

1. CWCM with **complete votes** is NP-complete.
2. CWCM with **top-truncated votes** is in P.

6 Conclusion and Future Work

The central theme of this paper was the reinstatement of combinatorial protections in single-peaked and nearly-single peaked electorates by allowing top-truncated voting. We observed this behavior first in the case of manipulation and showed how for different voting protocols manipulation with complete votes was in P whereas manipulation with top-truncated votes jumped to being NP-complete. These results were followed by the results for bribery where, again, similar behavior was observed. In studying the above two, we note that, to the best of our knowledge, we are the first to systematically look at the impact on complexity of manipulative actions when the electorate is single-peaked or nearly single-peaked and when top-truncated preferences are allowed. In addition to the above results, we also showed an instance of a natural voting system (eliminate(veto)) where, contrary to intuition, the complexity of manipulation, when top-truncated ballots are allowed, actually increases from being in P in the general case to being NP-complete in the single-peaked case. Finally, we concluded our discussion by proving the existence of a voting system where allowing top-truncated voting isn't beneficial in the sense that it actually results in a decrease in the complexity of manipulation.

There are many possible avenues for future work. Foremost would be look at some other interesting voting rules and also consider other types of partial preferences (like bottom orders, weak orders etc.) to try and see if similar behavior is observed in them. Second, in this paper we have considered only manipulation and bribery, but not control. Therefore, we feel that it would be worthwhile to see if similar observations can be made for the problem of control as well. Third, while considering nearly single-peaked preferences in this paper, we have essentially talked about only one notion of nearness, namely, the k-maverick notion. However, there are several other notions of nearness (see (Erdélyi, Lackner, and Pfandler 2013)) and seeing if we can obtain similar results for them as well would be interesting. Finally, we have considered only weighted elections in this work, but we believe that looking at the unweighted case would be very interesting and it is definitely something we consider as a future research direction.

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